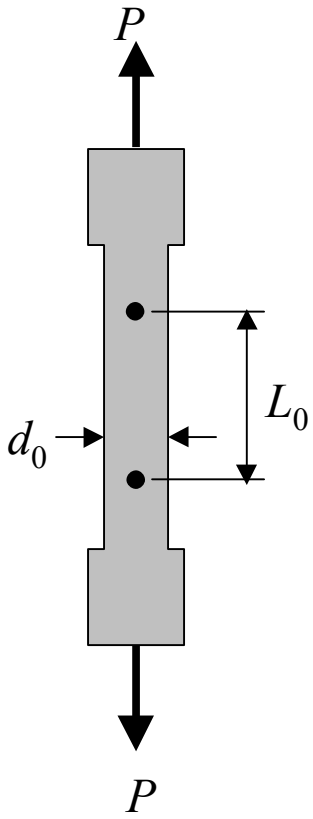
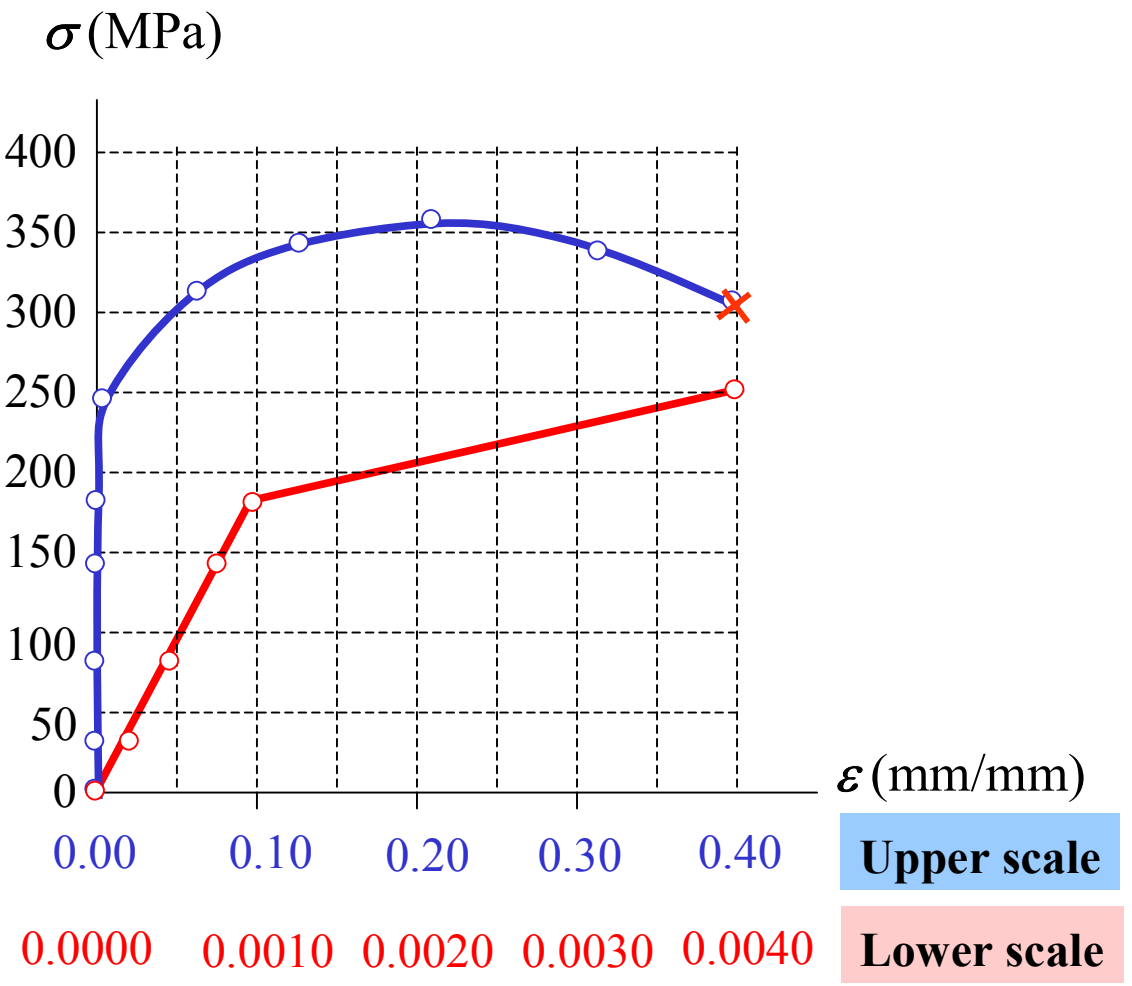
MECHANICAL PROPERTIES OF MATERIALS

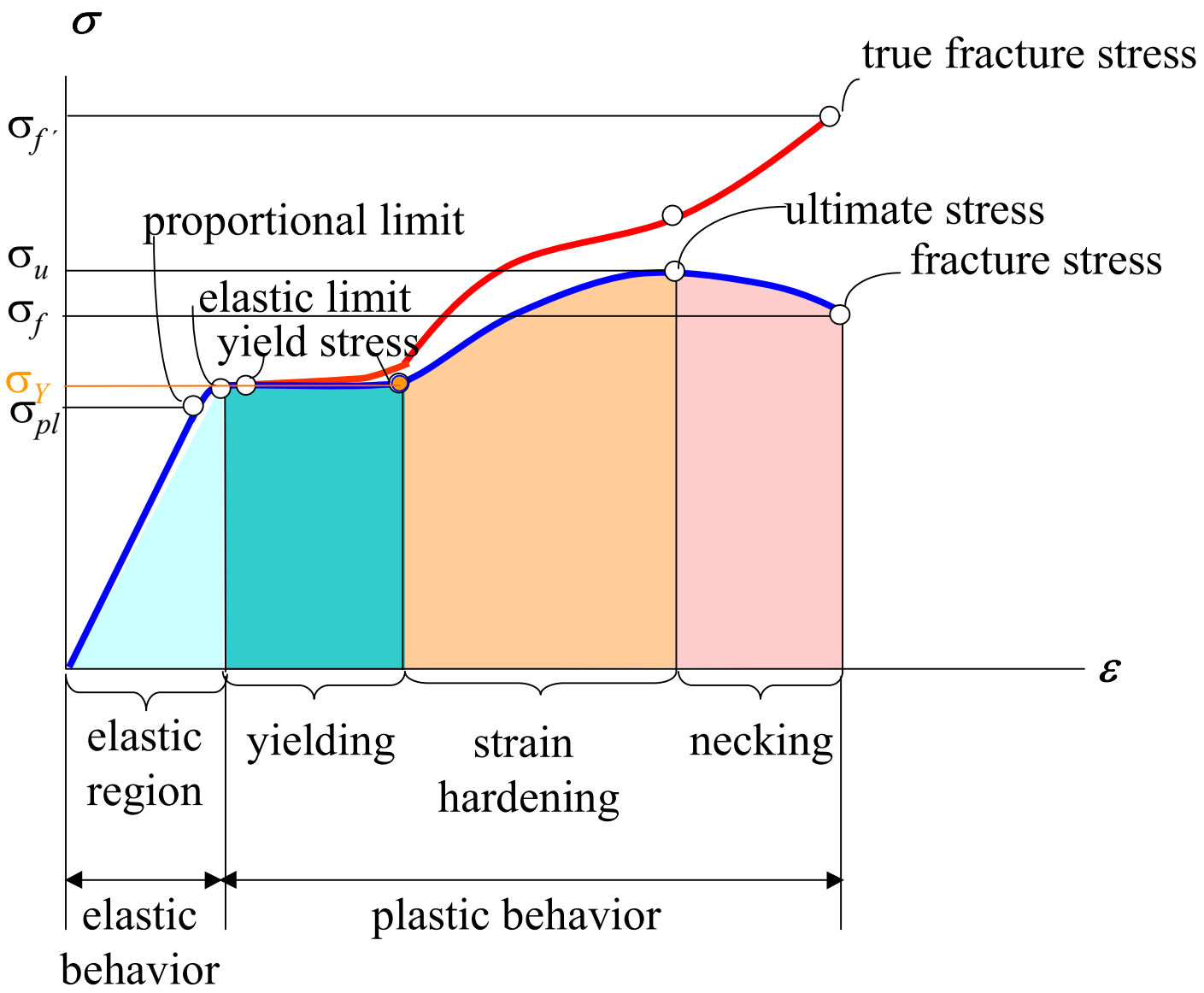
- Simple Tension Test
- The Stress-Strain Diagram
- Stress-Strain Behavior of Ductile and Brittle Materials
- Hooke's Law
- Strain Energy
- Poisson's Ratio
- The Shear Stress-Strain Diagram
- Failure of Materials Due to Creep and Fatigue

