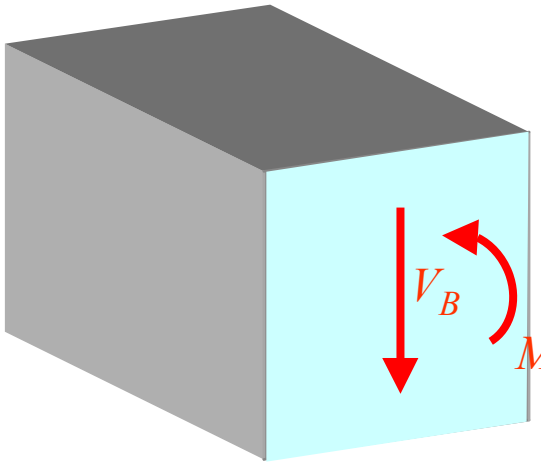
TRANVERSE SHEAR

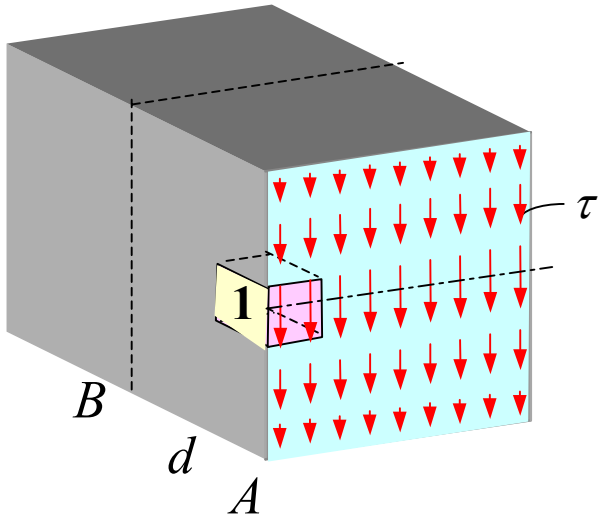
- Shear in Straight Members
- The Shear Formula
- Shear Stresses in Beams
- Shear Flow in Built-up Members

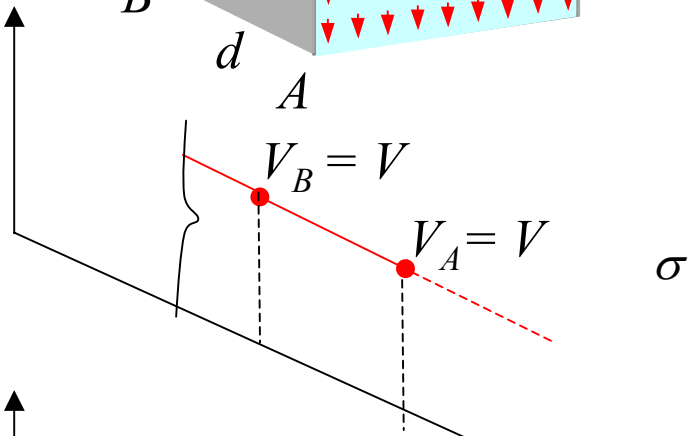
Shear in Straight Members

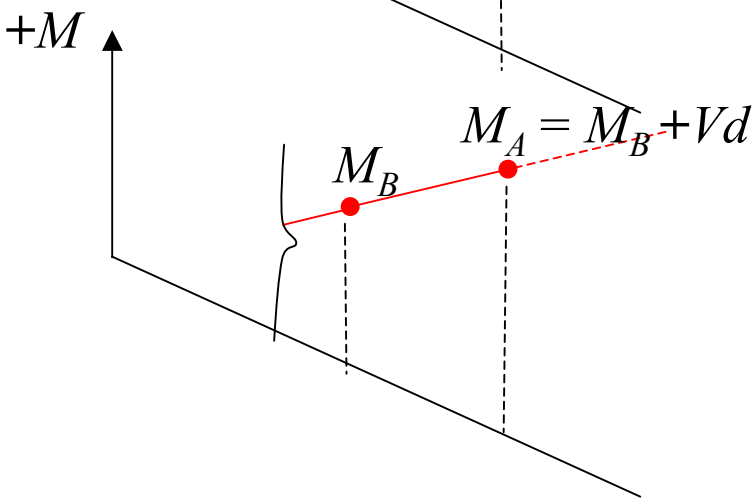


$$V = \int \tau \, dA = (\tau_{ave})A$$

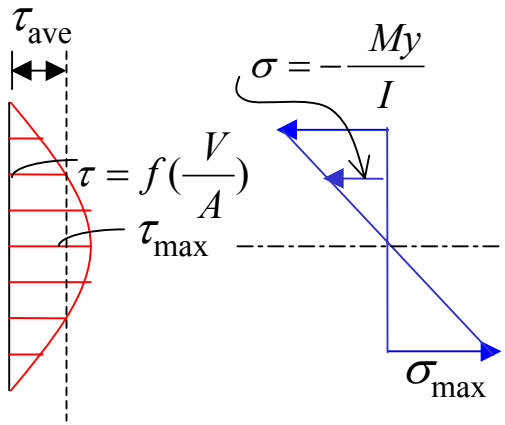
$$\tau_{ave} = \int \tau \, dA = \frac{V}{A}$$

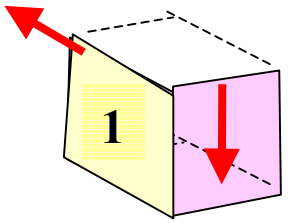




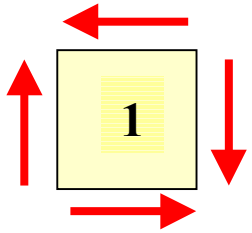


σ





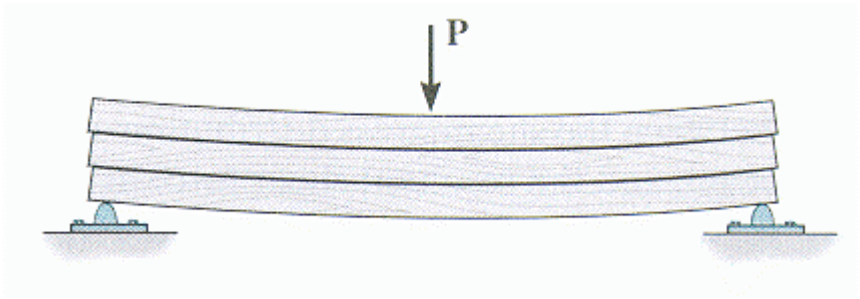
Shear Stress 3D



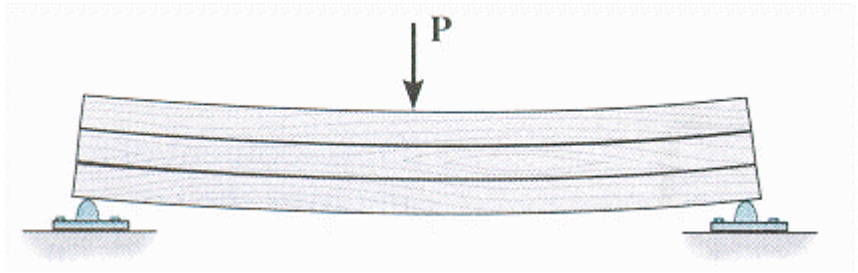
Shear Stress 2D

The Shear Stress Formula: Assumptions

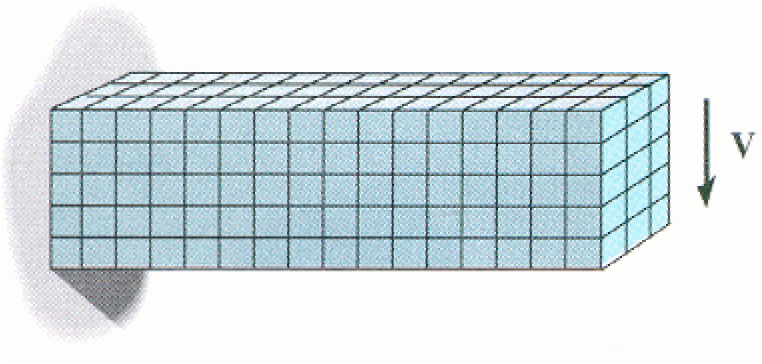
- Linear elastic
- Plane section remains plane