Stress Strain Relationship

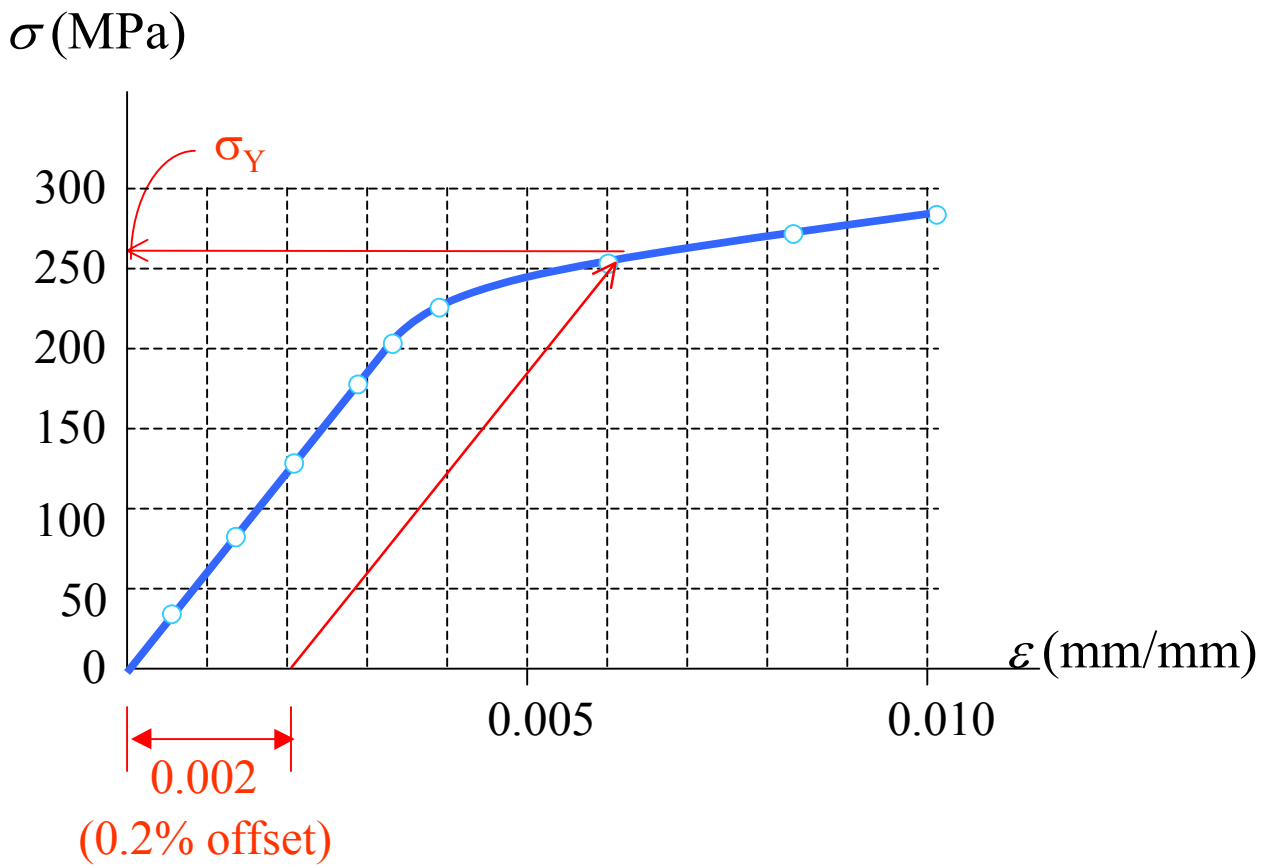




The Stress-Strain Diagram



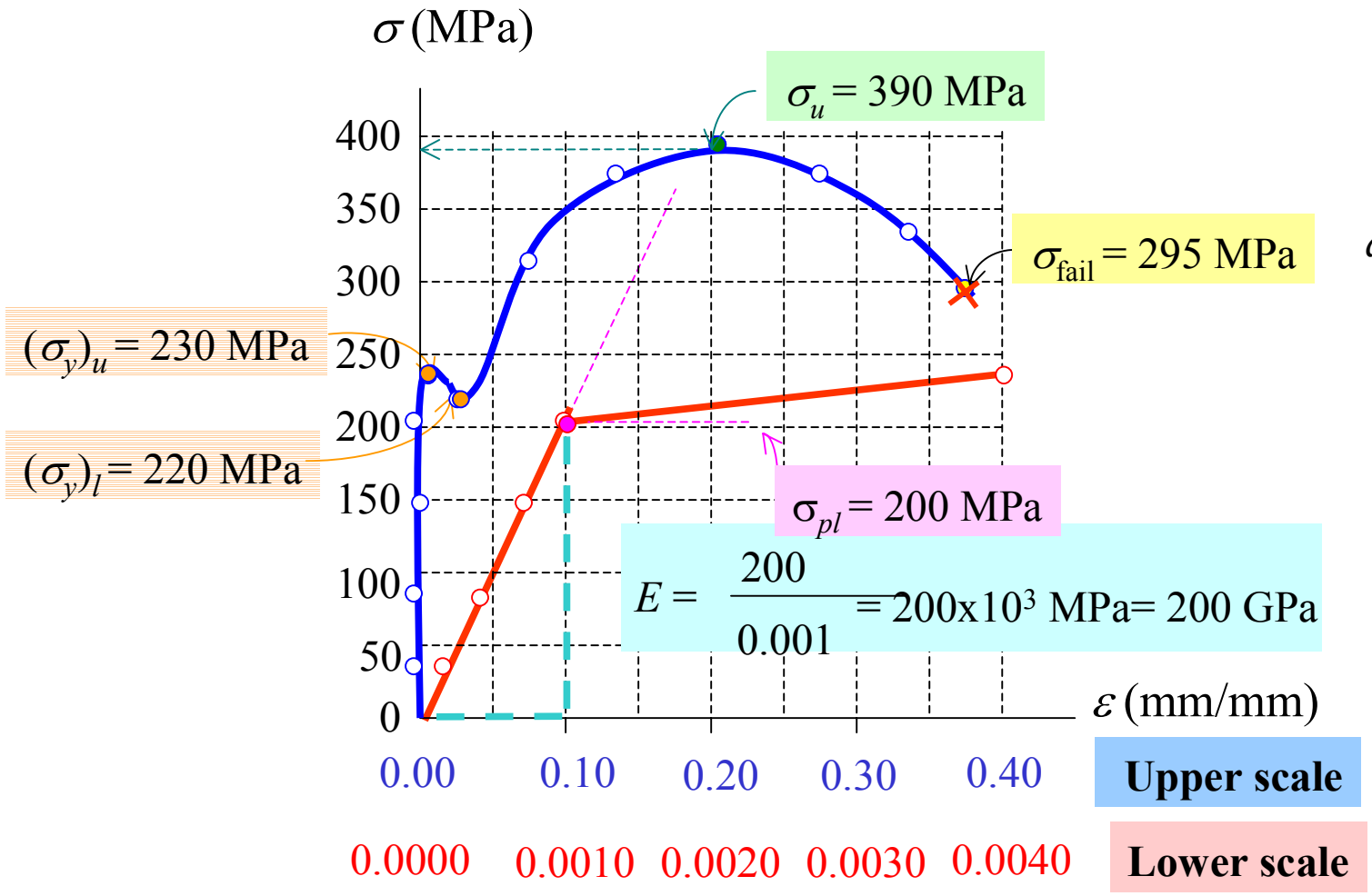
Offset Yield Stress

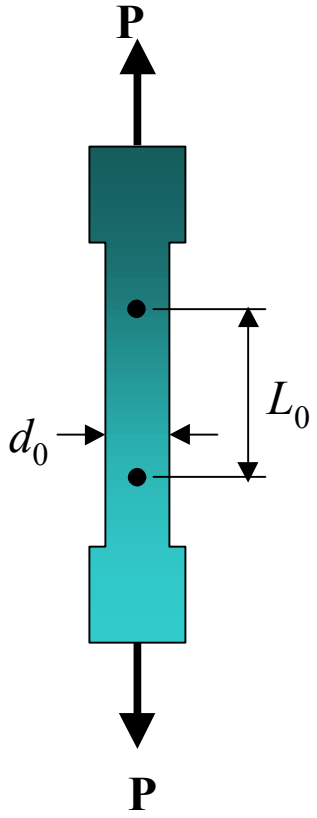


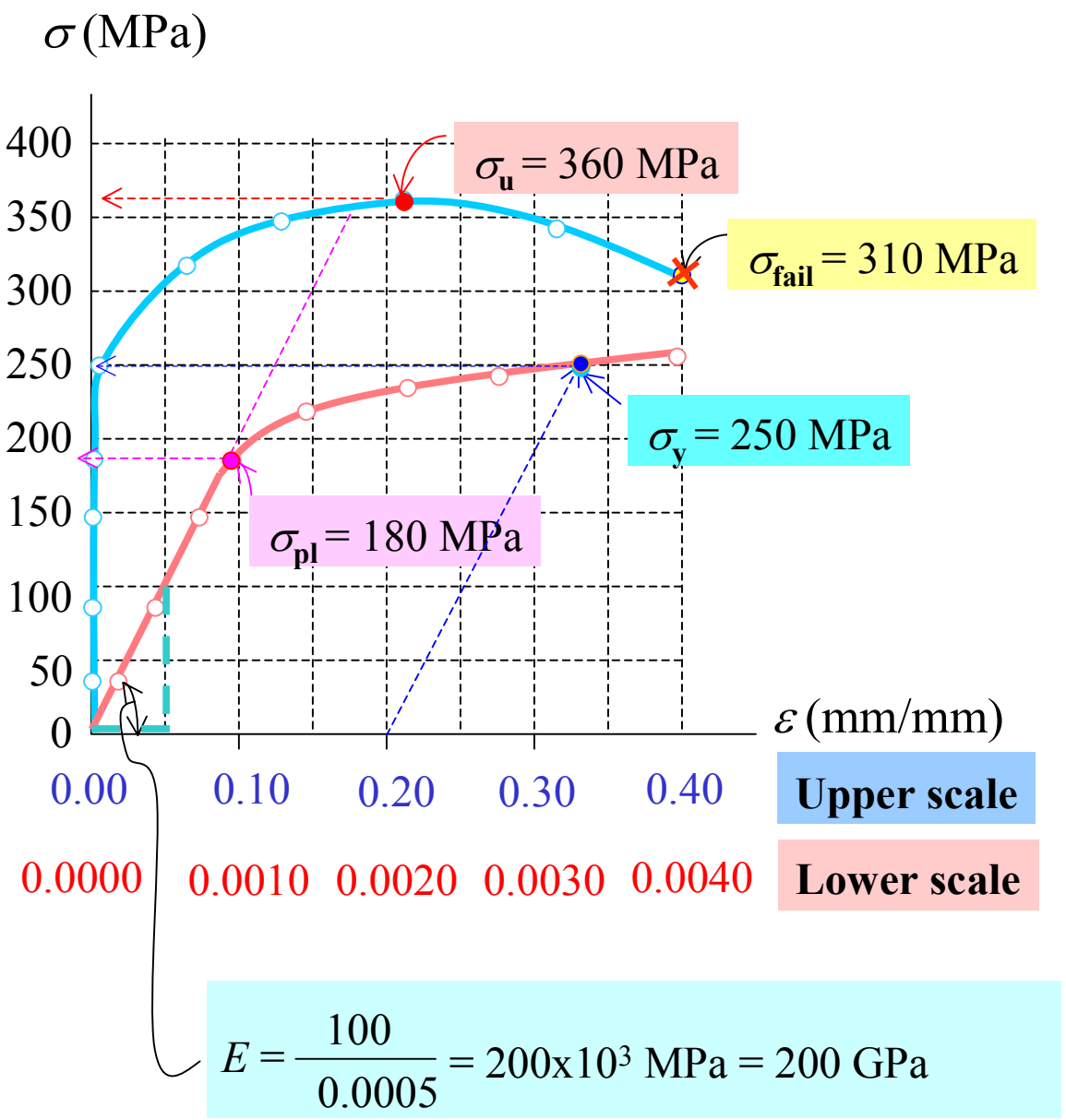
Offset yield strength for material with no yield points

$$\sigma = \frac{P}{A_0}$$

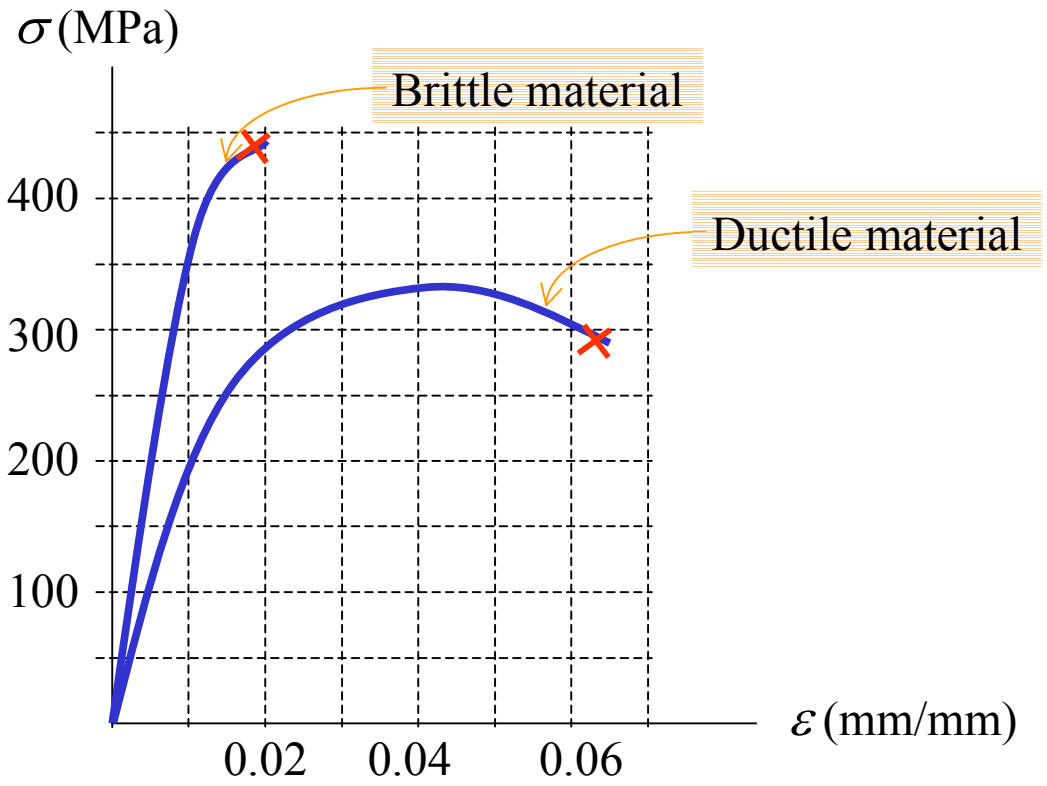
$$\varepsilon = \frac{\delta}{L_0}$$






σ (MPa)
400
350
300
250
200
150
100
50
0
σu = 360 MPa
σfail = 310 MPa
σy = 250 MPa
σpl = 180 MPa
ε (mm/mm)
0.00
0.10
0.20
0.30
0.40
Upper scale
0.0000
0.0010
0.0020
0.0030
0.0040
Lower scale
E = 100 / 0.0005 = 200x10³ MPa = 200 GPa

Stress-Strain Behavior of Ductile and Brittle Materials

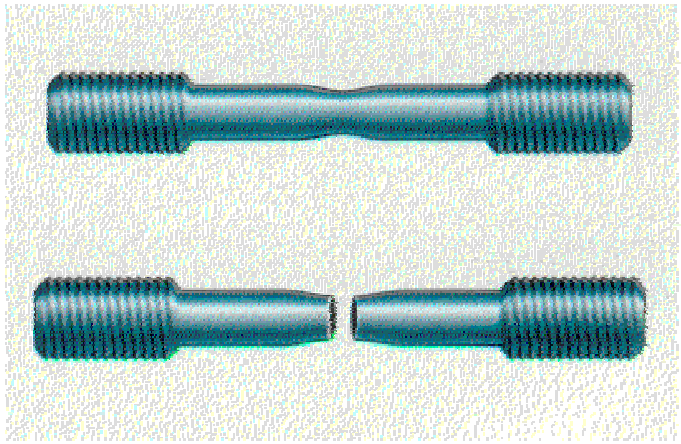


σ–ε diagrams for a methacrylate plastic

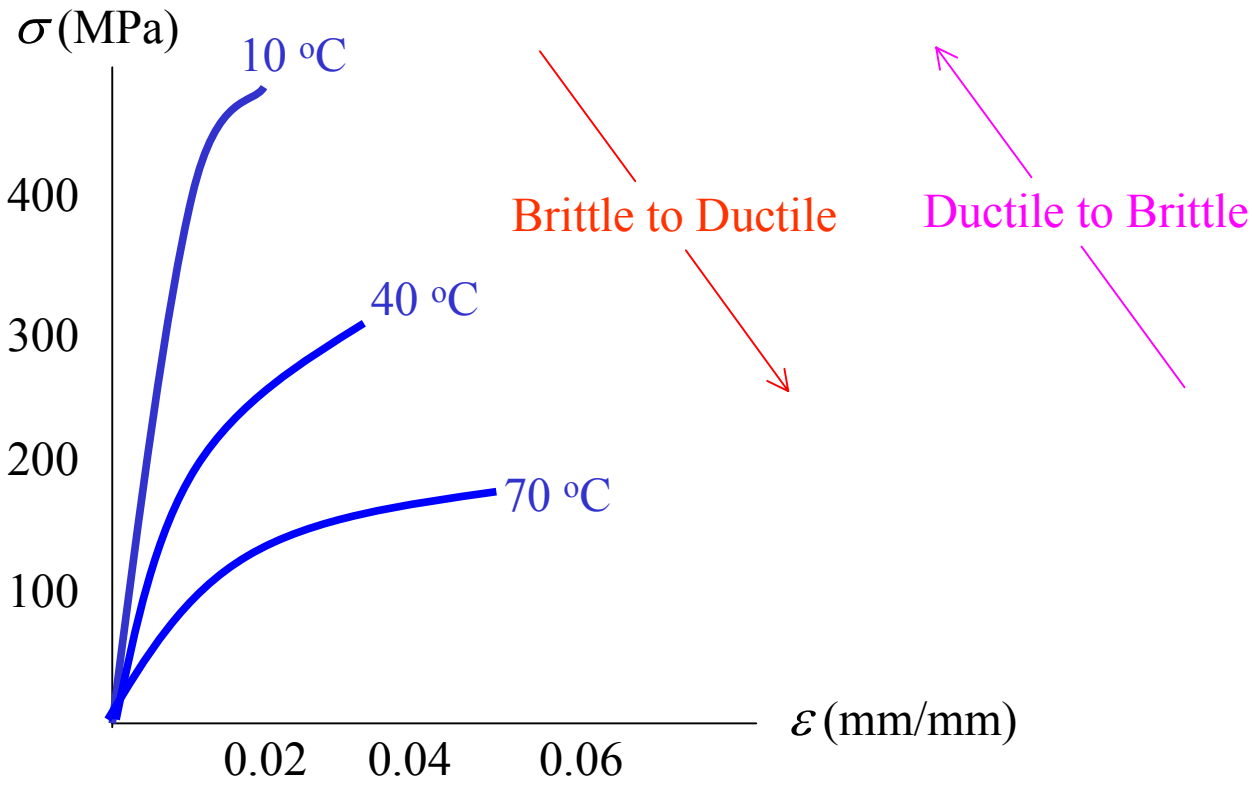
- **Elongation**

$$\textit{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%)$$

$$\textit{Percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%)$$

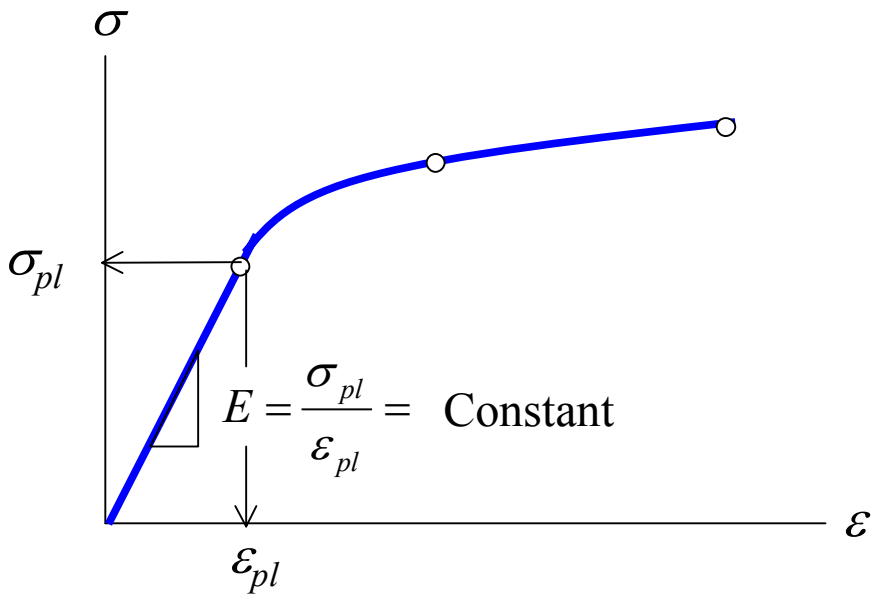


• **Temperature Effects:**



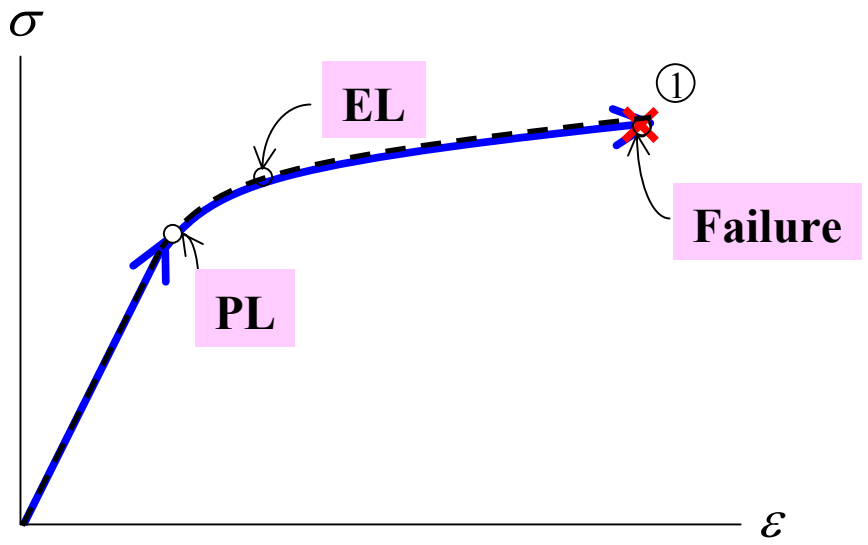
σ–ε diagrams for a methacrylate plastic

Hooke's Law

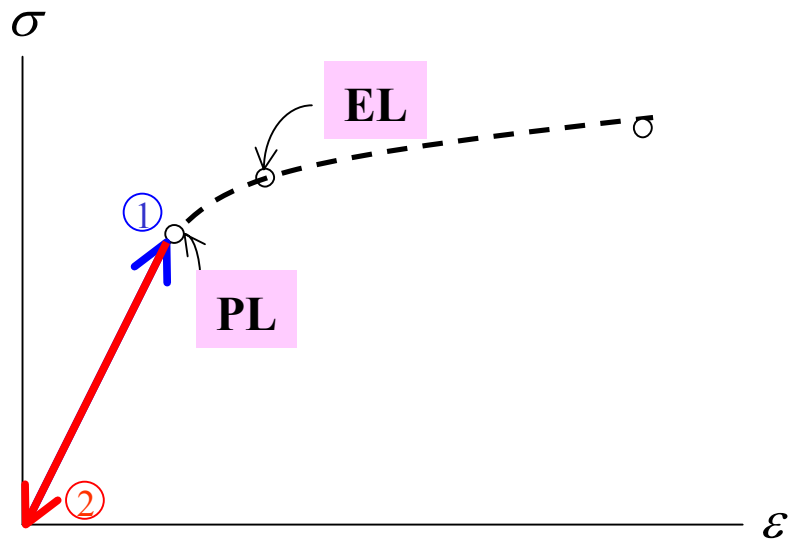

σ
σ_{pl}
$E = \frac{\sigma_{pl}}{\varepsilon_{pl}} =$ Constant
ε
ε_{pl}

• **Elastic and Plastic Behavior of Materials**

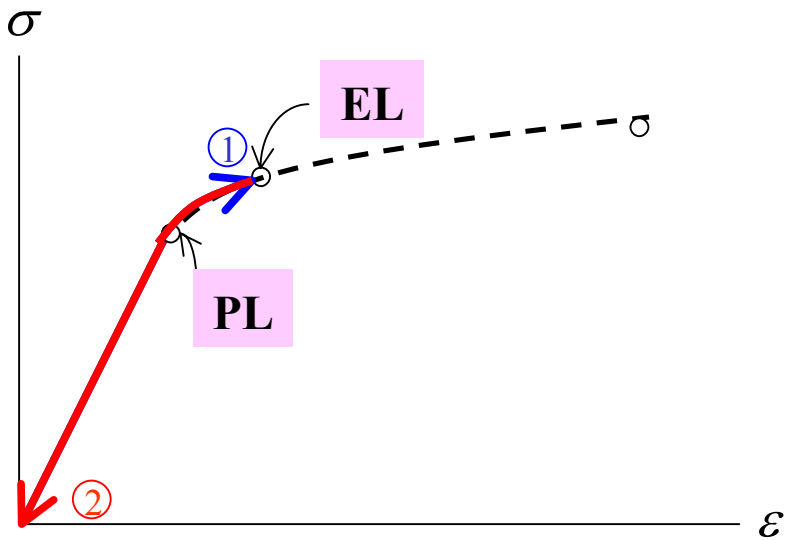
- Apply load to failure



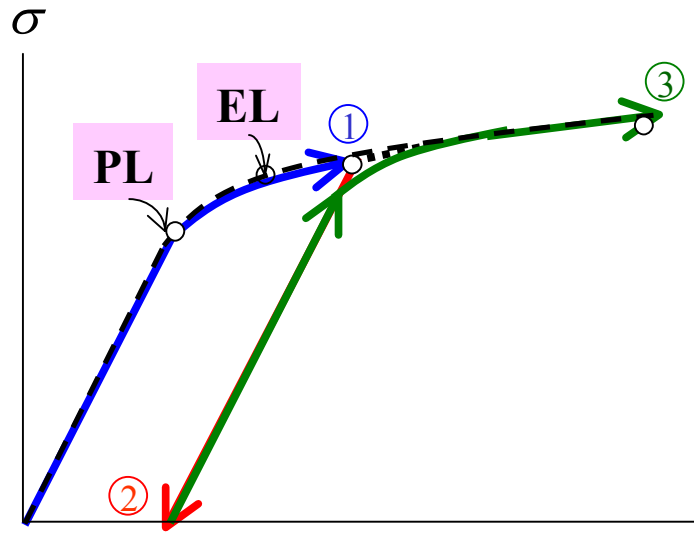
- Apply and release load



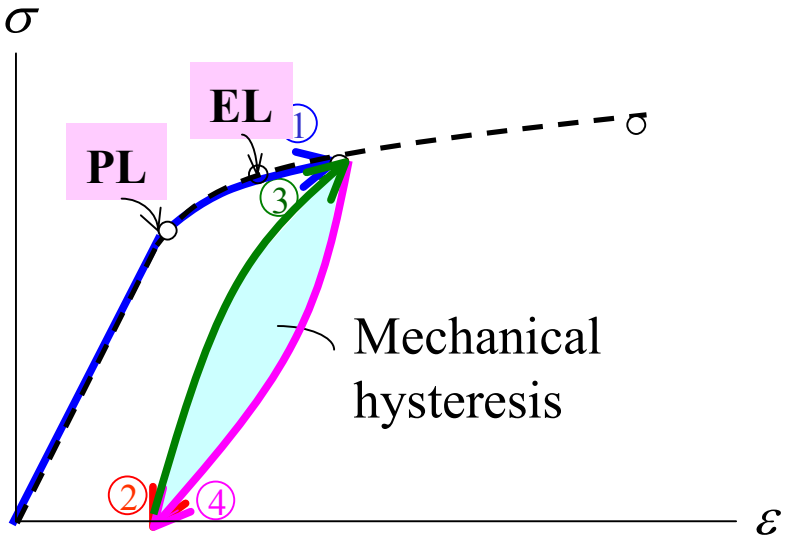
(a) *Load is less than proportional limit*



(b) *Load is more than proportional limit, but less than elastic limit*

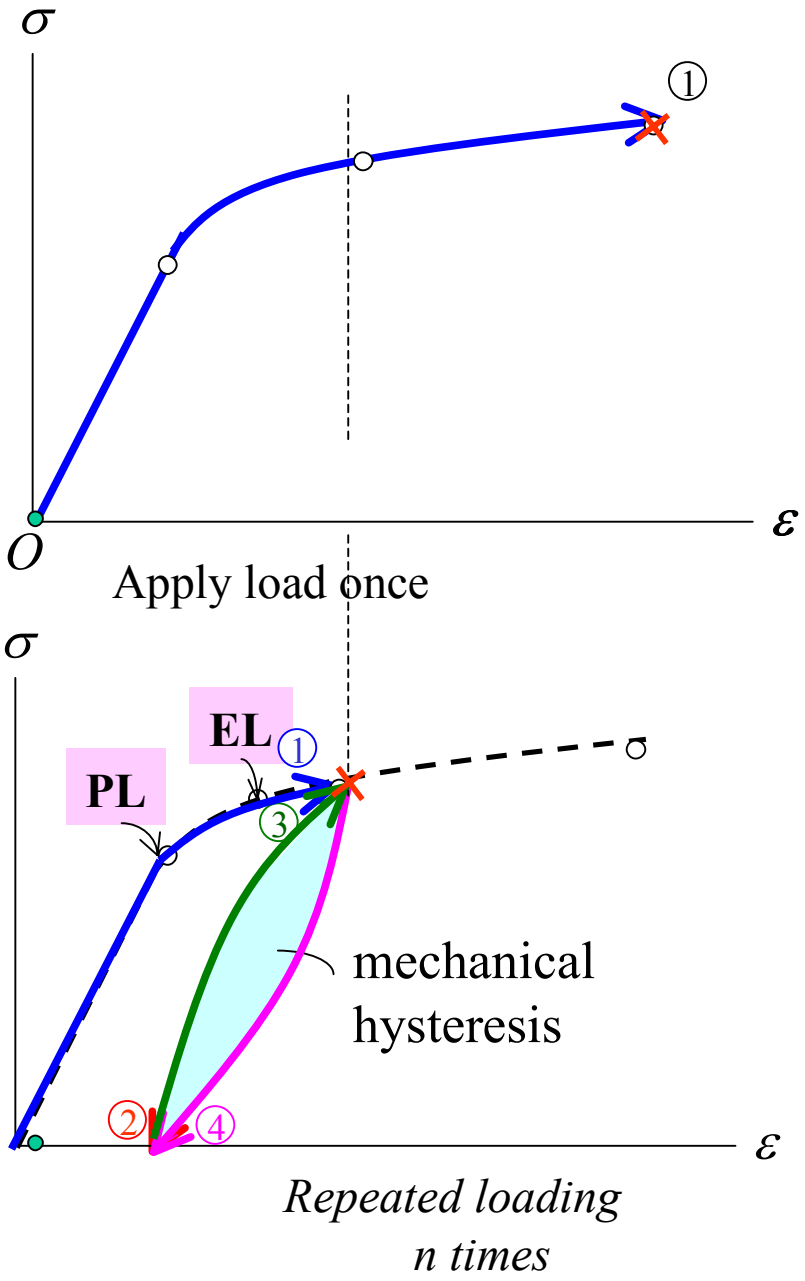


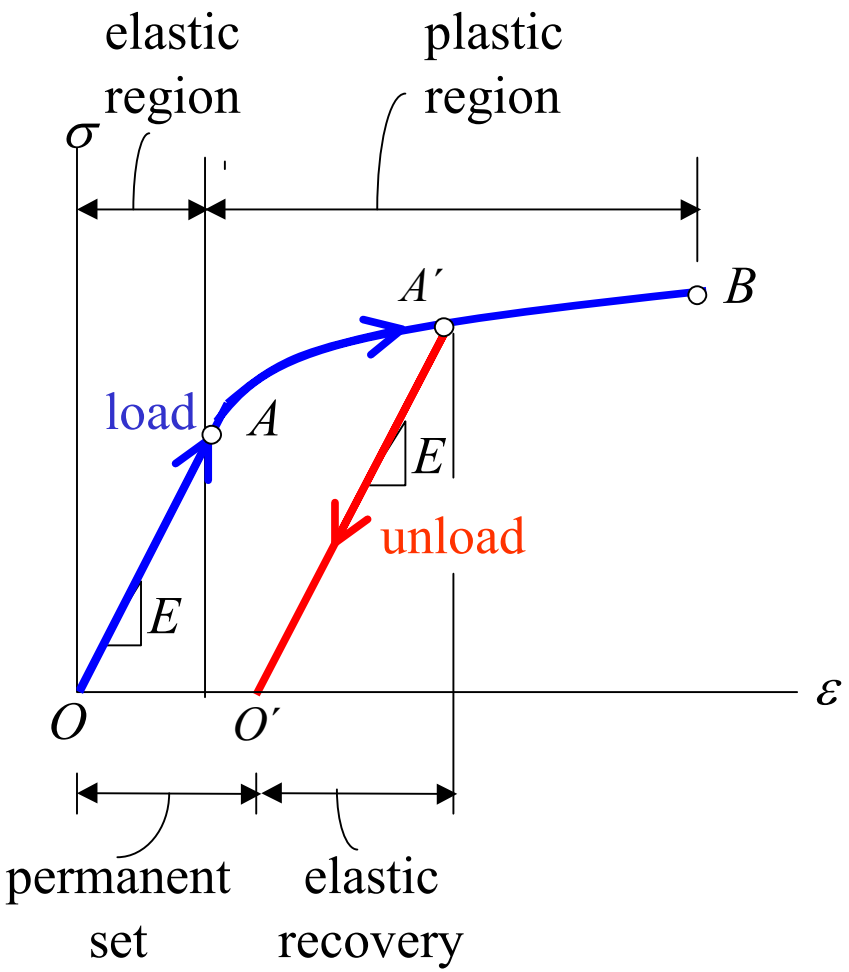
(c) *Load is more than elastic limit, and reaply*