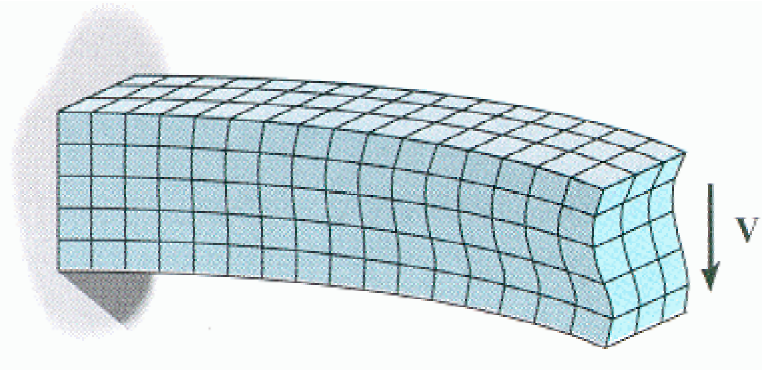
Boards not bonded together



Boards bonded together

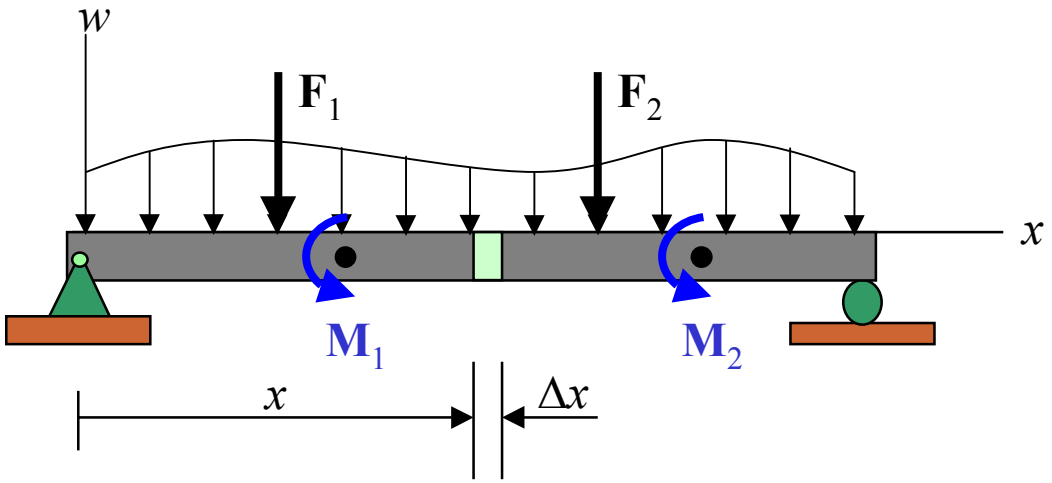


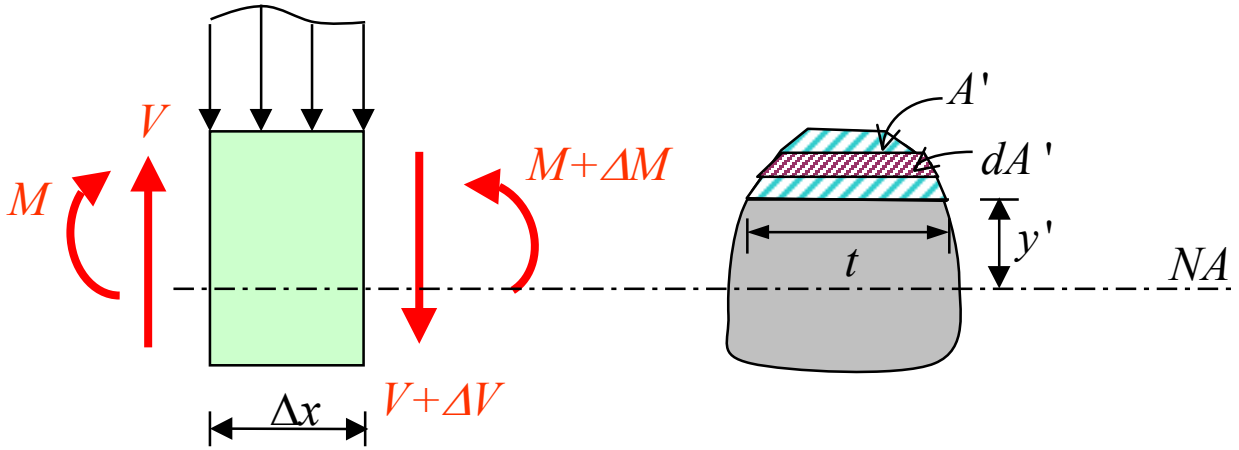
Before deformation

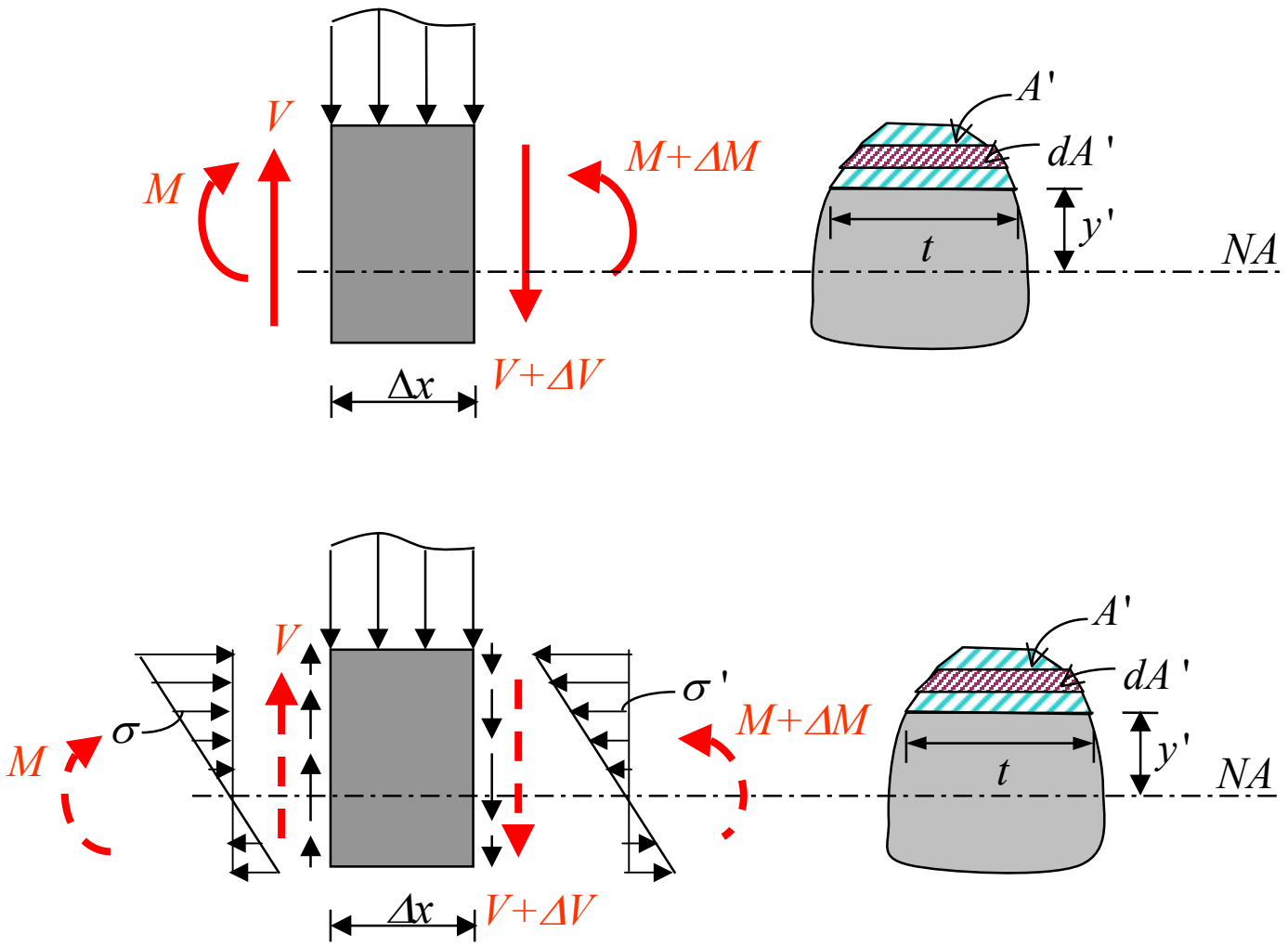


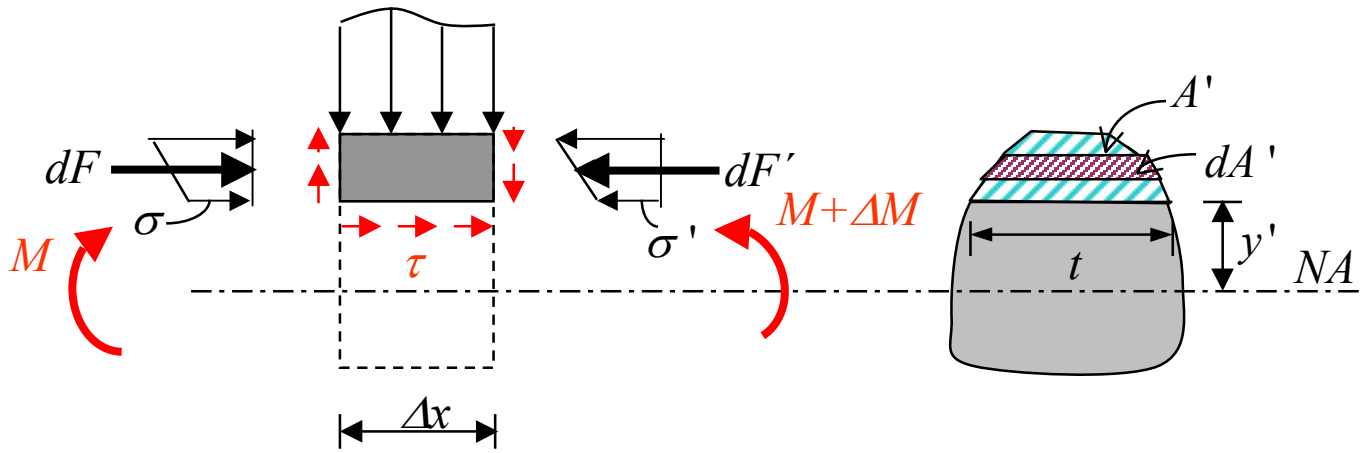
After deformation

The Shear Stress Formula






V
M
M+ΔM
A'
dA'
y'
t
NA
Δx
V+ΔV
σ
σ'



$$\overset{+}{\rightarrow} \Sigma F_x = 0:$$

$$\int_{A'} \sigma dA - \int_{A'} \sigma' dA + \tau (t\Delta x) = 0$$

$$\int_{A'} \frac{My}{I} dA - \int_{A'} \frac{(M + \Delta M) y}{I} dA + \tau (t\Delta x) = 0$$

$$\int_{A'} \frac{My}{I} dA - \int_{A'} \frac{My}{I} dA - \int_{A'} \frac{(\Delta M) y}{I} dA + \tau (t\Delta x) = 0 \quad -----(1)$$

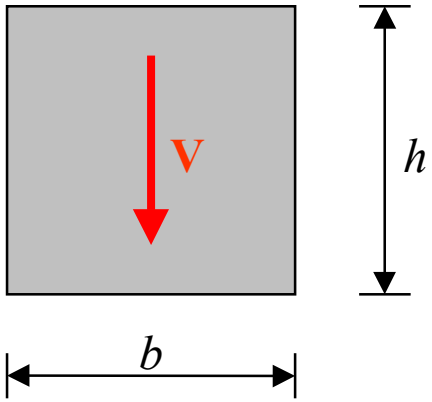
$$\tau = \frac{1}{t(\Delta x)} \int_{A'} \frac{(\Delta M) y}{I} dA = \frac{(\Delta M)}{It(\Delta x)} \int_{A'} y dA$$

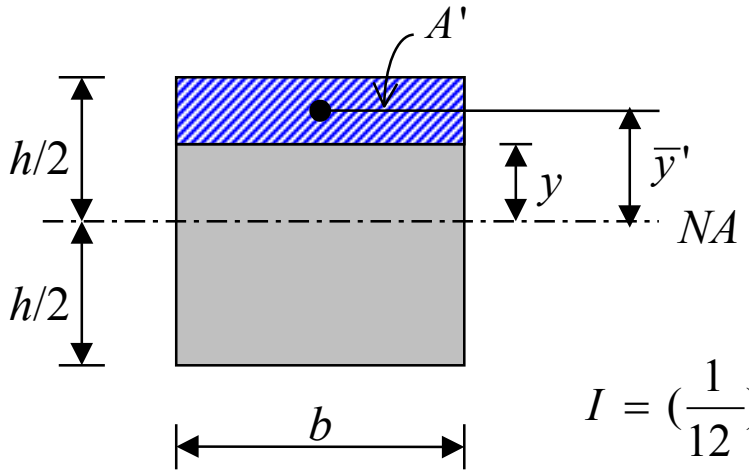
$$\Delta x \rightarrow 0; \qquad = \frac{(dM)}{It(dx)} \int_{A'} y dA \qquad V \qquad \bar{y}' A' = Q$$

$$\rightarrow \quad \boxed{\tau = \frac{VQ}{It}}$$

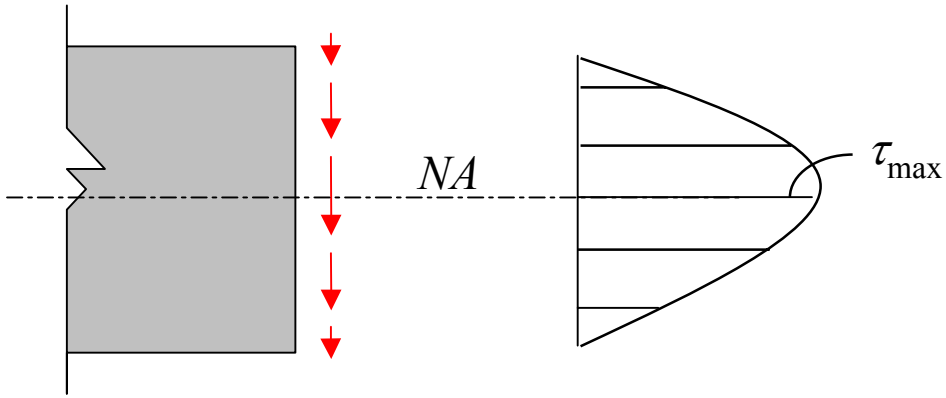
Shear Stress in Beam

- **Rectangular Cross Section**





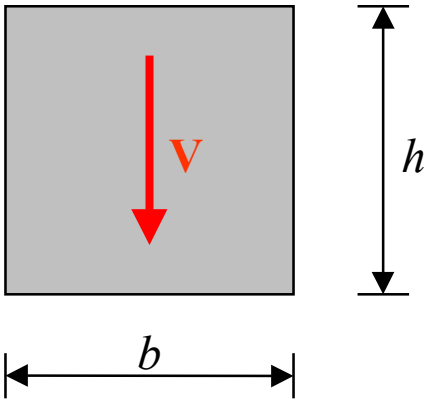
$$I = (\frac{1}{12})bh^3$$

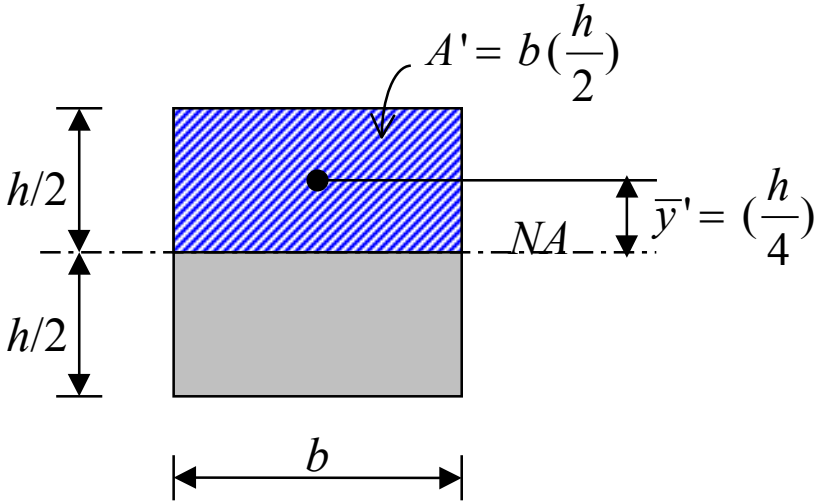


$$\tau = \frac{VQ}{It} = \frac{V(\bar{y}'A')}{It}$$

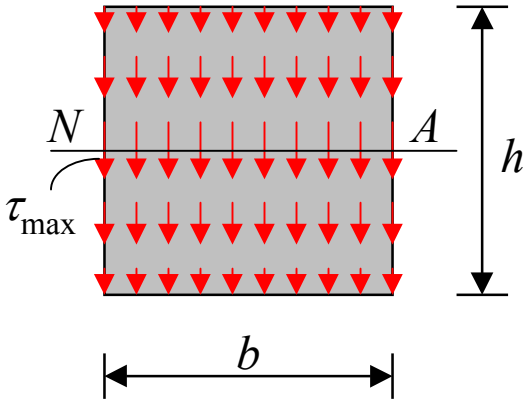
$$= \frac{V(\bar{y}'A')}{(\frac{1}{12})bh^3 b}$$

$\tau = \tau_{max}$ at the neutral axis, $y = 0$:





$$I = (\frac{1}{12})bh^3$$



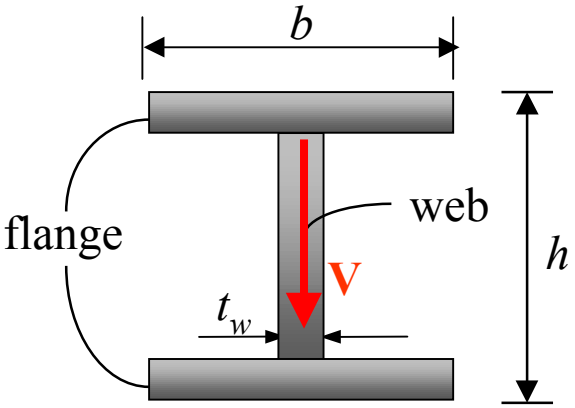
Shear-stress distribution

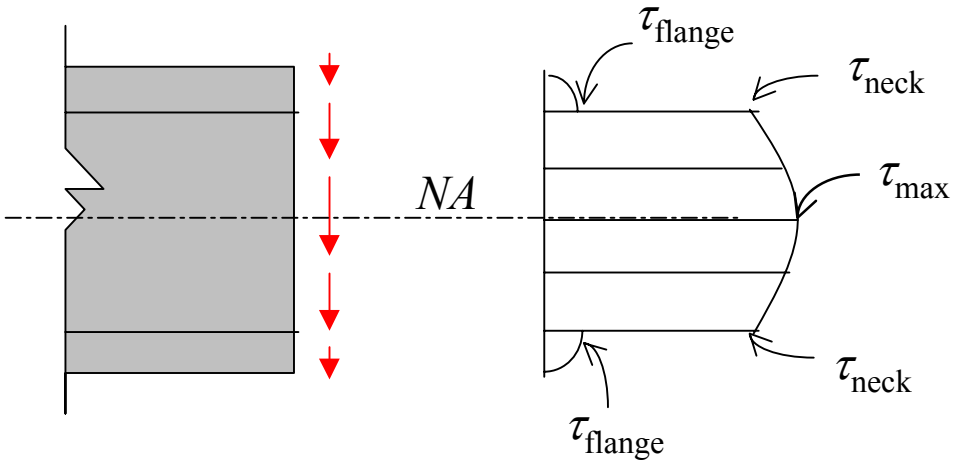
$$\tau_{max} = \frac{VQ_{maz}}{It} = \frac{V(\bar{y}'A')}{It}$$

$$= \frac{V(\frac{h}{4})(b\frac{h}{2})}{(\frac{1}{12})bh^3 b} = (\frac{12}{8})\frac{V}{bh} \quad (bh = A)$$

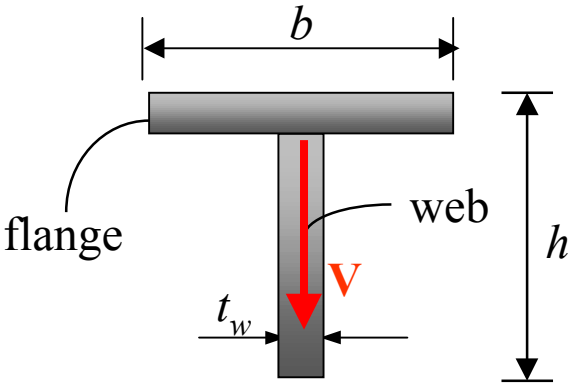
$$\tau_{max} = 1.5\frac{V}{A}$$

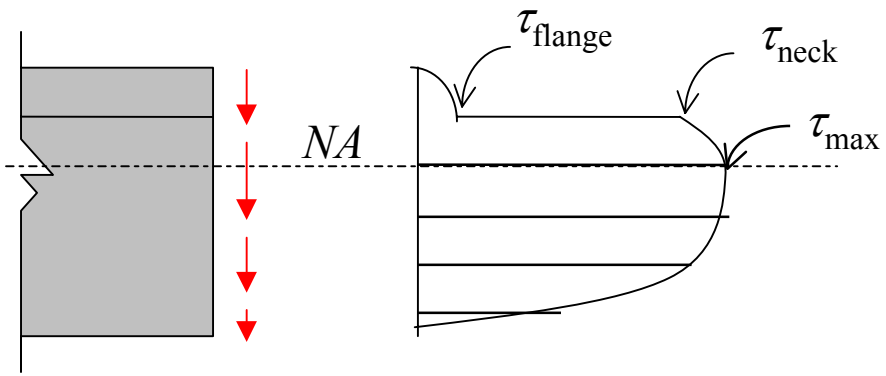
• Wide-Flange Beam





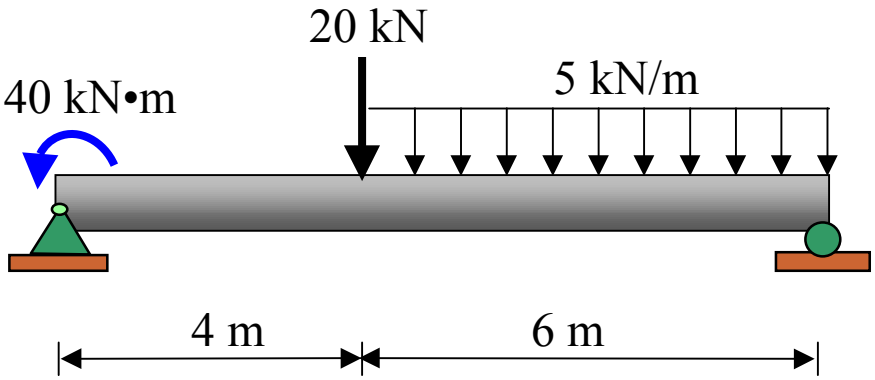
• T- Beam

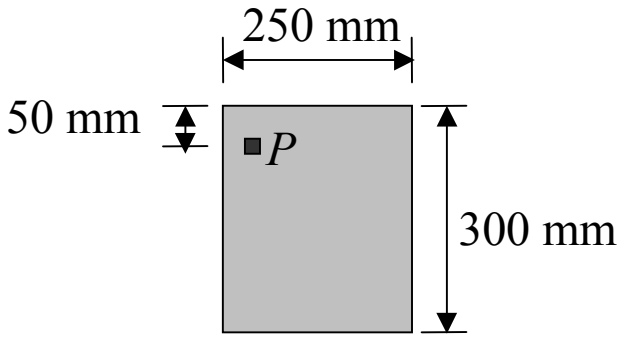




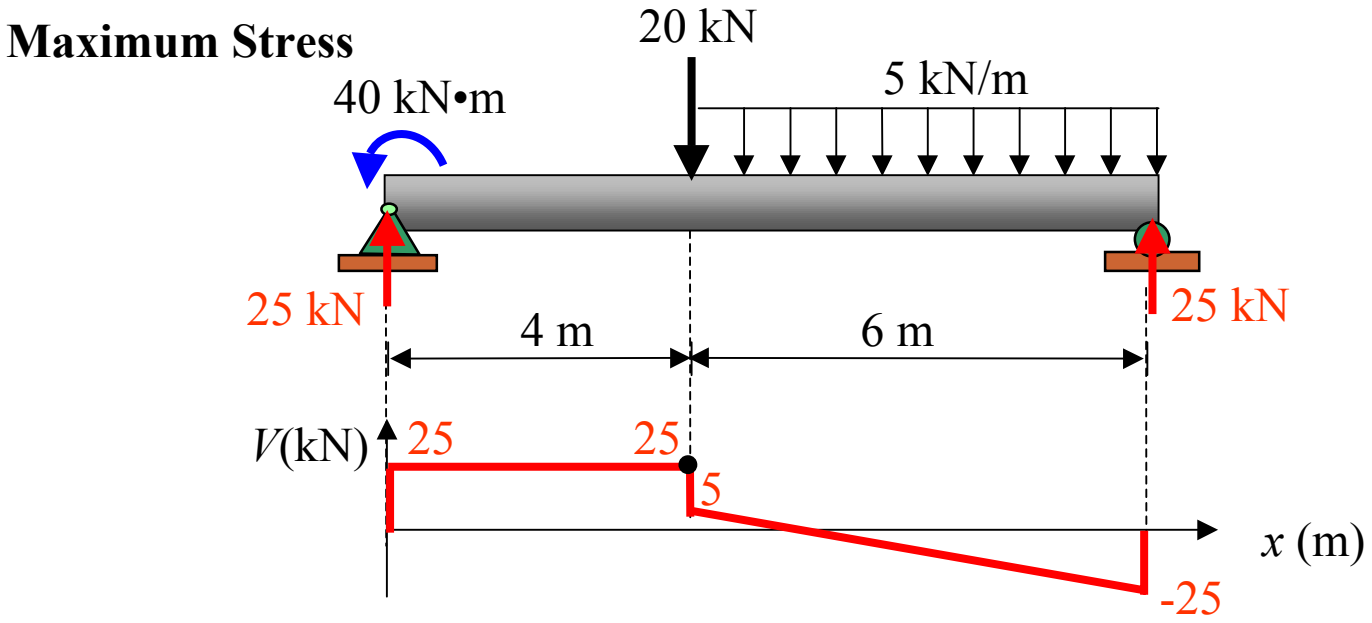
Example 1a

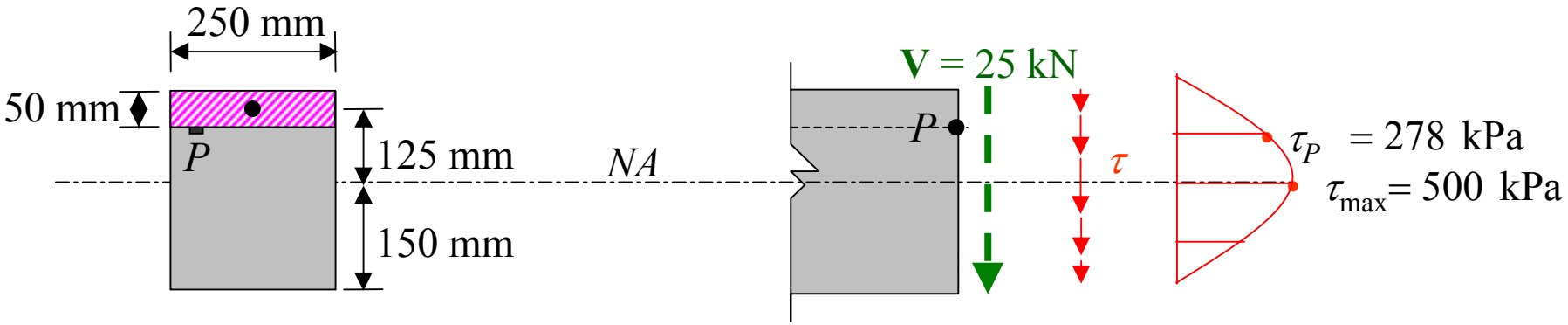
The beam shown is made of wood, determine the shear stress in the beam at point *P* and compute the maximum shear stress in the beam. Plot the shear-stress distribution acting over the beam's cross-sectional area.





- **Maximum Stress**





$$I = \frac{1}{12}(0.25)(0.3)^3$$

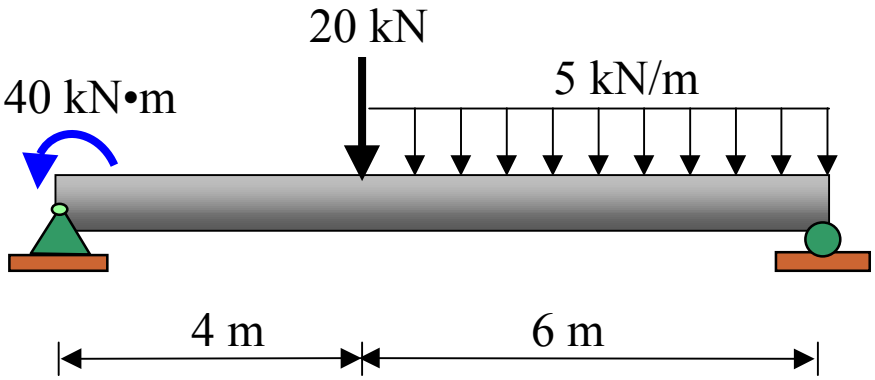
$$= 562.5 \times 10^{-6}\ \text{m}^4$$

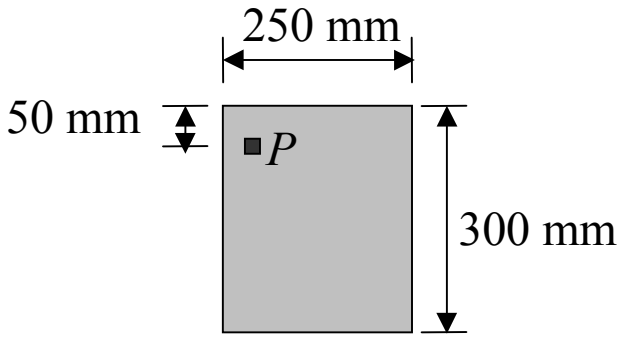
$$\tau_P = \frac{VQ}{It} = \frac{(25\ \text{kN})[(0.125\ \text{m})(0.05\ \text{m} \times 0.25\ \text{m})]}{(562.5 \times 10^{-6}\ \text{m}^4)(0.25\ \text{m})} = 278\ \text{kPa}$$

$$\tau_{\max} = 1.5\frac{V}{A} = 1.5\frac{25 \times 10^3\ \text{N}}{(0.25\ \text{m} \times 0.30\ \text{m})} = 500\ \text{kPa}$$