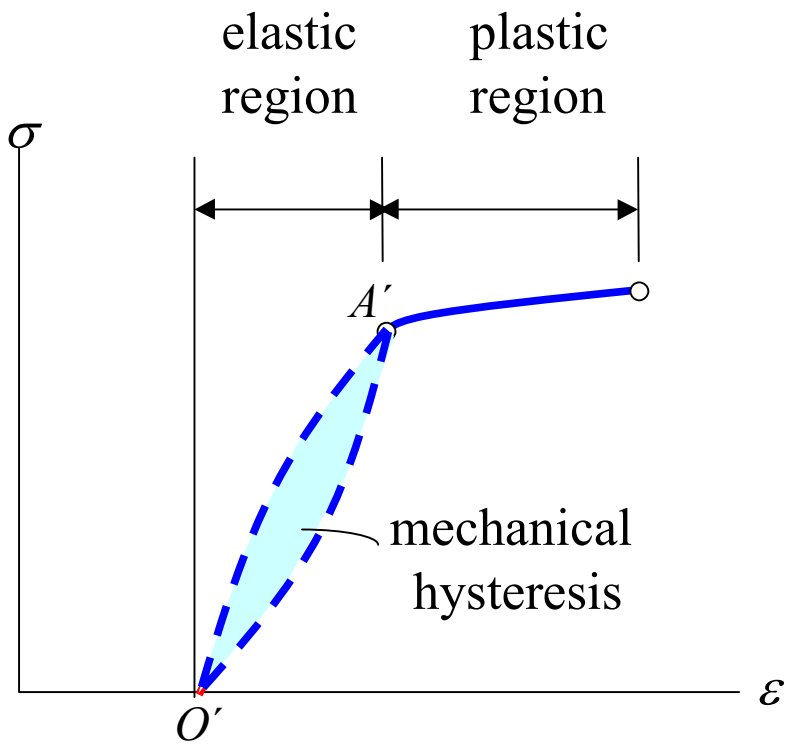


(d) *Repeated load is more than elastic limit loading*

- Comparison

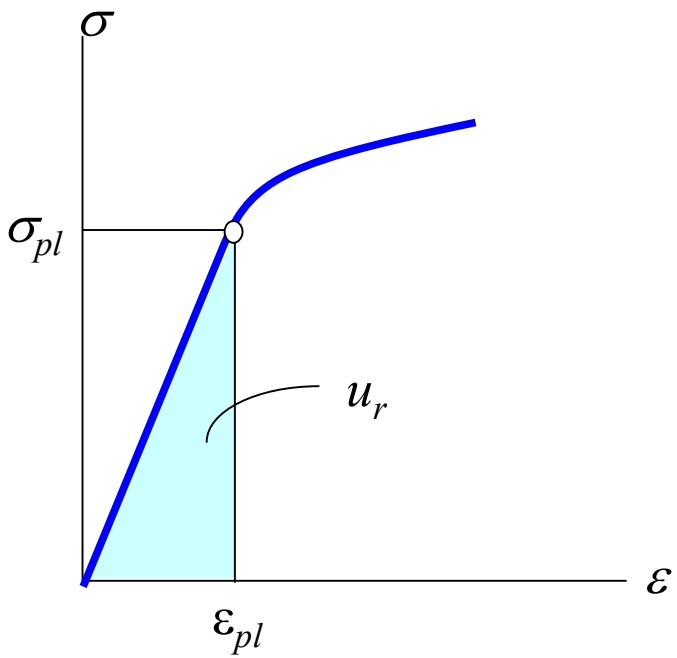

σ
①
ε
O
Apply load once
σ
EL
①
PL
③
mechanical
hysteresis
②
④
ε
Repeated loading
n times


elastic
region
plastic
region
σ
A′
B
load
A
E
unload
E
O
O′
ε
permanent
set
elastic
recovery


elastic
region
plastic
region
σ
A′
mechanical
hysteresis
O′
ε

Strain Energy

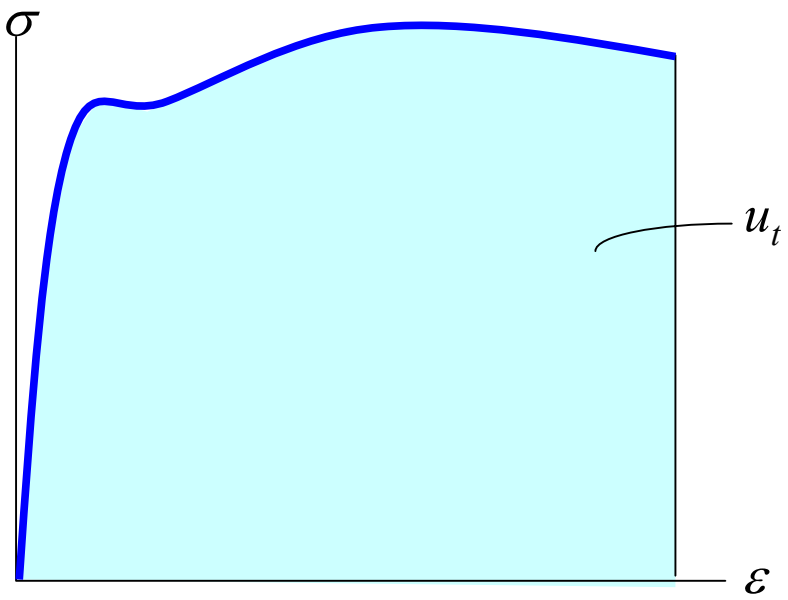
- **Modulus of Resilience**



Modulus of resilience u_r

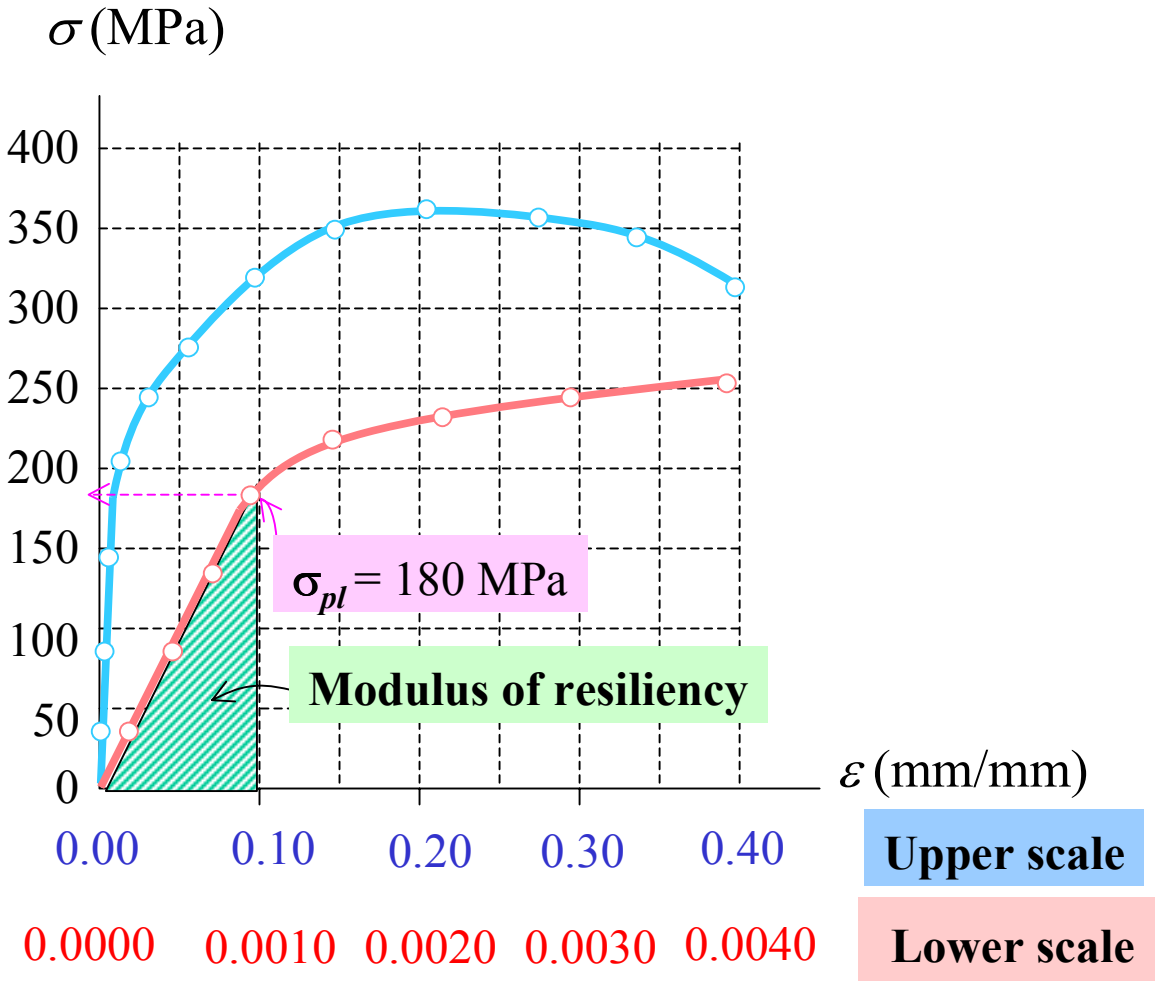
$$u_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}\frac{\sigma^2 pl}{E}$$