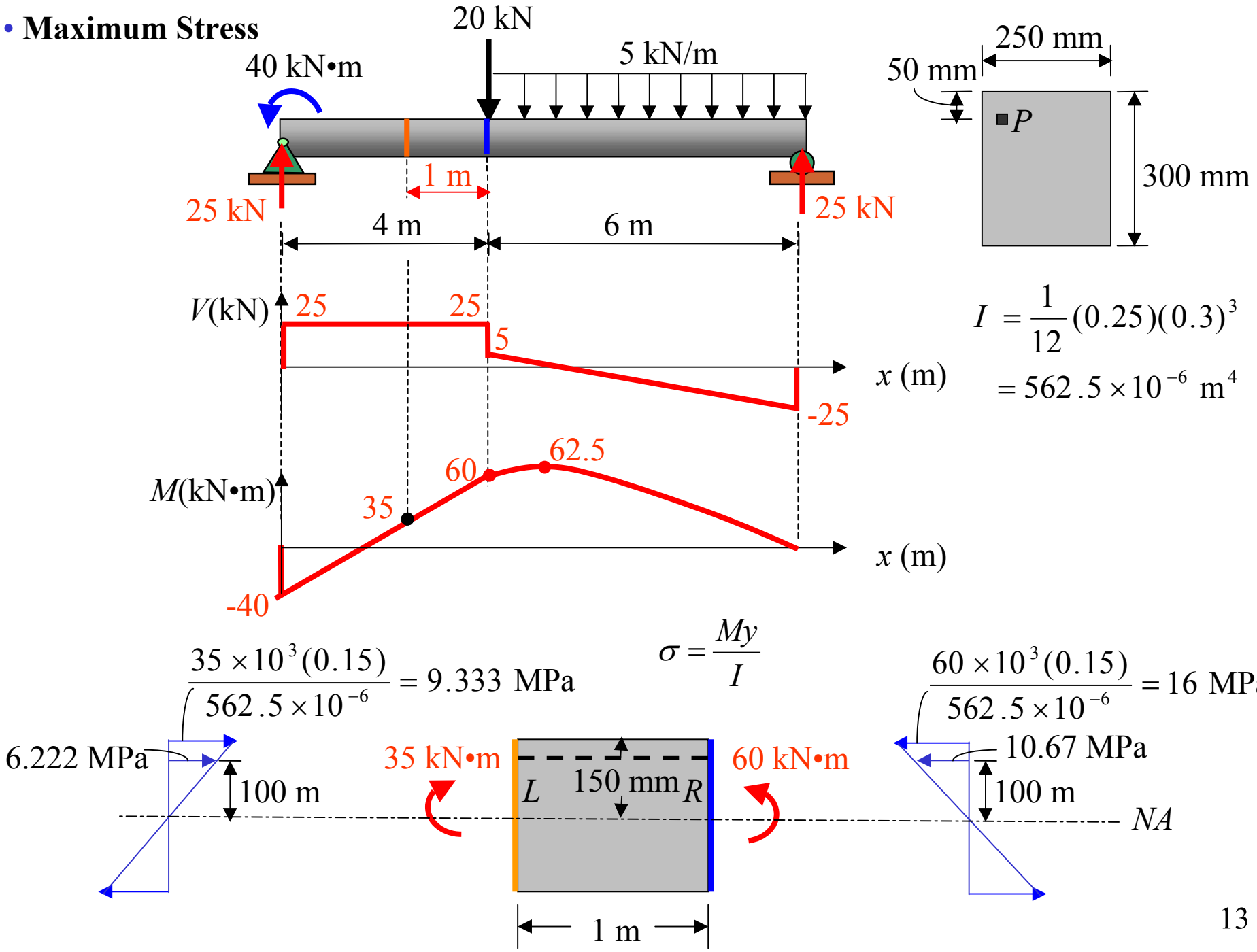
Example 1b

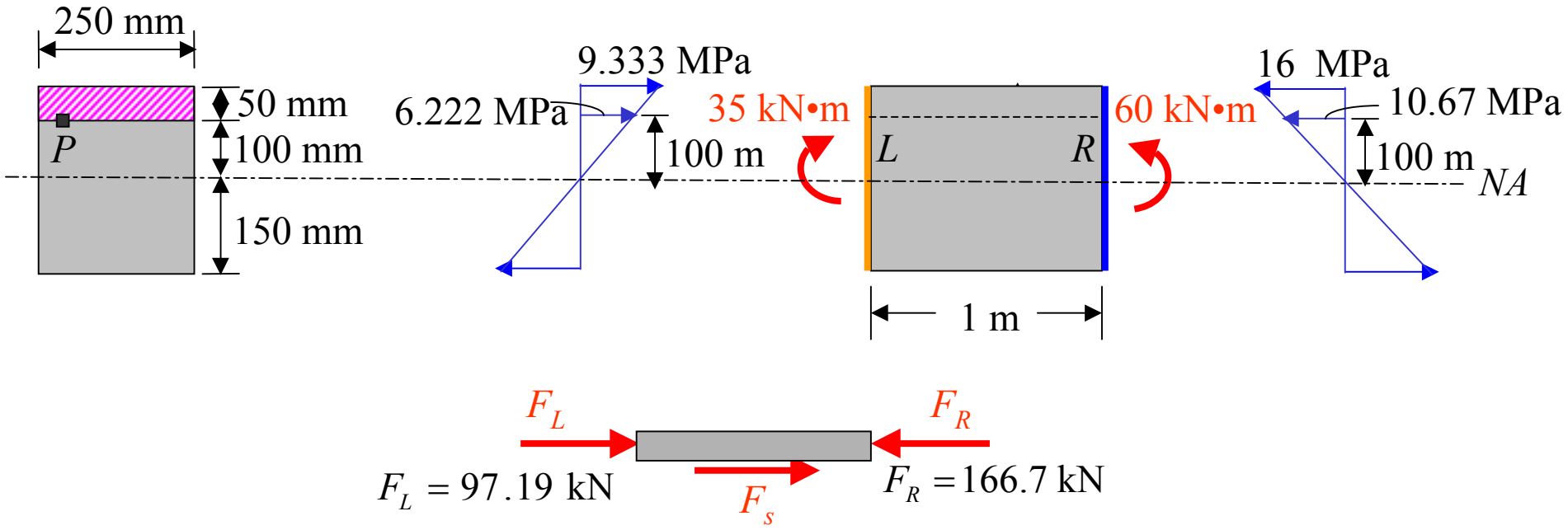
The simply supported beam has the cross-sectional area shown. Determine the shear flow in the beam at point *P*.





- **Maximum Stress**





- Force F_L on area A_L

$$F_L = \int_{A_L} \sigma dA$$

$$= (\sigma_{ave}) A_L$$

$$= [\frac{(9.33 + 6.22) \times 10^6}{2}](0.25 \times 0.05)$$

$$F_L = 97.19 \text{ kN}$$

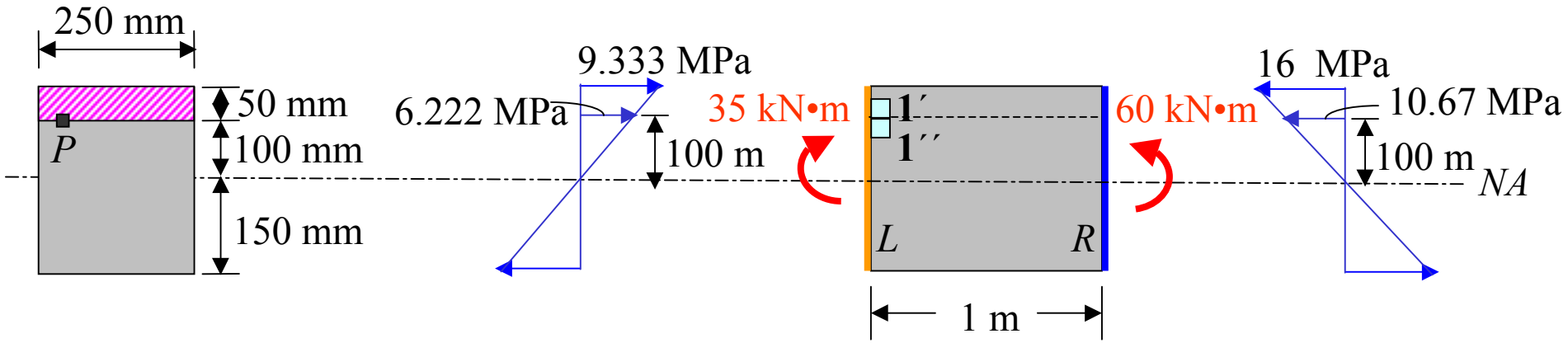
- Force F_R on area A_R

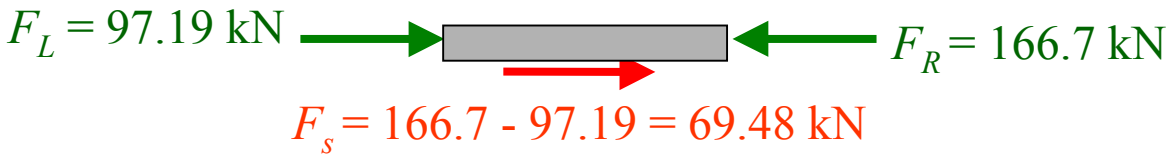
$$F_R = \int_{A_R} \sigma dA$$

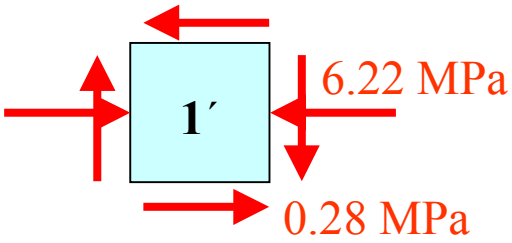
$$= (\sigma_{ave}) A_R$$

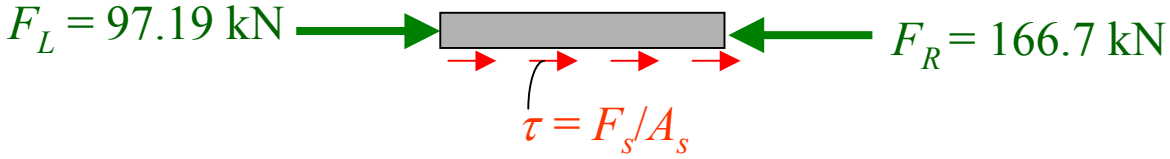
$$= [\frac{(16 + 10.67) \times 10^6}{2}](0.25 \times 0.05)$$

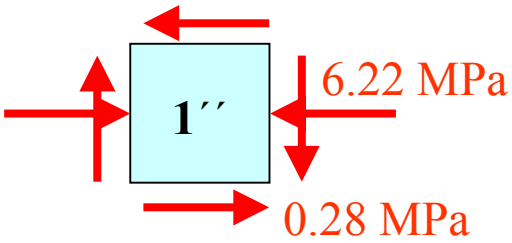
$$F_R = 166.7 \text{ kN}$$

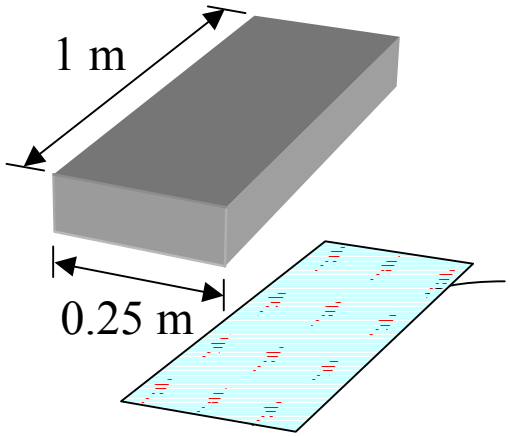










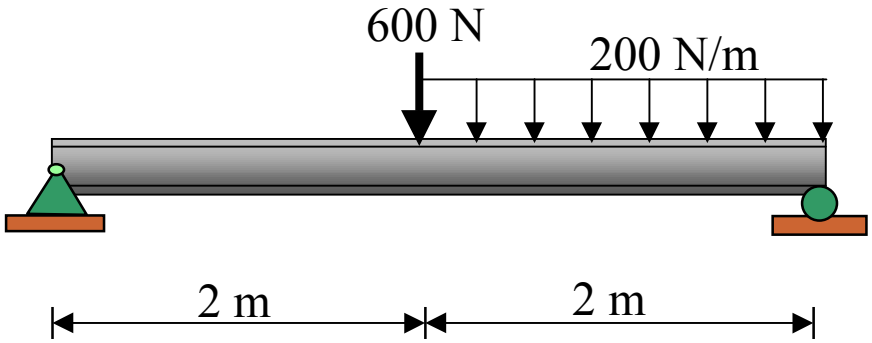


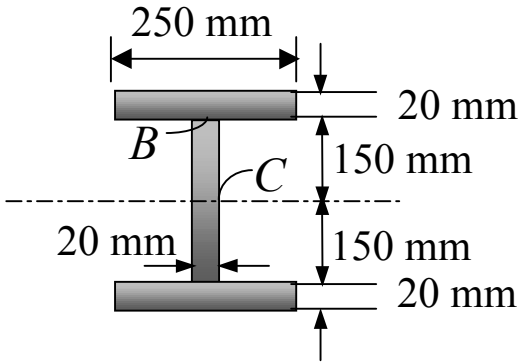
$$\tau = \frac{F_s}{A_s} = \frac{69.48 \times 10^3 \text{ N}}{(0.25 \text{ m})(1 \text{ m})}$$

$$= 278 \text{ kPa} \Leftarrow$$

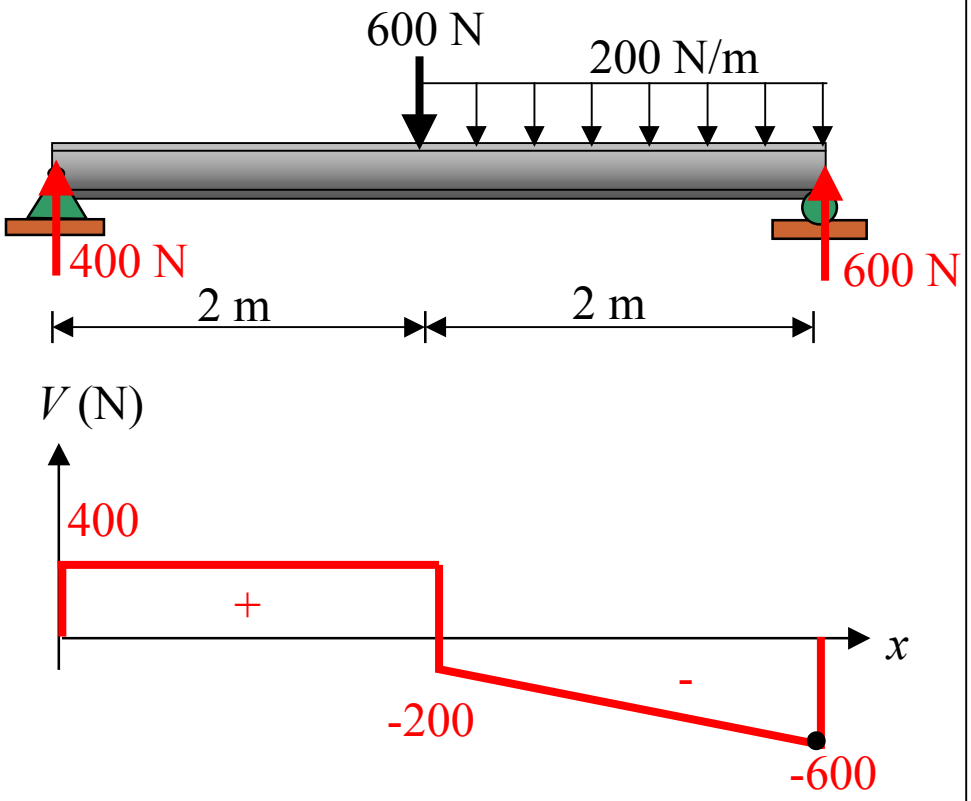
Example 2

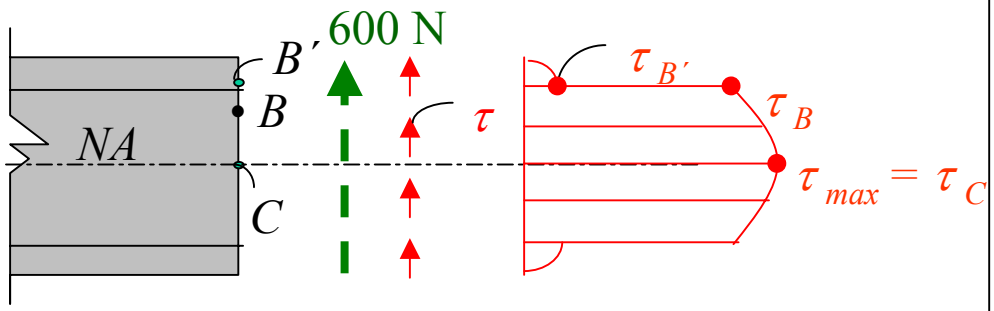
A steel wide-flange beam has the dimensions shown. Plot the shear-stress distribution acting over the beam´s cross-sectional area.