- **Modulus of Toughness**



Modulus of toughness u_t

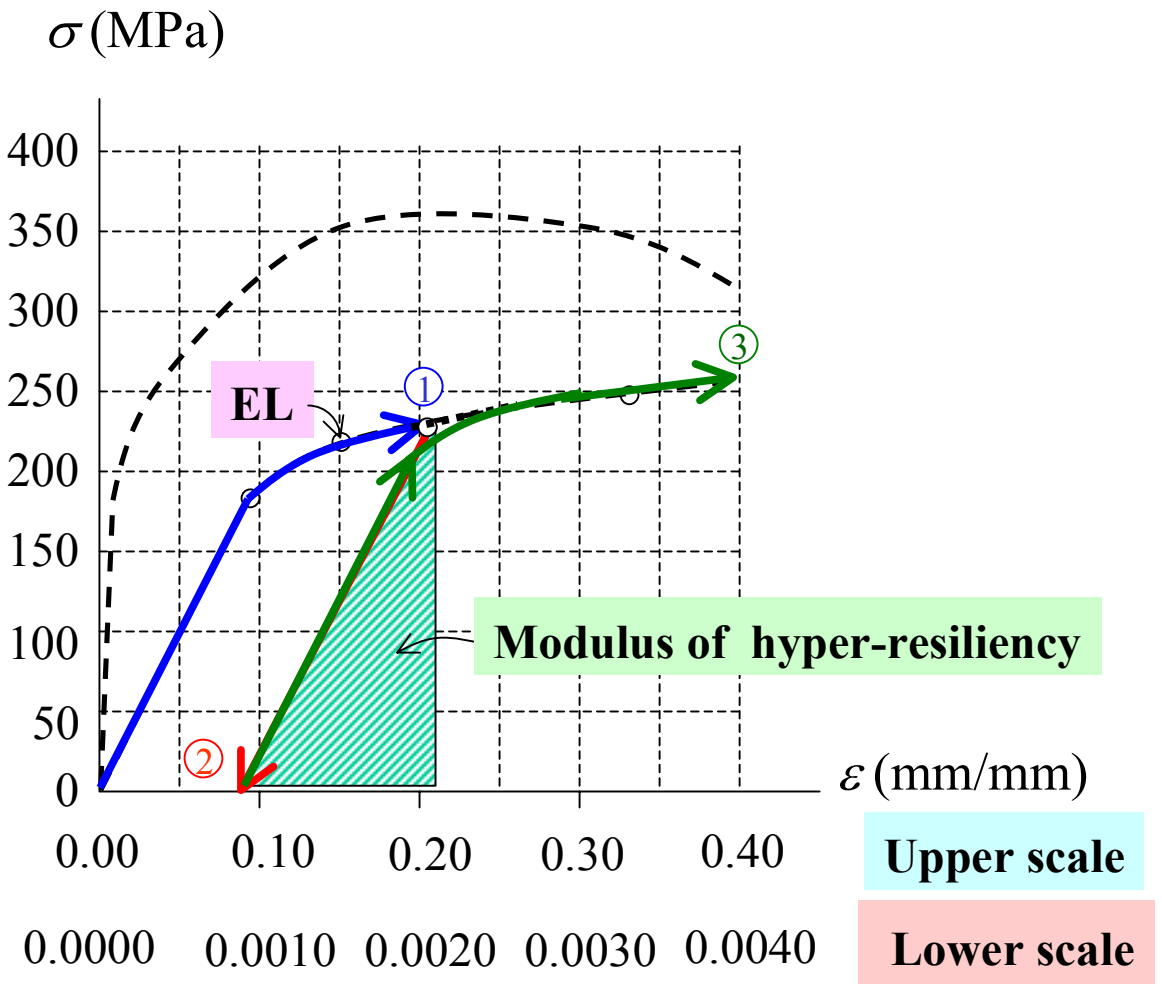
• **Modulus of Resilience**



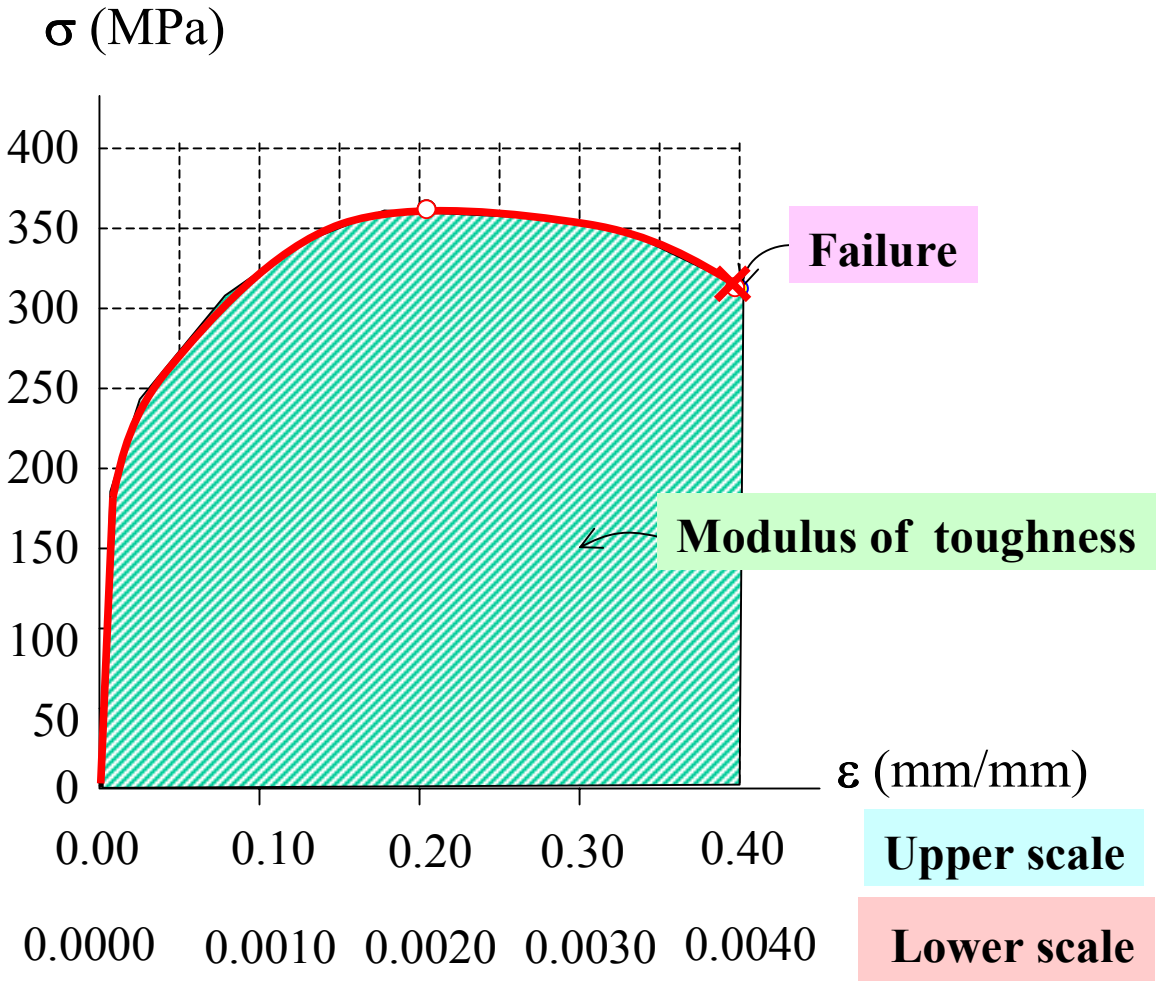
Modulus of resiliency (u_r)= Area under curve @ proportional limit

$u_r = (1/2)(0.001)(180\ \text{MPa}) = 90\ \text{kPa} = 90\ \text{kN/m}^2 = 90\ \text{kN•m/m}^3 = 90\ \text{kJ/m}^3$

Energy per unit volume $= (90\ \text{kN/m}^2)(1\ \text{m}^3) = 90\ \text{kN•m} = 90\ \text{kJ}$

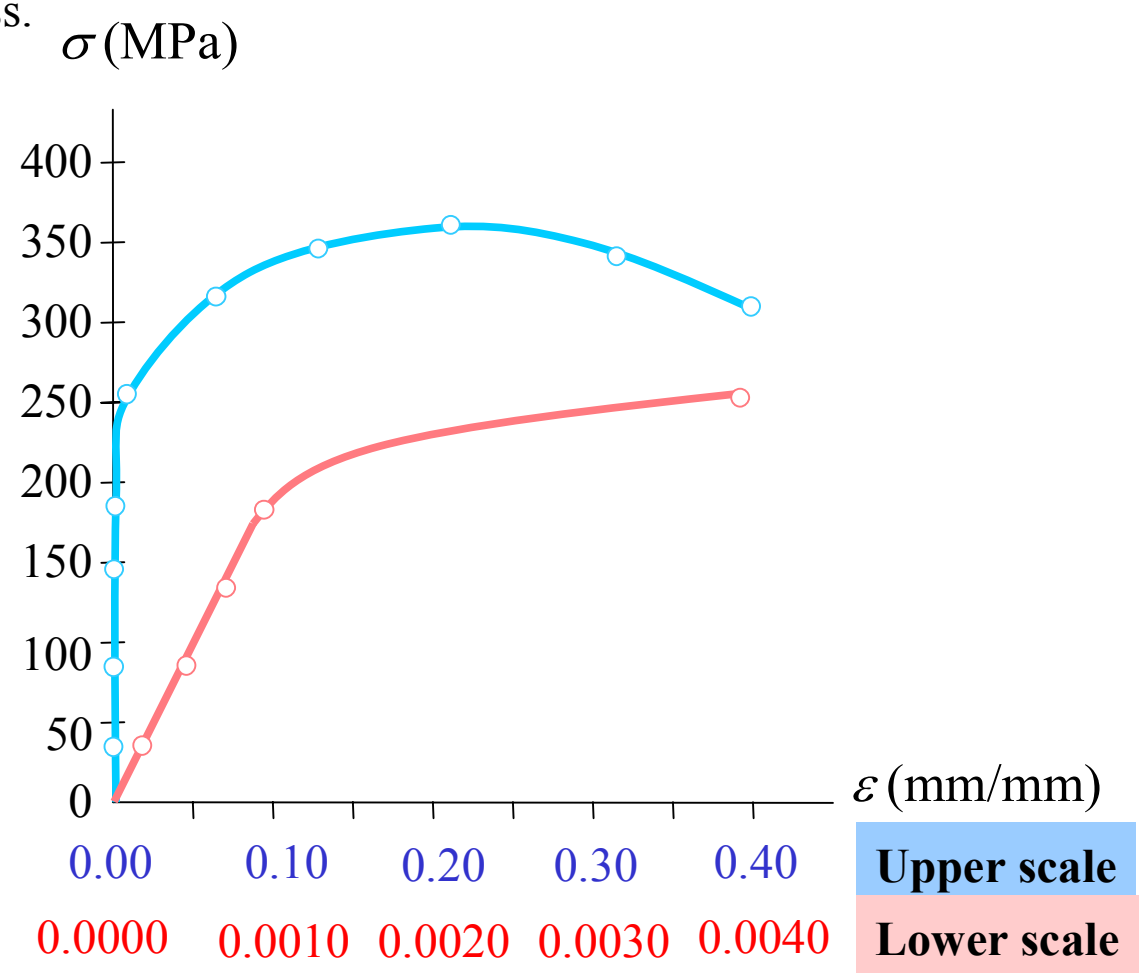

σ (MPa)
400
350
300
250
200
150
100
50
0
EL
①
②
③
Modulus of hyper-resiliency
ε (mm/mm)
0.00
0.10
0.20
0.30
0.40
0.0000
0.0010
0.0020
0.0030
0.0040
Upper scale
Lower scale

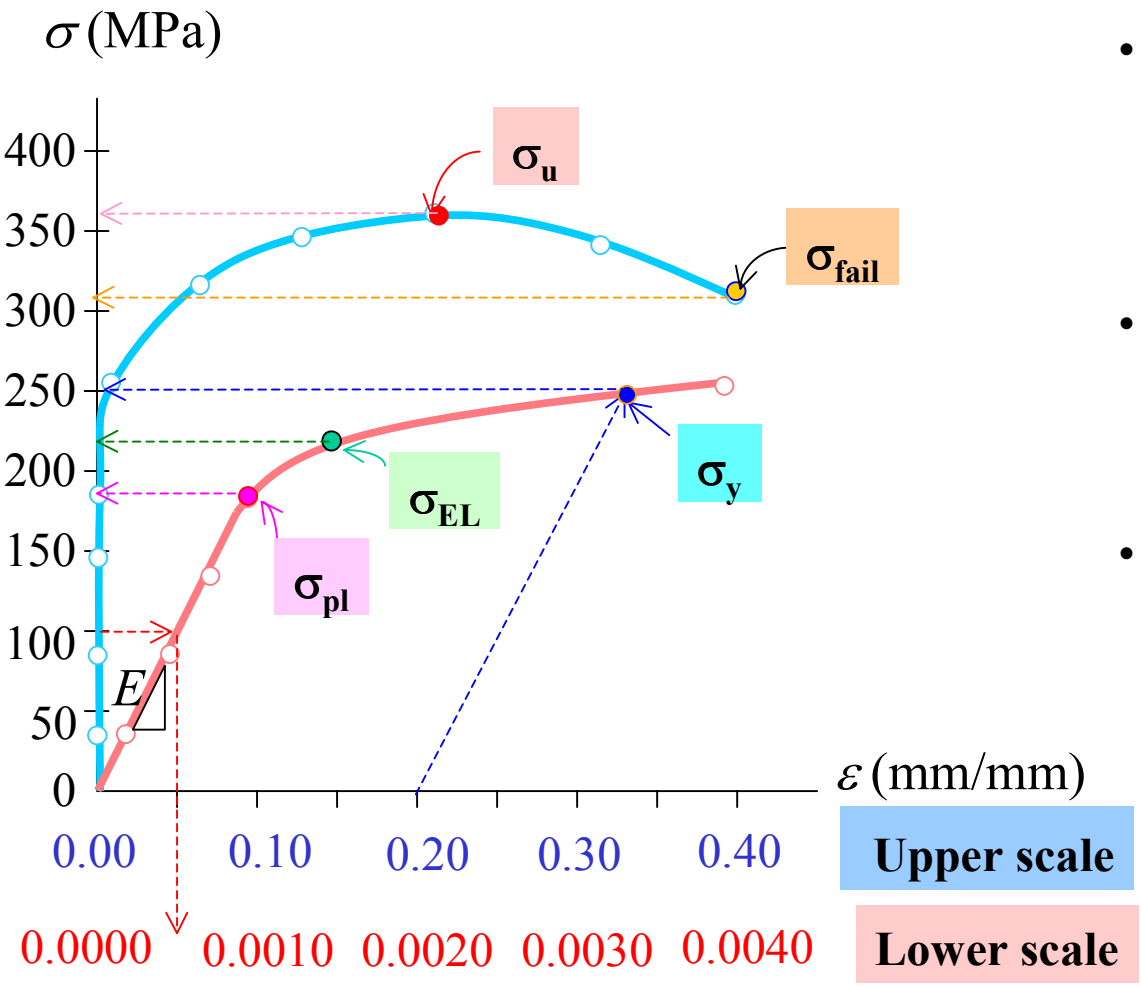
• **Modulus of Toughness**



Example 1

A tension test for a steel alloy results in the stress-strain diagram shown. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the proportional limit, elastic limit, ultimate stress and the fracture stress.