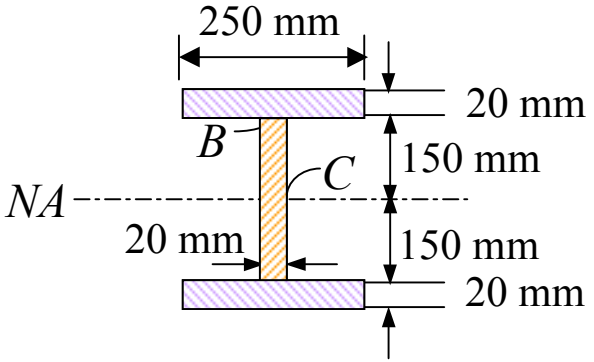


• **Internal Shear**





• **Section property**



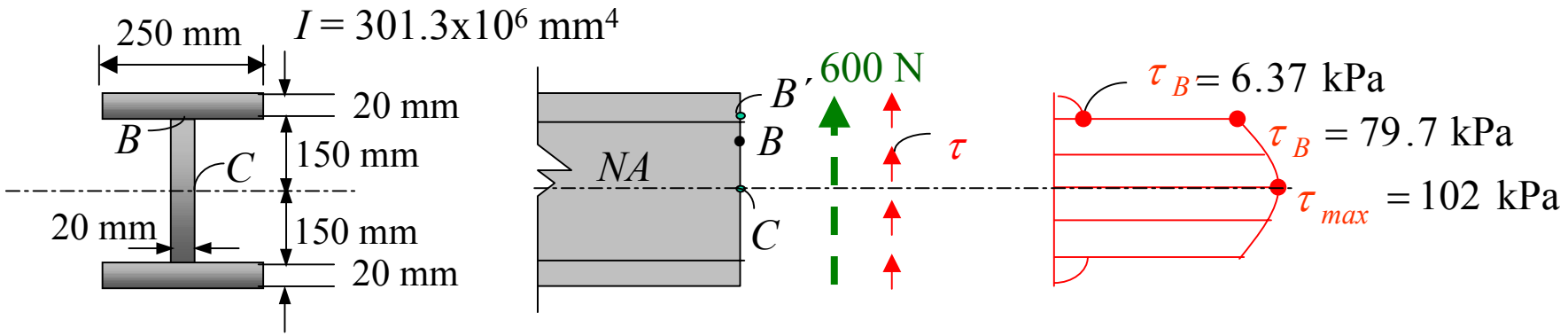
$$I = \Sigma(\bar{I} + Ad^2)$$

$$= (\bar{I} + Ad^2)_{\text{web}} + 2(\bar{I} + Ad^2)_{\text{flang}}$$

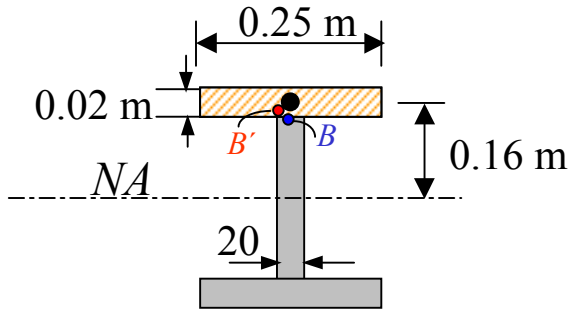
$$= [(1/12)(20)(300^3) + 0] +$$

$$2[(1/12)(250)(20^3) + (20\text{x}250)(160^2)]$$

$$= 301.3\text{x}10^6 \text{ mm}^4$$



- **Shear stress in beam:** $\tau = \dfrac{VQ}{It}$

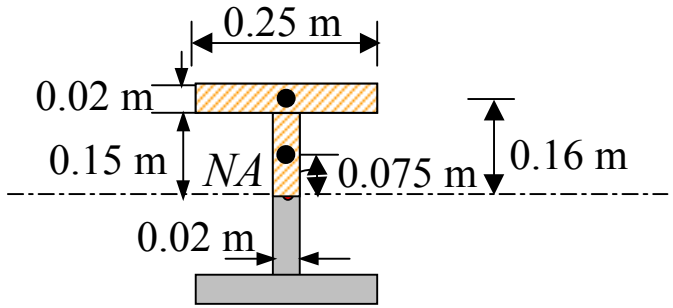


$$\tau_{B'} = \frac{(600\ \text{N})[(0.16\ \text{m})(0.25\ \text{m} \times 0.02\ \text{m})]}{(301.3 \times 10^{-6}\ \text{m}^4)(0.25\ \text{m})}$$

$$= 6.37\ \text{kPa} \Leftarrow$$

$$\tau_B = \frac{(600\ \text{N})[(0.16\ \text{m})(0.25\ \text{m} \times 0.02\ \text{m})]}{(301.3 \times 10^{-6}\ \text{m}^4)(0.02\ \text{m})}$$

$$= 79.7\ \text{kPa} \Leftarrow$$

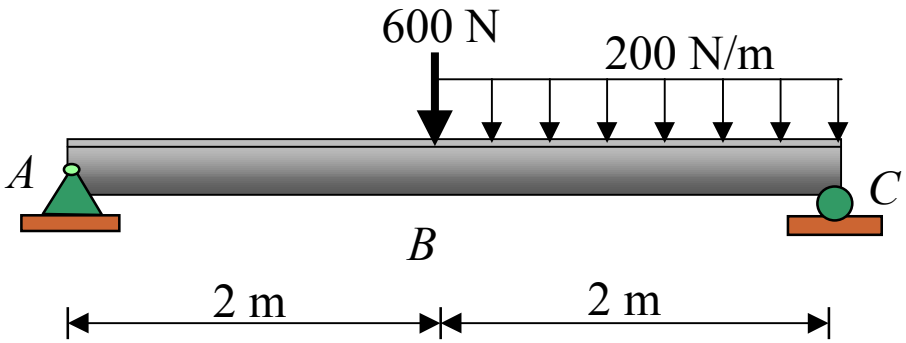


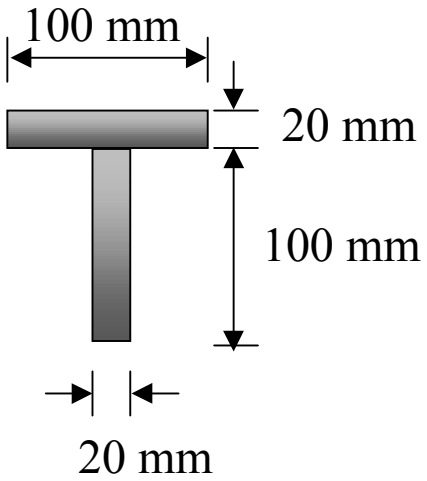
$$Q_{\max} = [\,(0.16)(0.25\text{x}0.02) + (0.075)(0.02\text{x}0.15)]$$
$$= 1.025\text{x}10^{-3}\ \text{m}^3$$

$$\tau_{\max} = \frac{(600\ \text{N})(1.025 \times 10^{-3}\text{m}^3)}{(301.3 \times 10^{-6}\ \text{m}^4)(0.02\ \text{m})} = 102\ \text{kPa} \Leftarrow$$

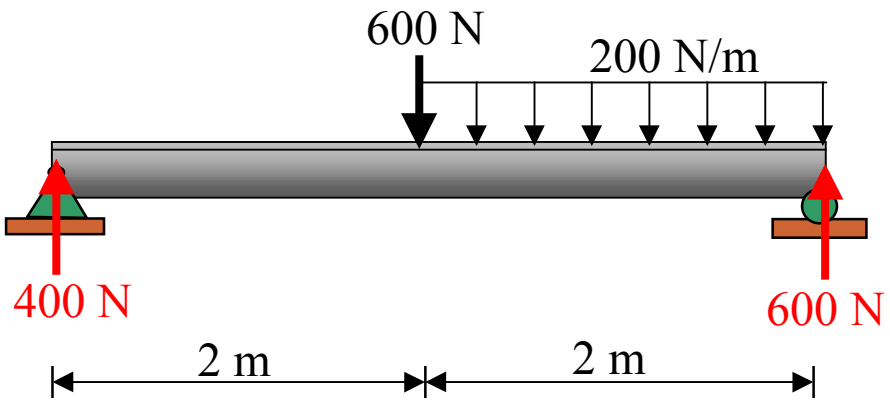
Example 3

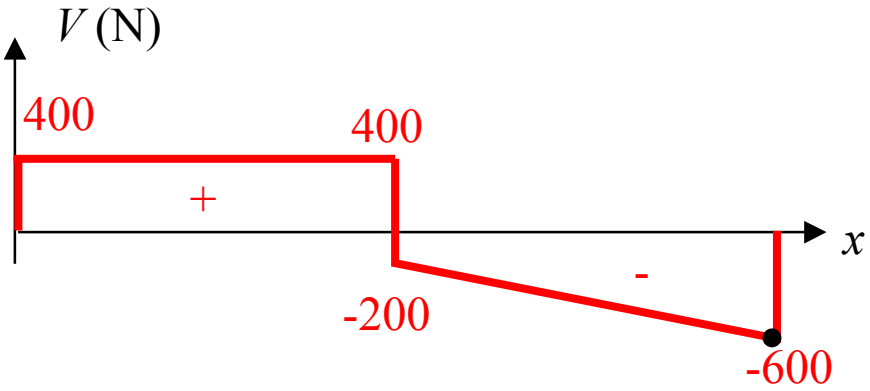
The beam show is made from two boards. Plot the shear-stress distribution acting over the beam's cross-sectional area.

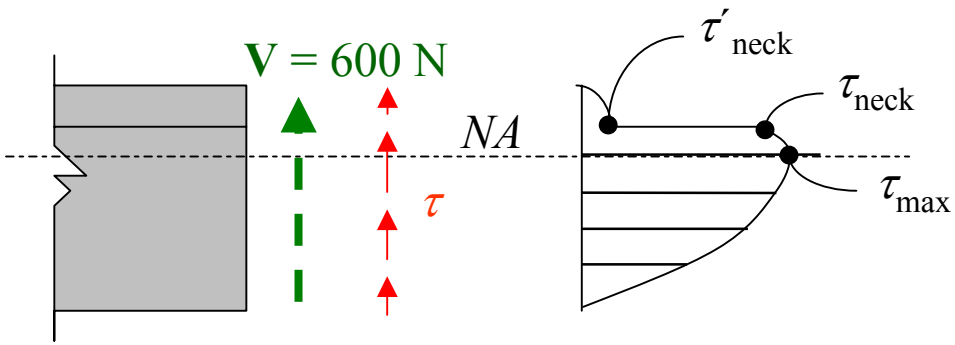




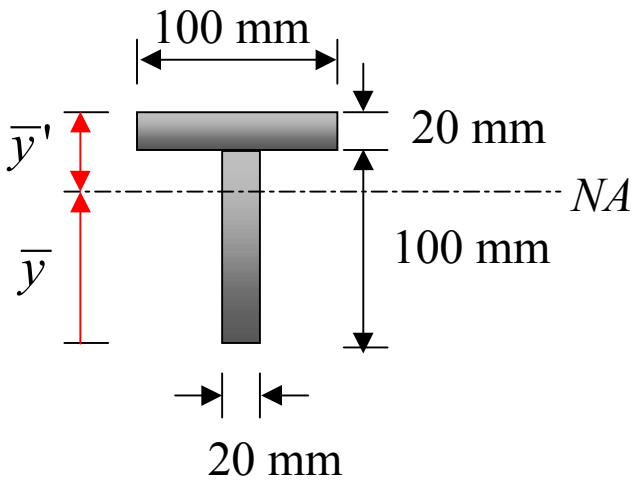
- **Internal loads**







- **Section property**



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

$$= \frac{(50)(100\times 20)+(110)(100\times 20)}{2(100\times 20)}$$

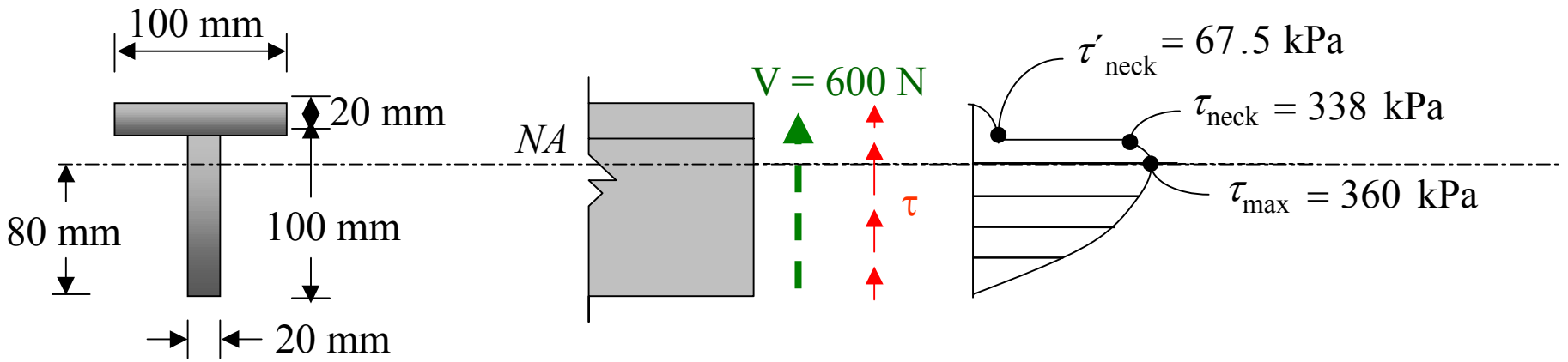
$$= 80 \text{ mm}$$

$$\bar{y}' = 120 - 80 = 40 \text{ mm}$$

$$I = \Sigma(\bar{I} + Ad^2)$$

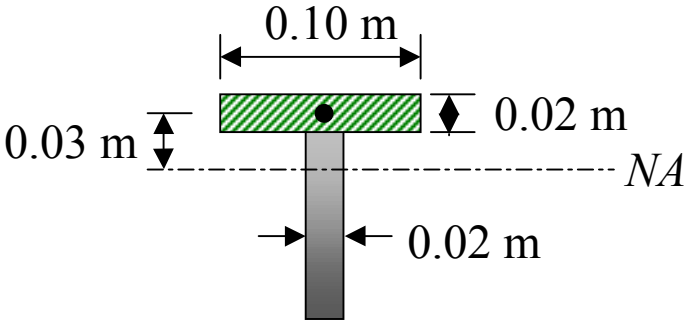
$$= [(1/12)(20)(100^3) + (20\text{x}100)(80\text{-}50)^2] + [(1/12)(100)(20^3) + (100\text{x}20)(40\text{-}10)^2]$$

$$= 5.333\text{x}10^6 \text{ mm}^4 = 5.333\text{x}10^{-6} \text{ m}^4$$



$I = 5.333 \times 10^{-6}\ \text{m}^4$

- **Shear stress in beam:** $\tau = \dfrac{VQ}{It}$

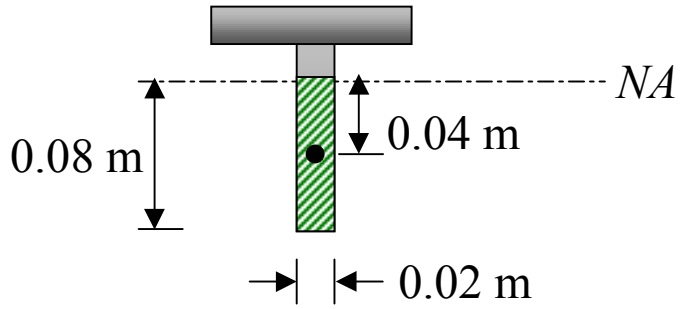


$$\tau'_{neck} = \frac{(600\ \text{N})[(0.03\ \text{m})(0.10\ \text{m} \times 0.02\ \text{m})]}{(5.333 \times 10^{-6}\ \text{m}^4)(0.10\ \text{m})}$$

$$= 67.5\ \text{kPa} \Leftarrow$$

$$\tau_{neck} = \frac{(600\ \text{N})[(0.03\ \text{m})(0.10\ \text{m} \times 0.02\ \text{m})]}{(5.333 \times 10^{-6}\ \text{m}^4)(0.02\ \text{m})}$$

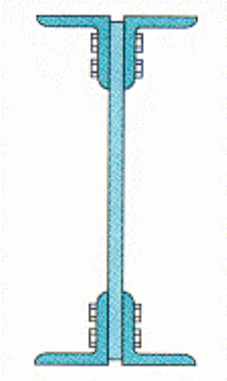
$$= 338\ \text{kPa} \Leftarrow$$



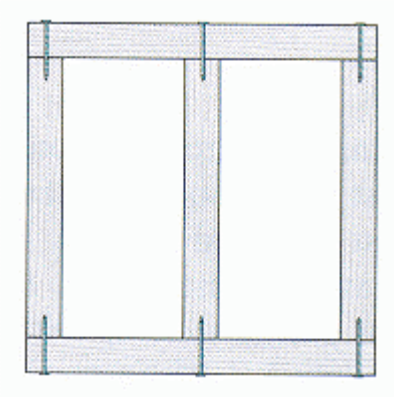
$$\tau_{max} = \frac{(600\ \text{N})[(0.04\ \text{m})(0.08\ \text{m} \times 0.02\ \text{m})]}{(5.333 \times 10^{-6})(0.02\ \text{m})}$$

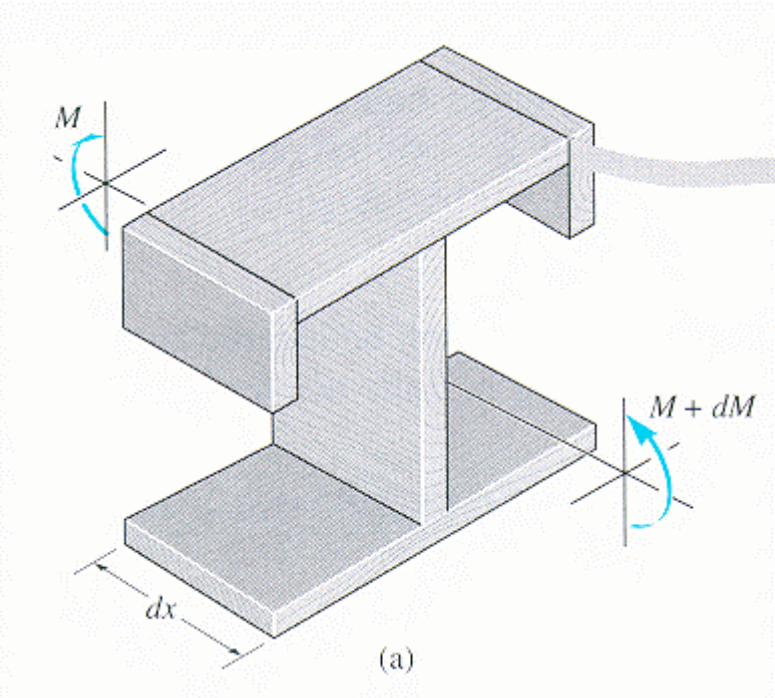
$$= 360\ \text{kPa} \Leftarrow$$

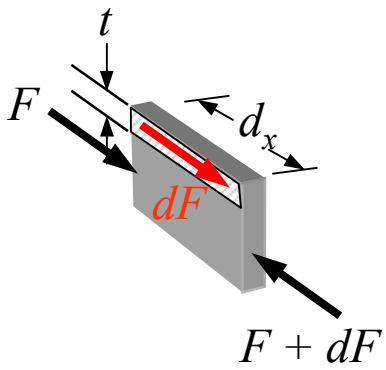
Shear Flow (q) in Built-Up Members











$$dF = \frac{dM}{I}\int_{A'} y\,dA$$

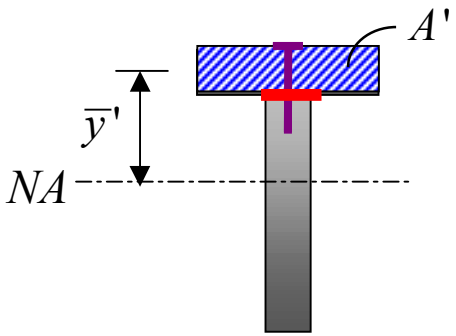
$$q = \frac{dF}{dx} = \frac{dM}{I\,dx}\int_{A'} y\,dA$$

V

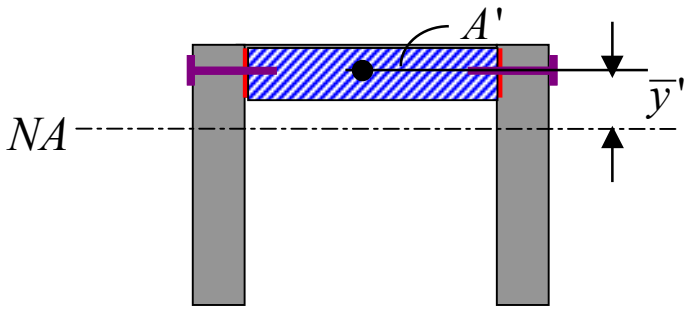
Q

$$q = \frac{VQ}{I} = \tau t$$

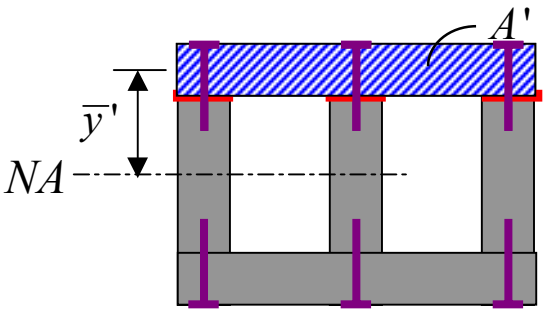
(a) The shear flow is resisted at *one* surfaces.



(b) The shear flow is resisted at *two* surfaces.

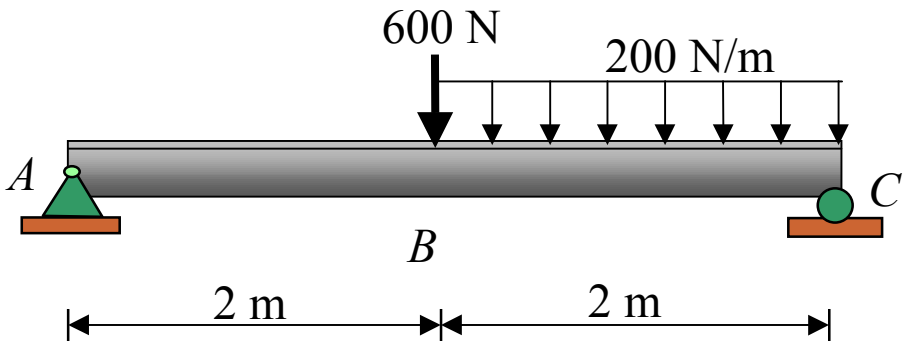


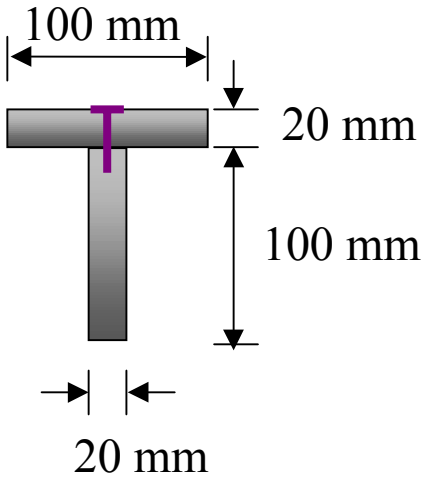
(c) The shear flow is resisted at *three* surfaces.



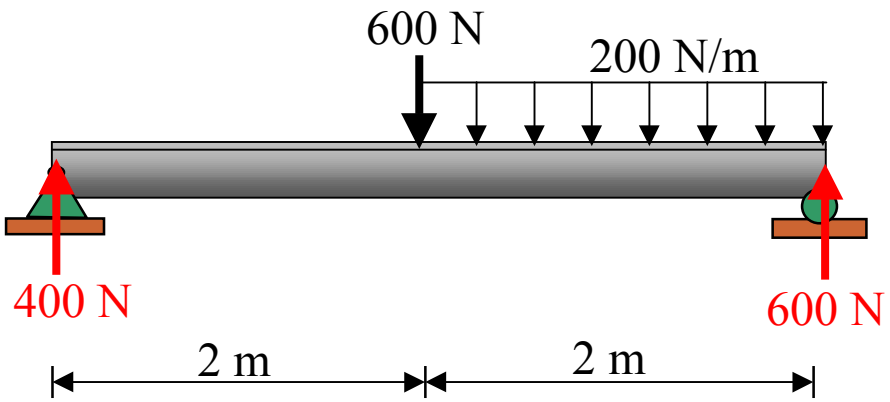
Example 4

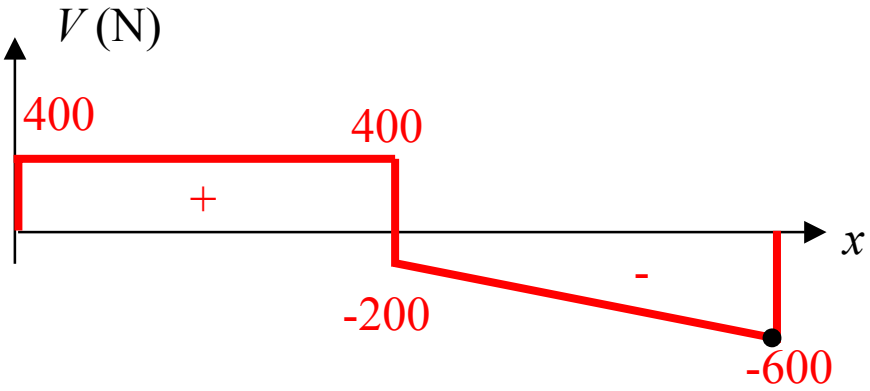
The beam show is made from two boards. Determine the **maximum shear in nail** if diameter of nail is 3 mm and spacing is 200 mm.