- **Modulus of Elasticity**

$$E = \frac{100\ MPa}{0.0005\ mm/mm} = 200\ GPa$$

- **Yield Strength**

$\sigma_y = 250$ MPa

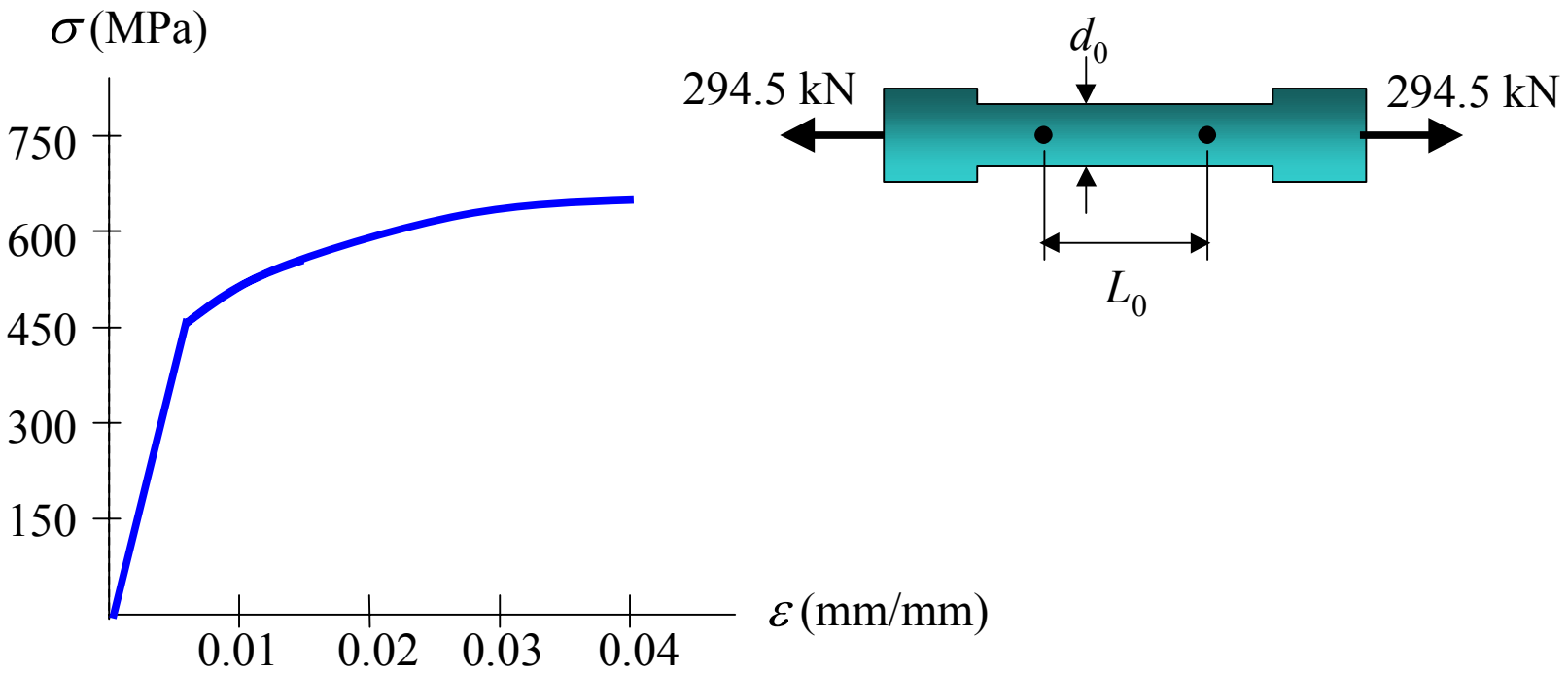
- **Proportional Limit**

$\sigma_{pl} = 180$ MPa

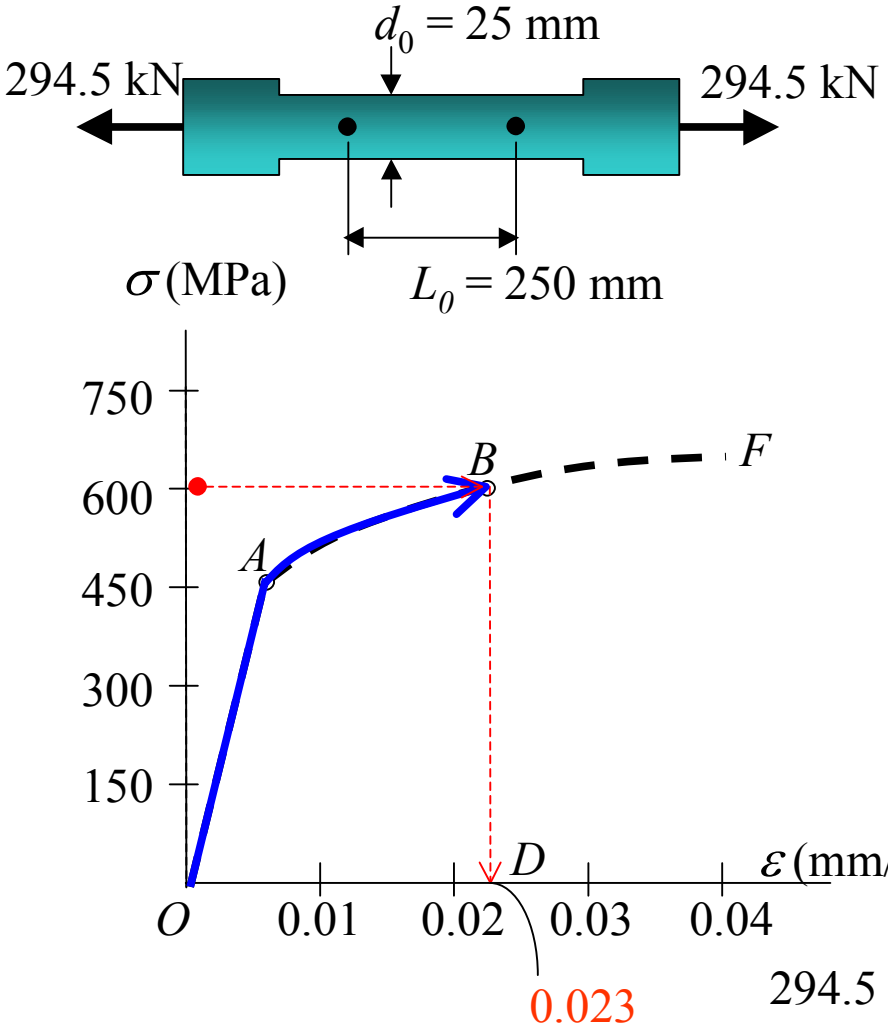
- **Elastic Limit** $\sigma_{pl} = 220$ MPa
- **Ultimate Stress** $\sigma_u = 365$ MPa
- **Fracture Stress** $\sigma_f = 310$ MPa

Example 2

An aluminum specimen shown has a diameter of $d_0 = 25$ mm, a gauge length of $L_0 = 250$ mm and is subjected to an axial load of 294.5 kN. If a portion of the stress-strain diagram for the material is shown, determine the approximate elongation of the rod when the load is applied. If the load is removed, does the rod return to its original length? Also, compute the modulus of resilience both before and after the load application.



• **The Load is *Applied***



- Normal Stress

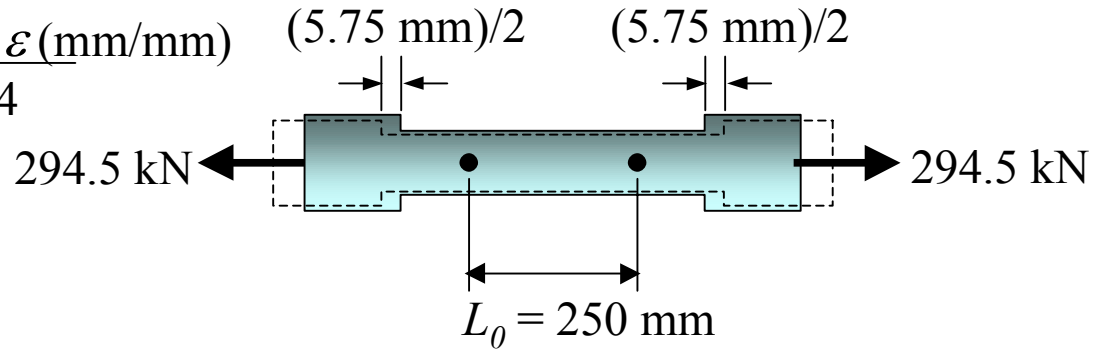
$$\sigma = \frac{P}{A_0} = \frac{294.5 \times 10^3\ N}{(\pi/4)(0.025\ m)^2} = 600\ MPa$$

- The Strain

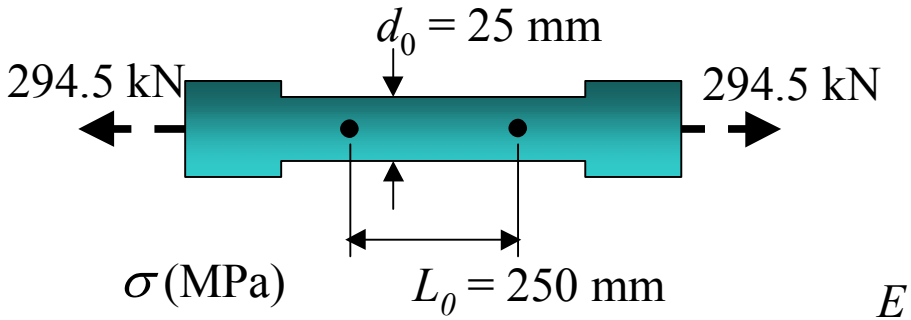
$$\varepsilon = 0.023 \text{ mm/mm}$$

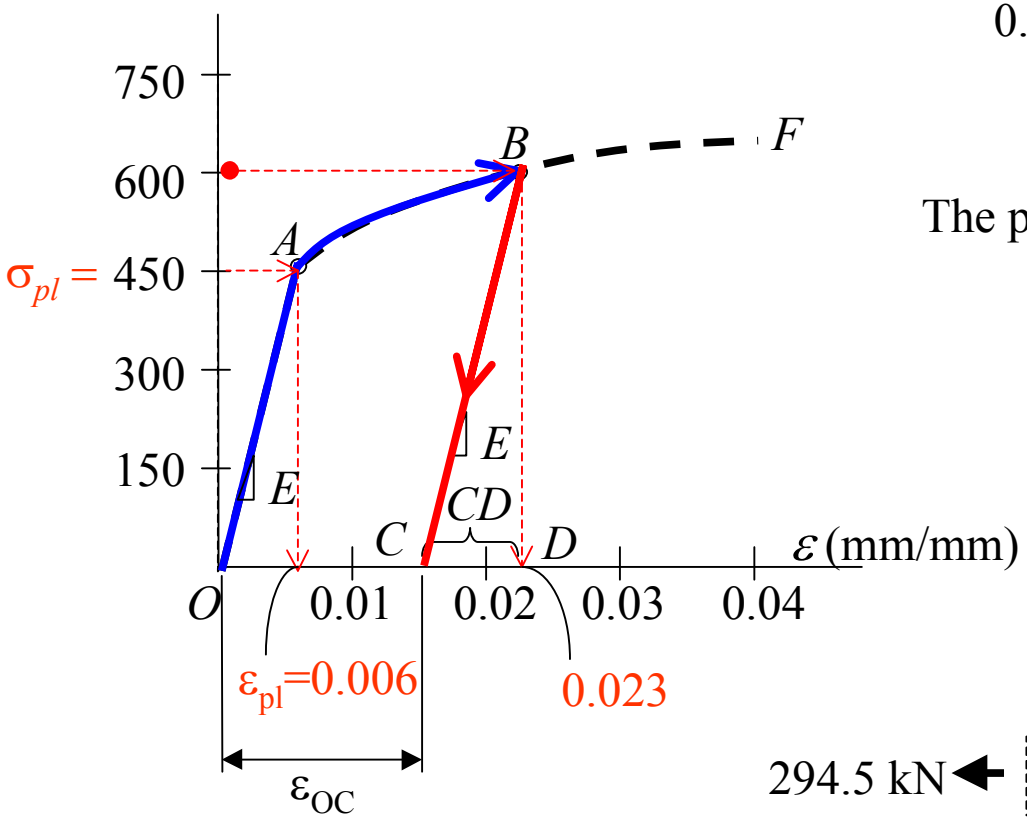
- The Elongation

$$\delta = \varepsilon L = (0.023 \text{ mm/mm})(250 \text{ mm}) = 5.75 \text{ mm}$$



• **The Load is *Removed***





- Normal Stress

$$\sigma = \frac{P}{A_0} = \frac{294.5 \times 10^3\ N}{(\pi/4)(0.025\ m)^2} = 600\ MPa$$

- Permanent Strain

$$E = \frac{450\ MPa}{0.006\ mm/mm} = 75.0\ GPa \quad = \quad \frac{600\ MPa}{CD}$$

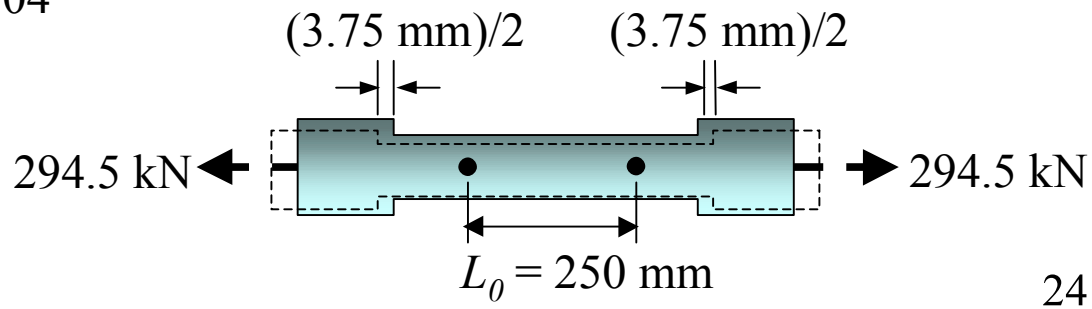
$$CD = 0.008 \text{ mm/mm}$$

The permanent strain, $\varepsilon_{OC} = 0.023 - CD$

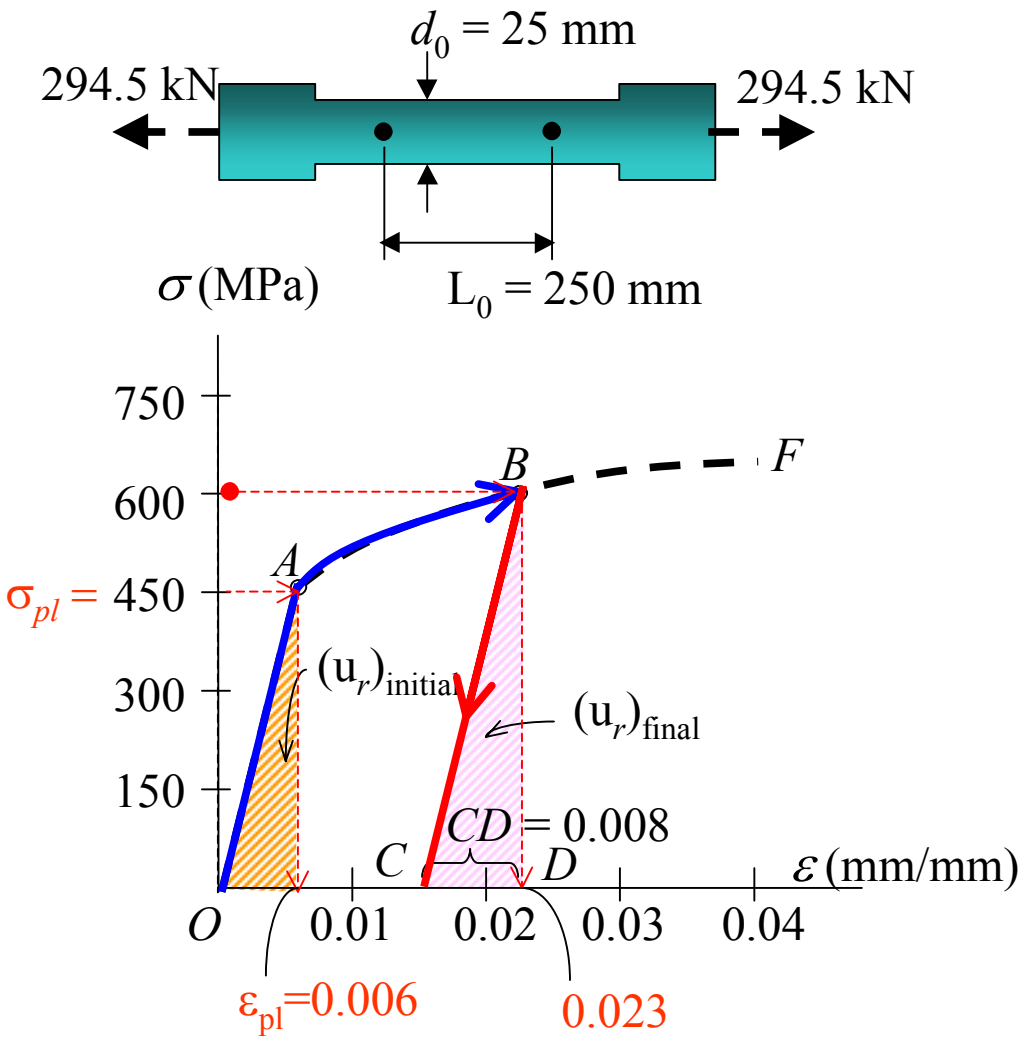
$$\varepsilon_{OC} = 0.023 - 0.008 = 0.015 \text{ mm/mm}$$