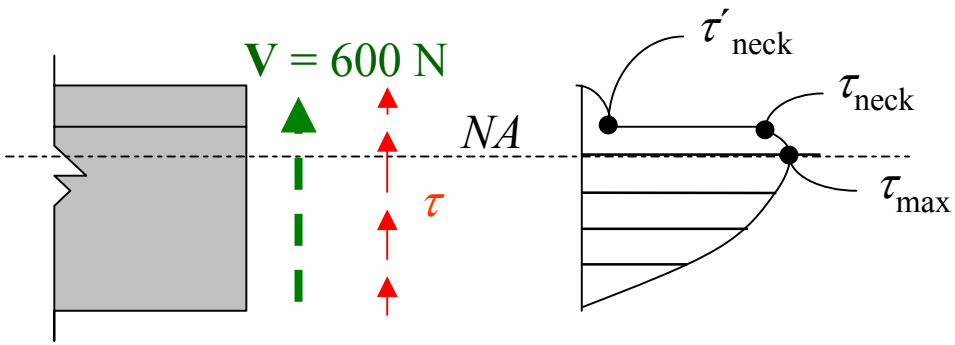




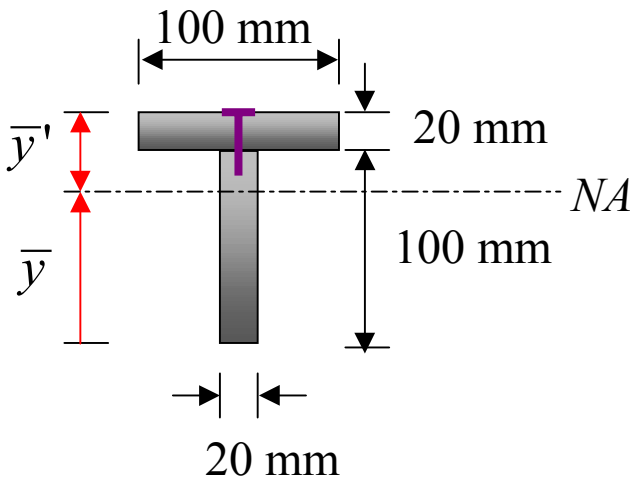
- **Internal loads**







- **Section property**



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

$$= \frac{(50)(100\times 20)+(110)(100\times 20)}{2(100\times 20)}$$

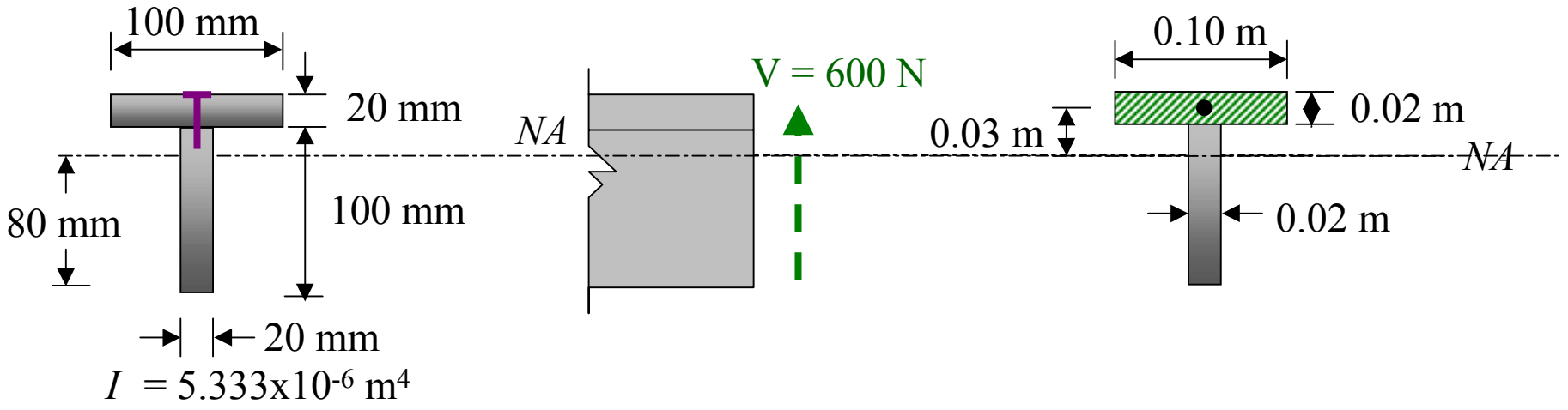
$$= 80 \text{ mm}$$

$$\bar{y}' = 120 - 80 = 40 \text{ mm}$$

$$I = \Sigma(\bar{I} + Ad^2)$$

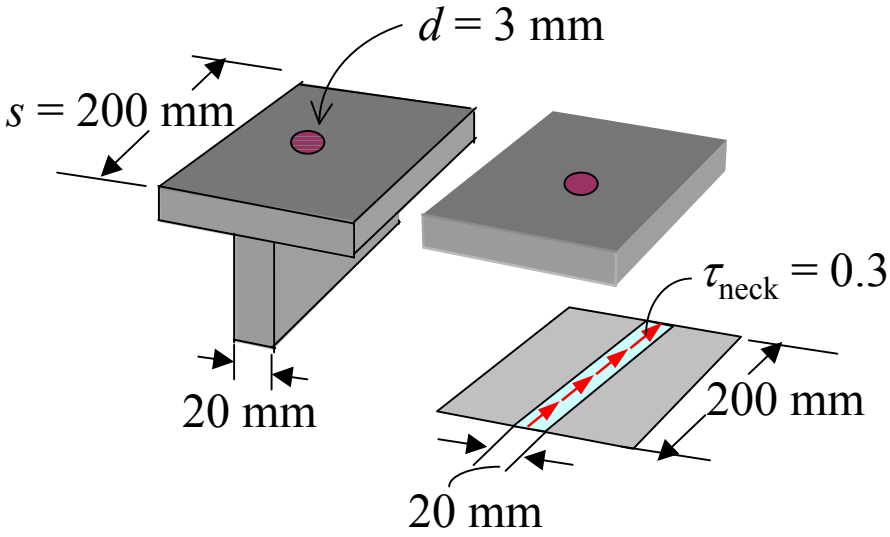
$$= [(1/12)(20)(100^3) + (20\text{x}100)(80\text{-}50)^2] + [(1/12)(100)(20^3) + (100\text{x}20)(40\text{-}10)^2]$$

$$= 5.333\text{x}10^{6} \text{ mm}^4 = 5.333\text{x}10^{-6} \text{ m}^4$$



- **Shear stress in beam at neck:** $\tau = \dfrac{VQ}{It}$

$$\tau_{neck} = \frac{(600\ \text{N})[(0.03\ \text{m})(0.10\ \text{m} \times 0.02\ \text{m})]}{(5.333 \times 10^{-6}\ \text{m}^4)(0.02\ \text{m})} = 0.3375\ \text{MPa}$$



$$(F_s)_{nail} = (\tau_{@\,C\,(neck)})(A_{shear})$$

$$= (0.3375\ \text{MPa})(0.02\ \text{m} \times 0.2\ \text{m})$$

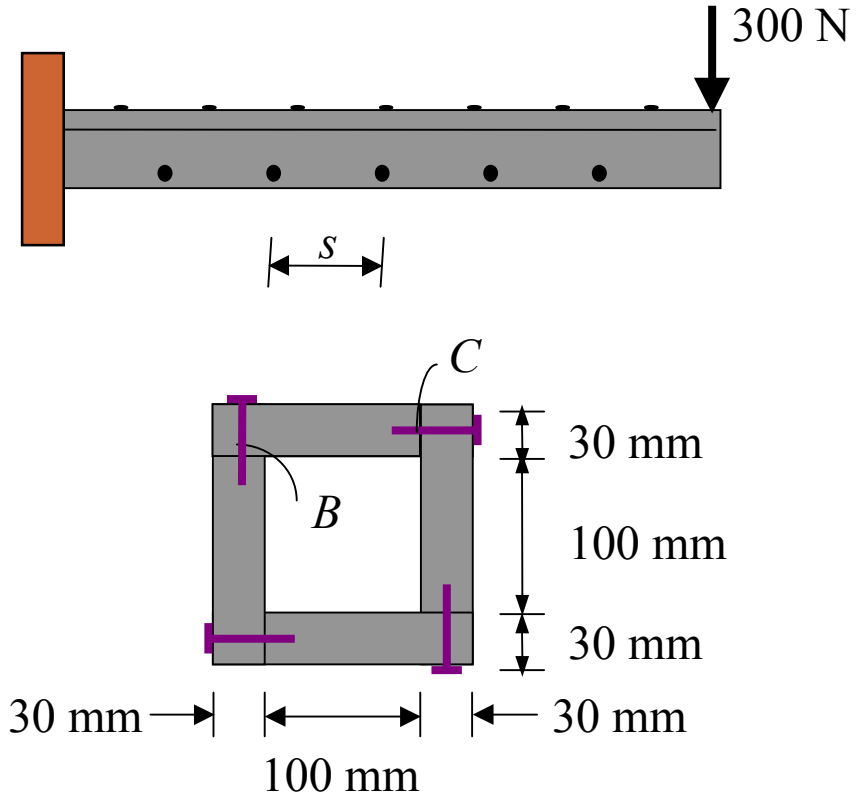
$$= 1350\ \text{N}$$

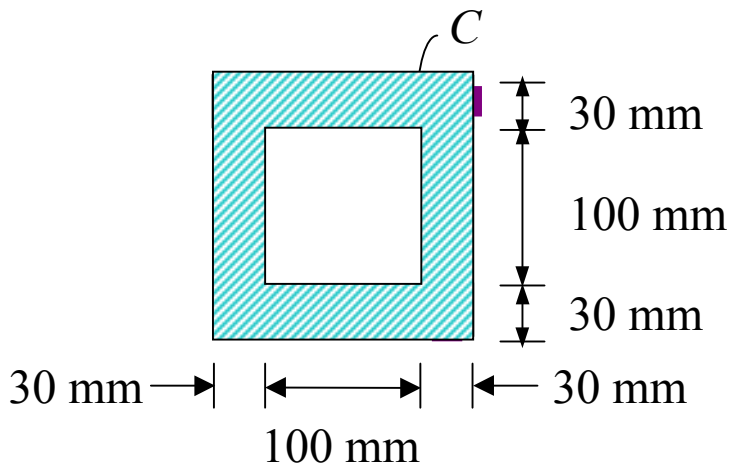
$$\tau_{nail} = \frac{(F_s)_{nail}}{A_{nail}}$$

$$= \frac{1350\ \text{N}}{(\pi / 4)(0.003\ \text{m})^2} = 191\ \text{MPa} \Leftarrow$$

Example 5

A box beam is to be constructed from four boards nailed together as shown. If each nail can support a shear force of 200 N, determine the maximum spacing *s* of nails at *B* and at *C* so that the beam will support the vertical force of 300 N.



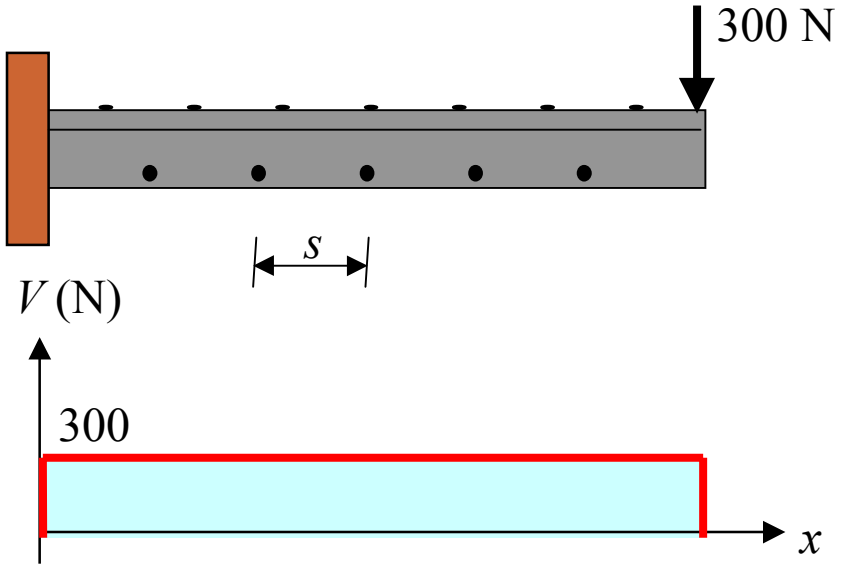


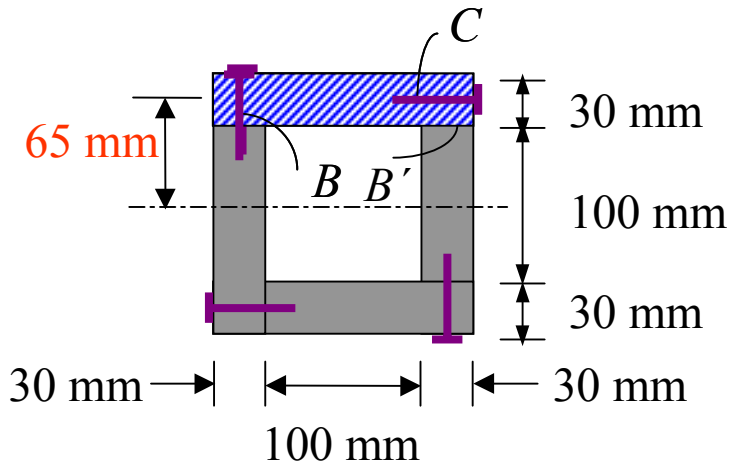
- ***Section property***

$$I = \left(\frac{1}{12}bh^3\right)_{out} - \left(\frac{1}{12}bh^3\right)_{in}$$

$$= (1/12)(0.16)(0.16^3) - (1/12)(0.1)(0.1^3)$$

$$= 46.28 \times 10^{-6} \ m^4$$

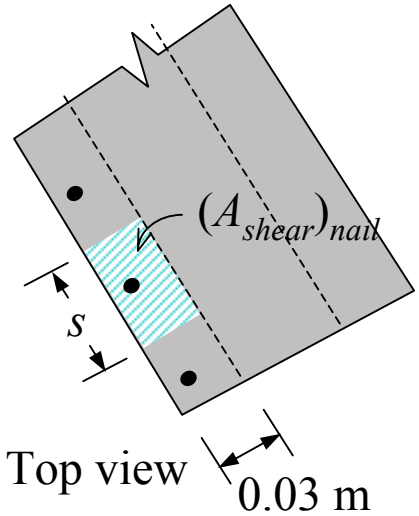




$I = 46.28\text{x}10^{-6}\ \text{m}^4$

- ***Maximum spacing s of nails at B***

$$\tau_B = \frac{VQ_B}{It} = \frac{V\bar{y}'A'}{It} = \frac{(300)(0.065)(0.16\times 0.03)}{(46.28\times 10^{-6})(2)(0.03)} = 33.71\,k\text{Pa}$$

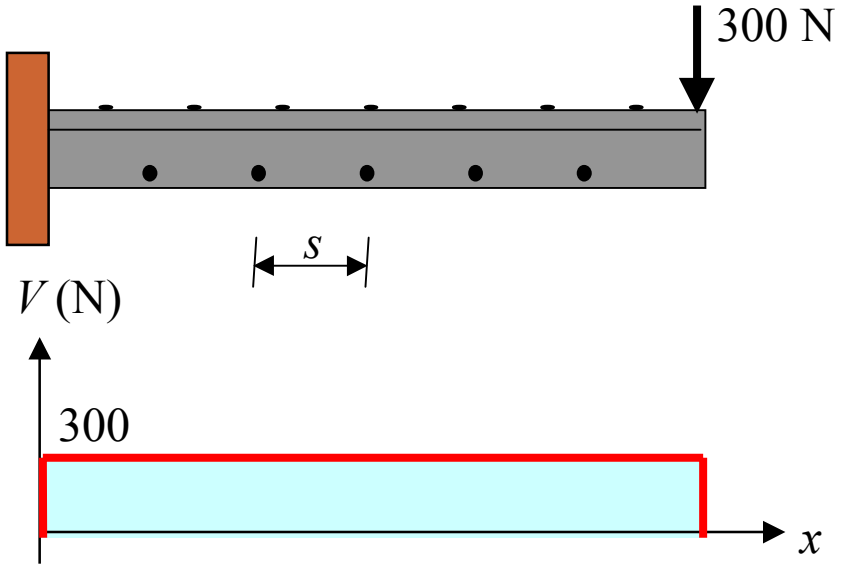


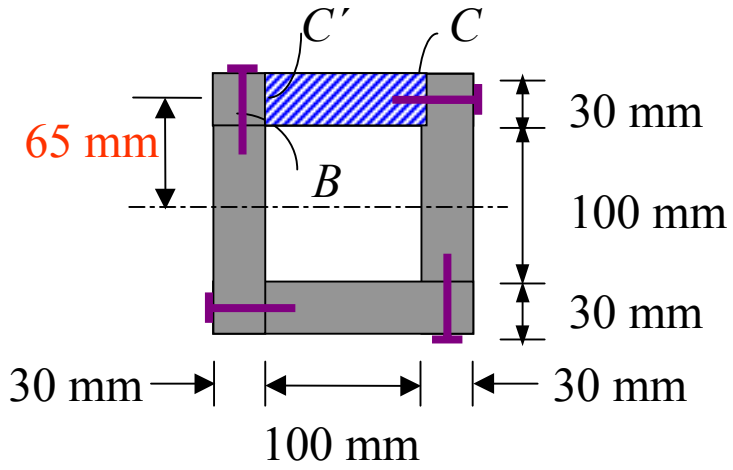
$$F_{nail} = \tau_B (A_{shear})_{nail}$$