- The Permanent Elongation

$$\delta = \varepsilon L = (0.015 \text{ mm/mm})(250 \text{ mm}) = 3.75 \text{ mm}$$



• **Modulus of Resilience**



- Normal Stress

$$\sigma = \frac{P}{A_0} = \frac{294.5 \times 10^3\ N}{(\pi/4)(0.025\ m)^2} = 600\ MPa$$

- Modulus of Resilience

$$(u_r)_{initial} = \frac{1}{2}\sigma_{pl}\varepsilon_{pl}$$

$$= \frac{1}{2}(450\ MPa)(0.006\ mm/mm)$$

$$= 1.35\ MJ/m^3$$

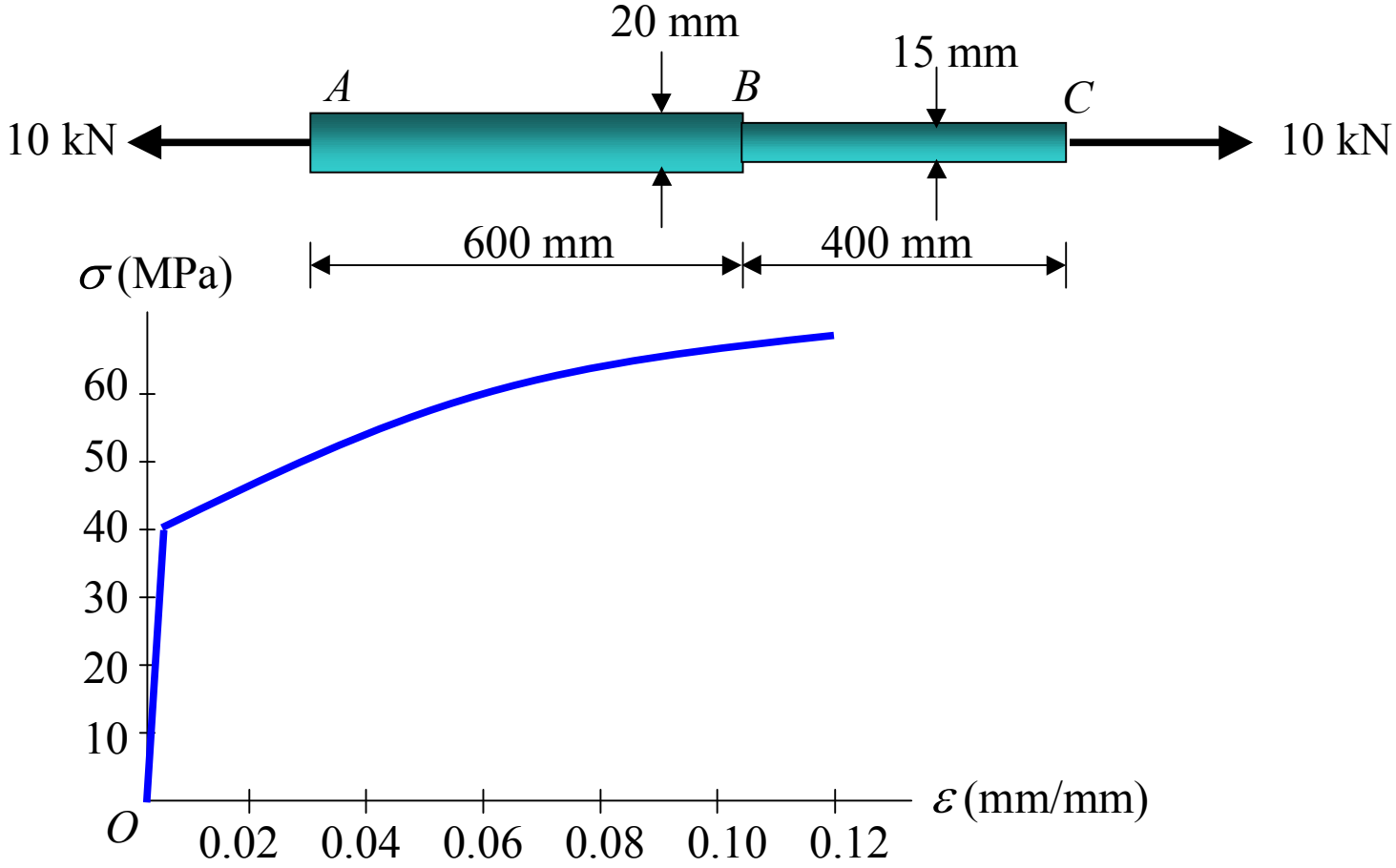
$$(u_r)_{final} = \frac{1}{2}\sigma_{pl}\varepsilon_{pl}$$

$$= \frac{1}{2}(600\ MPa)(0.008\ mm/mm)$$

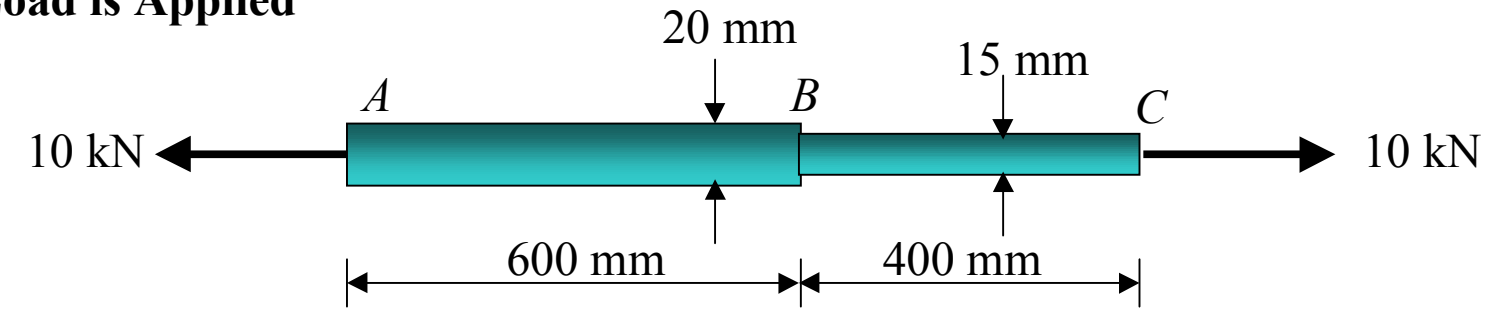
$$= 2.40\ MJ/m^3$$

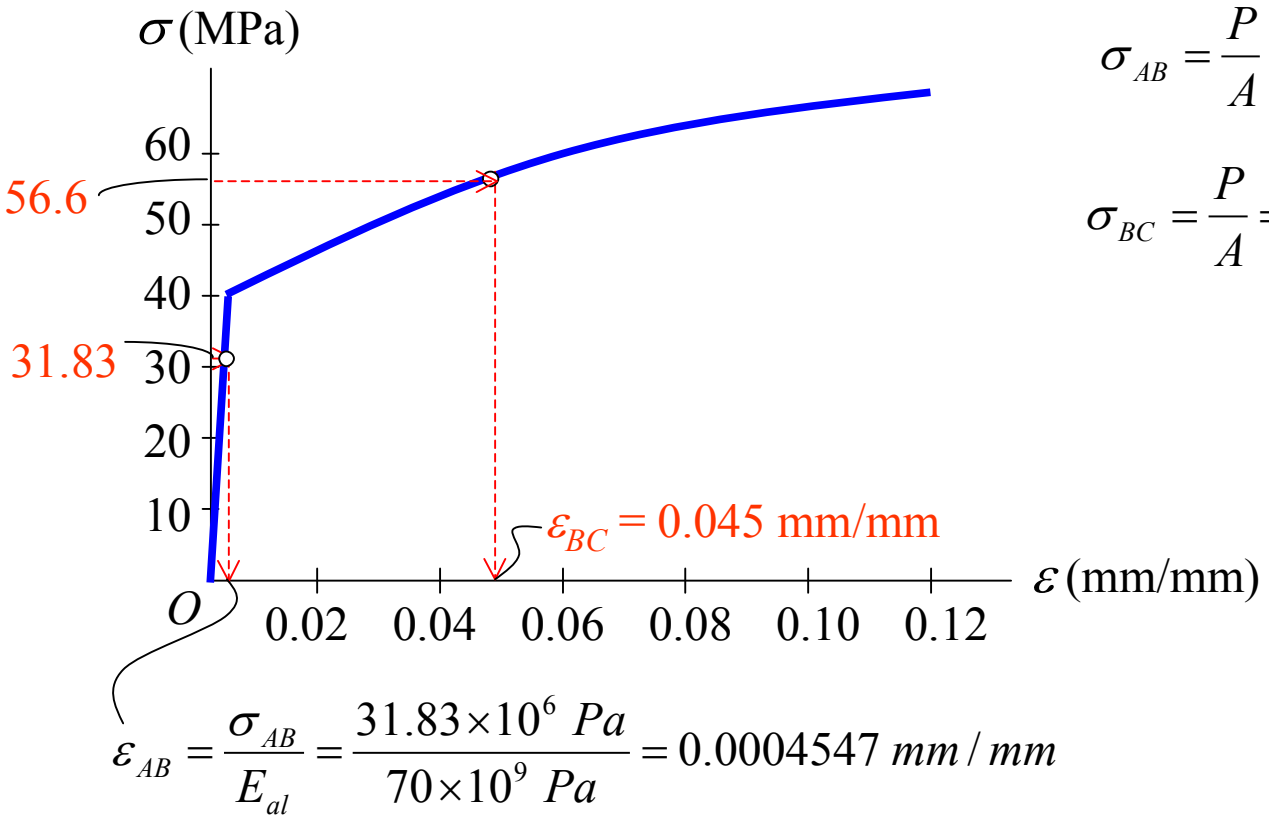
Example 3

An aluminum rod shown has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress-strain diagram for the material is shown, determine the approximate elongation of the rod when the load is applied. If the load is removed, does the rod return to its original length? Take E_{al} = 70 GPa.



• **The Load is Applied**





$$\sigma_{AB} = \frac{P}{A} = \frac{10\ kN}{\pi(0.01\ m)^2} = 31.83\ MPa$$

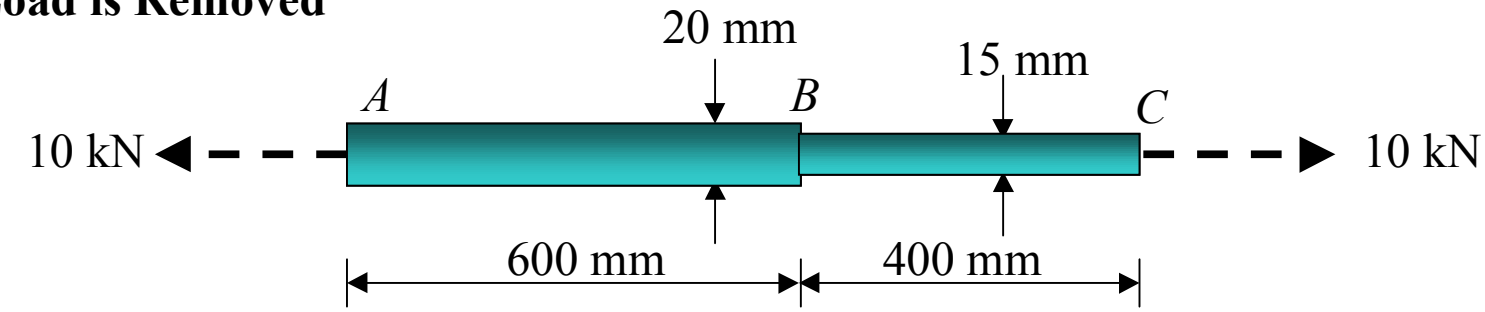
$$\sigma_{BC} = \frac{P}{A} = \frac{10\ kN}{\pi(0.0075\ m)^2} = 56.6\ MPa$$

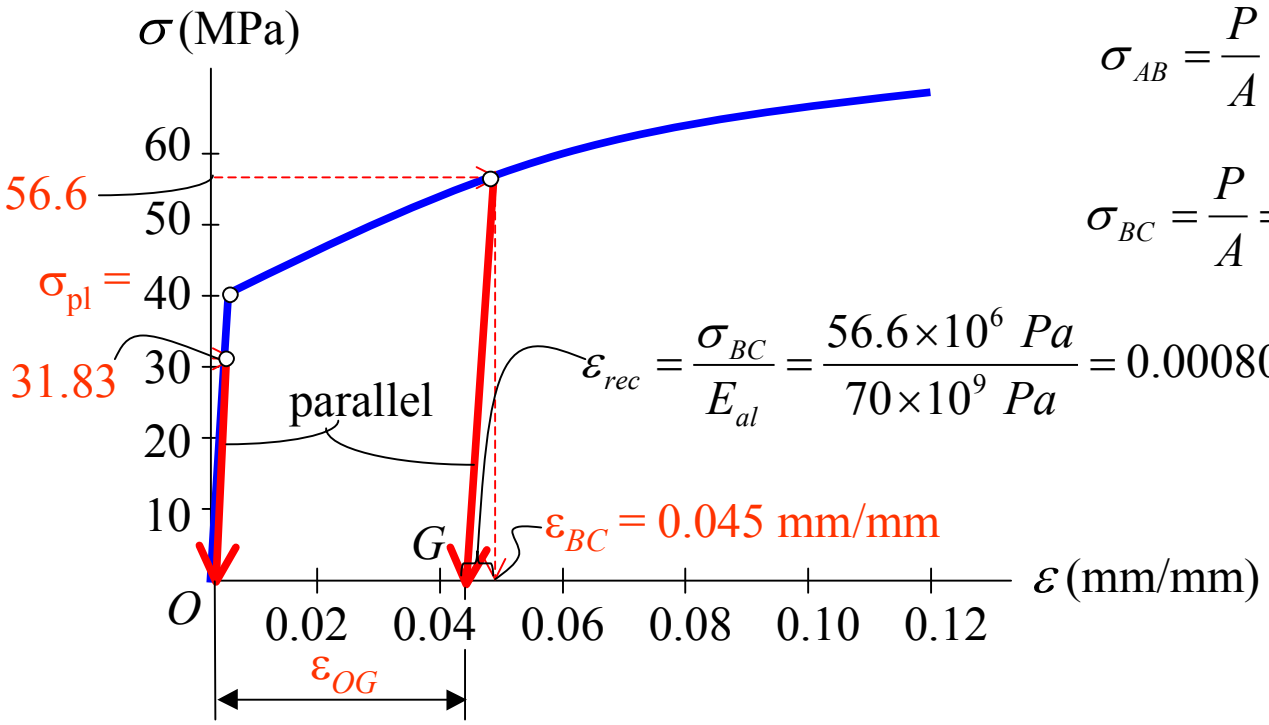
$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E_{al}} = \frac{31.83\times10^6\ Pa}{70\times10^9\ Pa} = 0.0004547\ mm/mm$$