$$200 = 33.71(10^3)s(0.03)$$

$$s = 0.198\ \text{m}$$

Use $s_B = 200$ mm

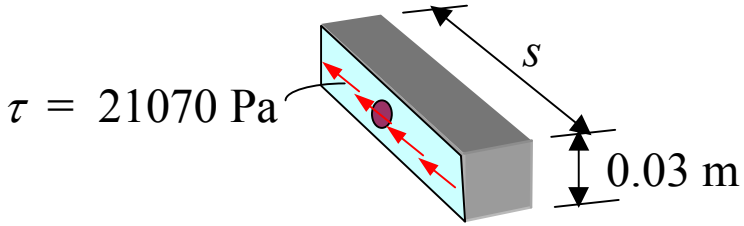




$I = 46.28\text{x}10^{-6}\ \text{m}^4$

- ***Maximum spacing s of nails at C***

$$\tau_B = \frac{VQ_C}{It} = \frac{V\bar{y}'A'}{It} = \frac{(300)(0.065)(0.10\times 0.03)}{(46.28\times 10^{-6})(2)(0.03)} = 21.07\ k\text{Pa}$$



$$F_{nail} = \tau_B A$$

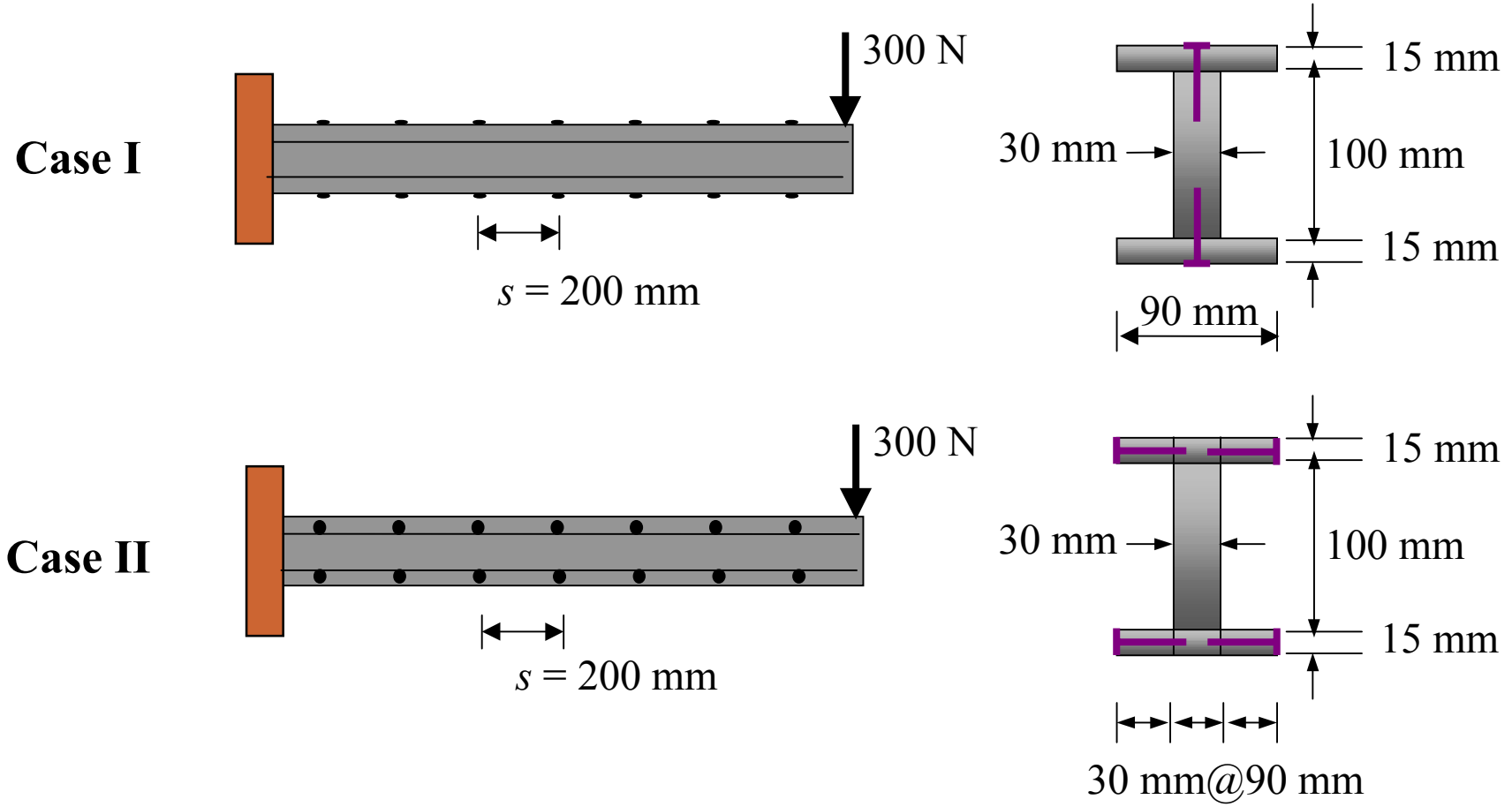
$$200 = 21.07(10^3)s(0.03)$$

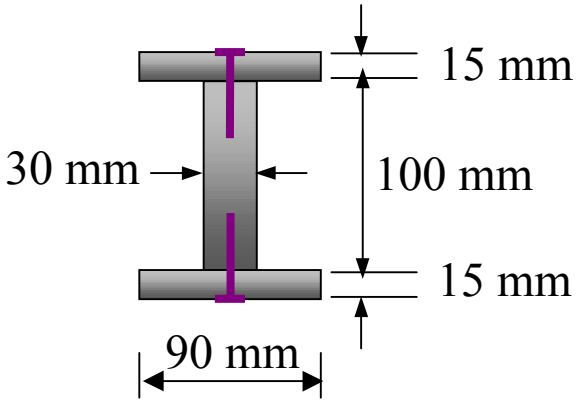
$$s = 0.316\ \text{m}$$

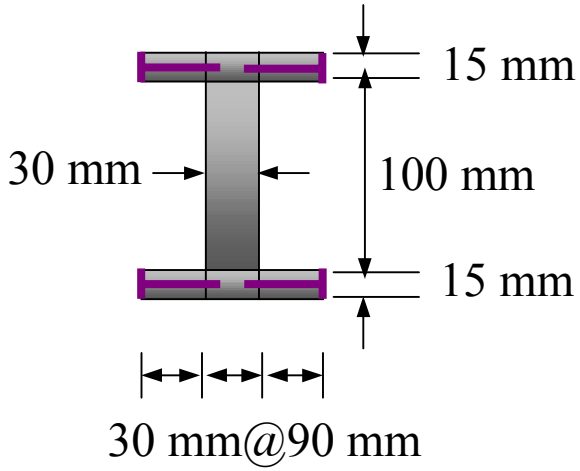
Use $s_B = 300$ mm

Example 6

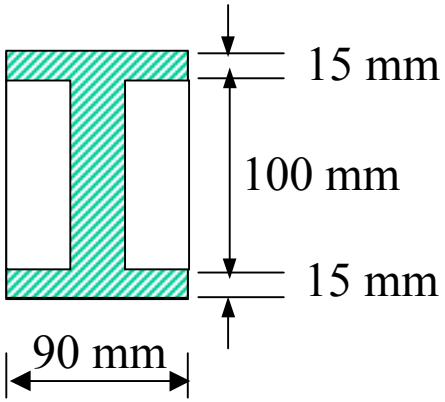
Nails are used in a beam that can be constructed either as in Case I or as in Case II. If the nails are spaced at 200 mm, determine the shear force in the nail for each case so this type of failure will not occur.







- ***Section property for both case***

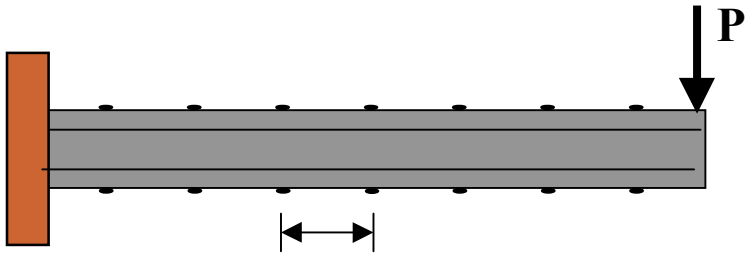


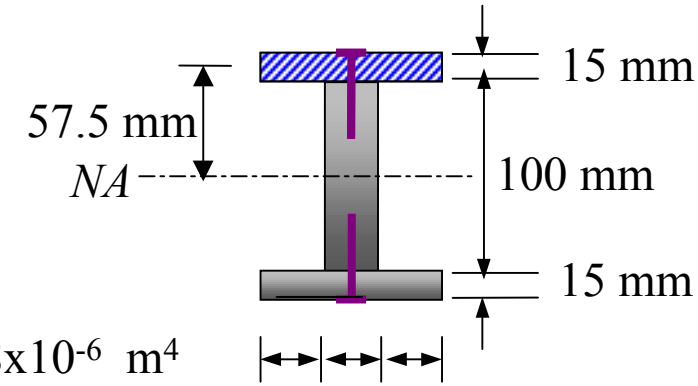
$$I = \left(\frac{1}{12}bh^3\right)_{out} - 2\left(\frac{1}{12}bh^3\right)_{in}$$

$$= (1/12)(0.09)(0.13^3) - 2(1/12)(0.03)(0.1^3)$$

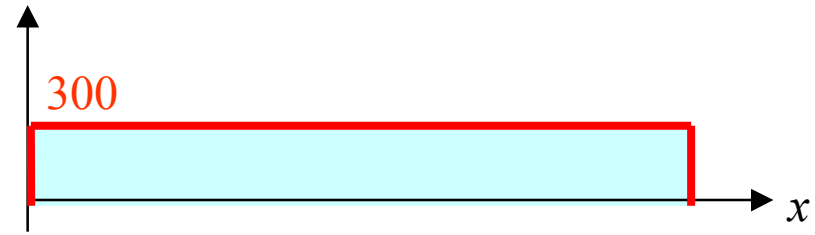
$$= 11.48\text{x}10^{-6}\ \text{m}^4$$

Case I

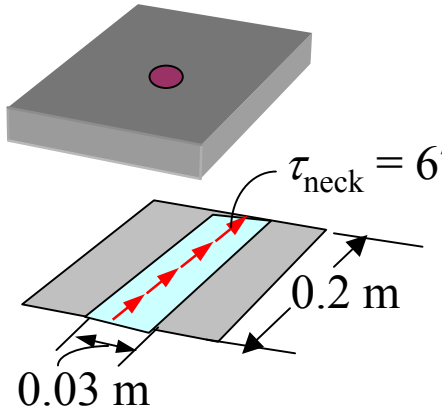




$I = 11.48\text{x}10^{-6}\ \text{m}^4$



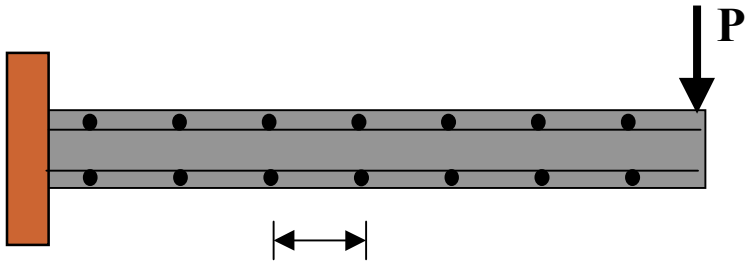
- ***The largest vertical shear for case I***

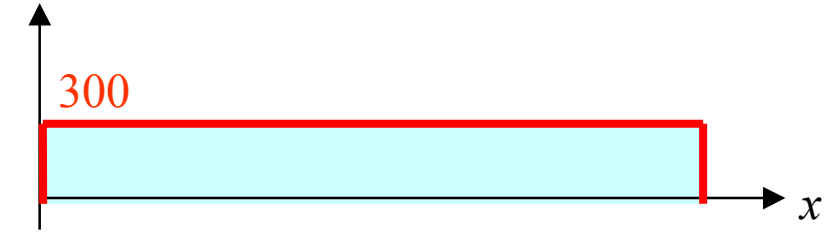


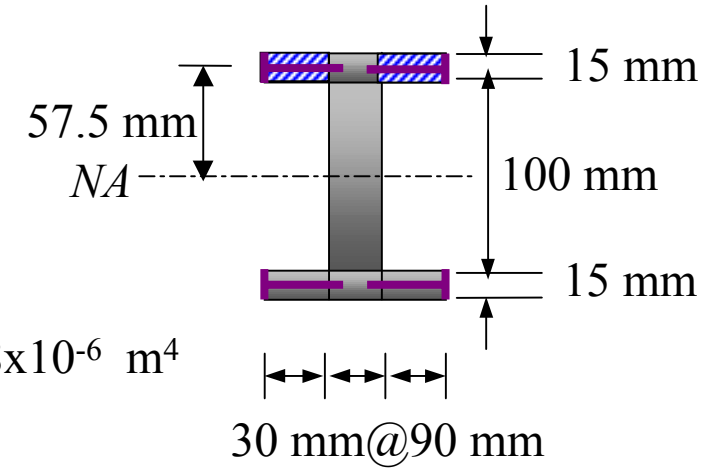
$$\tau = \frac{VQ}{It} = \frac{300[0.0575(0.09 \times 0.015)]}{(11.48 \times 10^{-6})(0.03)} = 67.62\ \text{kPa}$$

$$F_{nail} = \tau A = (67.62 \times 10^{3})(0.03 \times 0.2) = 406\ \text{N} \Leftarrow$$

Case II

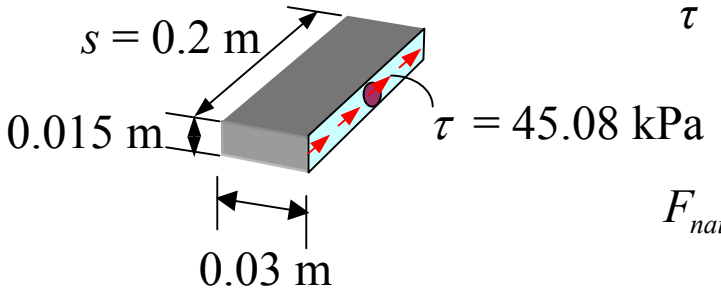






$I = 11.48\text{x}10^{-6}\ \text{m}^4$

- ***The largest vertical shear for case II***



$$\tau = \frac{VQ}{It} = \frac{300[0.0575 \times 2(0.03 \times 0.015)]}{(11.48 \times 10^{-6}) \times 2(0.015)} = 45.08\ \text{kPa}$$

$$F_{nail} = \tau A = (45.08 \times 10^{3})(0.015 \times 0.2) = 135\ \text{N} \Leftarrow$$