The elongation of the rod is

$$\delta = \Sigma\ \varepsilon L = \varepsilon_{AB}L_{AB} + \varepsilon_{BC}L_{BC} = (0.0004547)(600\text{ mm}) + (0.045)(400\text{ mm}) = 18.3\text{ mm}$$

• **The Load is Removed**





$$\sigma_{AB} = \frac{P}{A} = \frac{10\ kN}{\pi(0.01\ m)^2} = 31.83\ MPa$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10\ kN}{\pi(0.0075\ m)^2} = 56.6\ MPa$$

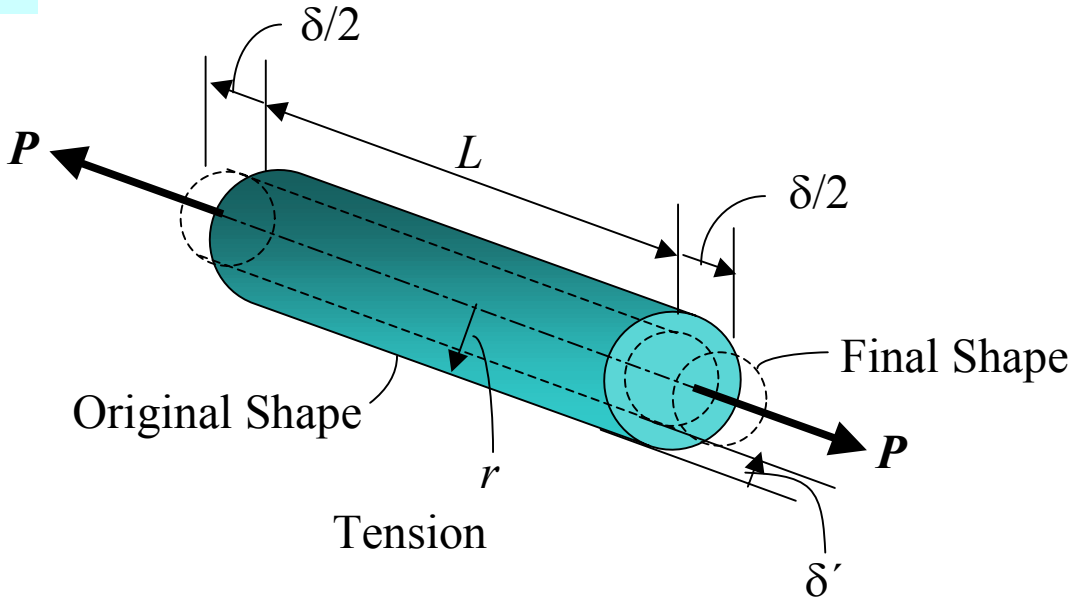
$$\varepsilon_{rec} = \frac{\sigma_{BC}}{E_{al}} = \frac{56.6\times10^6\ Pa}{70\times10^9\ Pa} = 0.000808\ mm/mm$$

The permanent strain, $\varepsilon_{OG} = 0.0450 - 0.000808 = 0.0442$ mm/mm

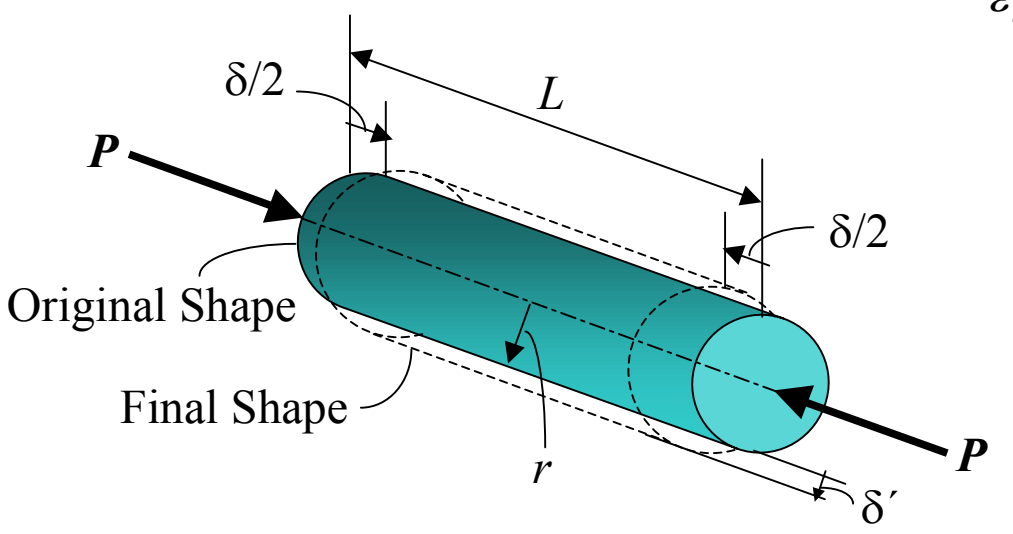
The elongation of the rod is

$$\delta' = \Sigma\ \varepsilon L = 0 + \varepsilon_{OG} L_{BC} = 0.0442(400\ \text{mm}) = 17.7\ \text{mm}$$

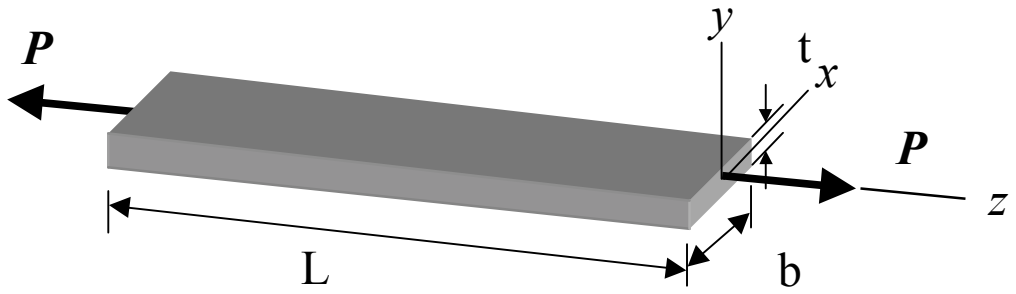
Poisson's Ratio



$$\varepsilon_{long} = \frac{\delta}{L} \quad and \quad \varepsilon_{lat} = \frac{\delta'}{r}$$



$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$



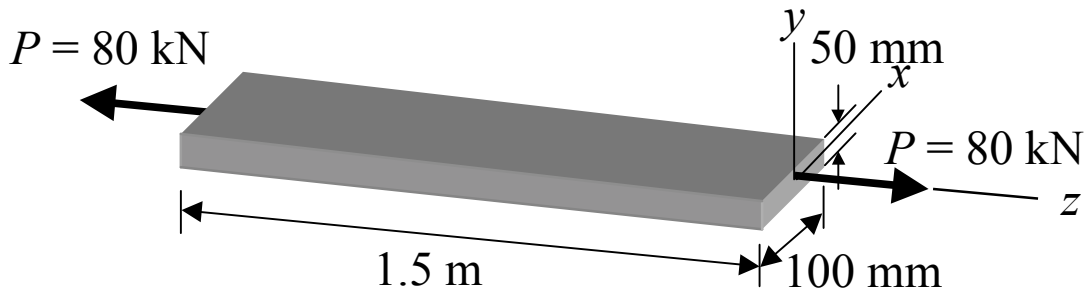
$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

Assumption:

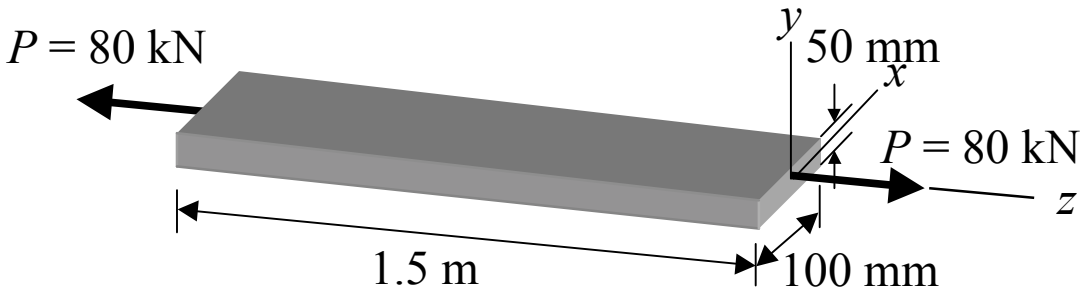
- Homogeneous
- Isotropic
- Elastic

Example 4

A bar made of A-36 steel has the dimensions shown. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically. Take $E = 200$ GPa and $\nu_{st} = 0.32$.



• **The change in the bar's length (δ)**



- *z* direction

$$\sigma_z = \frac{P}{A} = \frac{80\ kN}{(0.1\ m)(0.05\ m)} = 16\ MPa$$

$$E_{st} = \frac{\sigma_z}{\varepsilon_z}$$

$$200(10^9)\ Pa = \frac{16(10^6)\ Pa}{\varepsilon_z}$$

$$\varepsilon_z = 80(10^{-6})\ mm/mm = 80\mu$$

$\delta_z = \varepsilon_z L_z = [80(10^{-6})(1.5\ m)] = 120\ \mu m$

- *x* and *y* direction

$$Poisson's\quad Ratio:$$

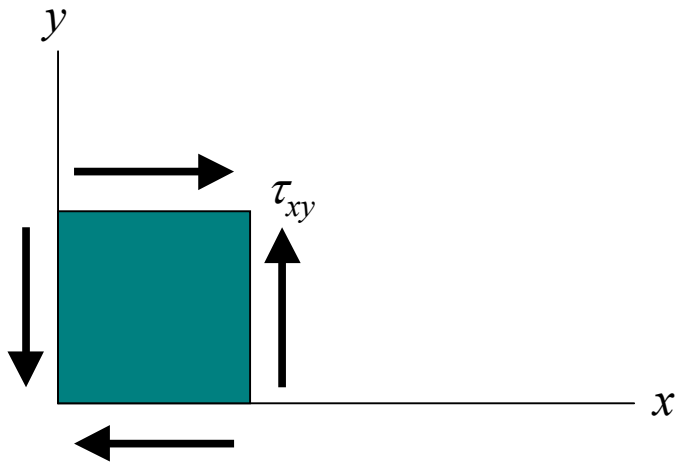
$$\nu_{st} = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

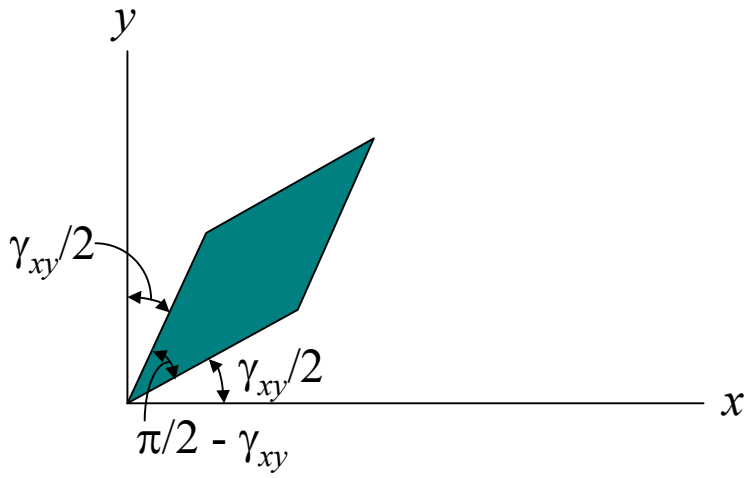
$\varepsilon_x = \varepsilon_y = -\nu_{st}\varepsilon_z = -0.32[(80(10^{-6})] = -25.6\ \mu$

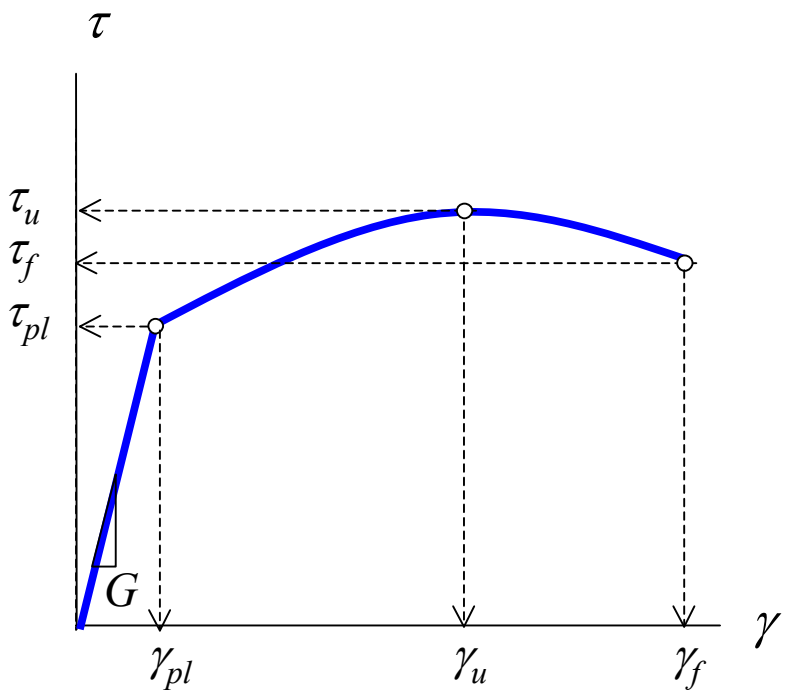
$\delta_x = -\varepsilon_x L_x = -[25.6(10^{-6})(0.1\ m) = -2.56\ \mu m$

$\delta_y = -\varepsilon_y L_y = -[25.6(10^{-6})(0.05\ m) = -1.28\ \mu m$

The Shear Stress-Strain Diagram





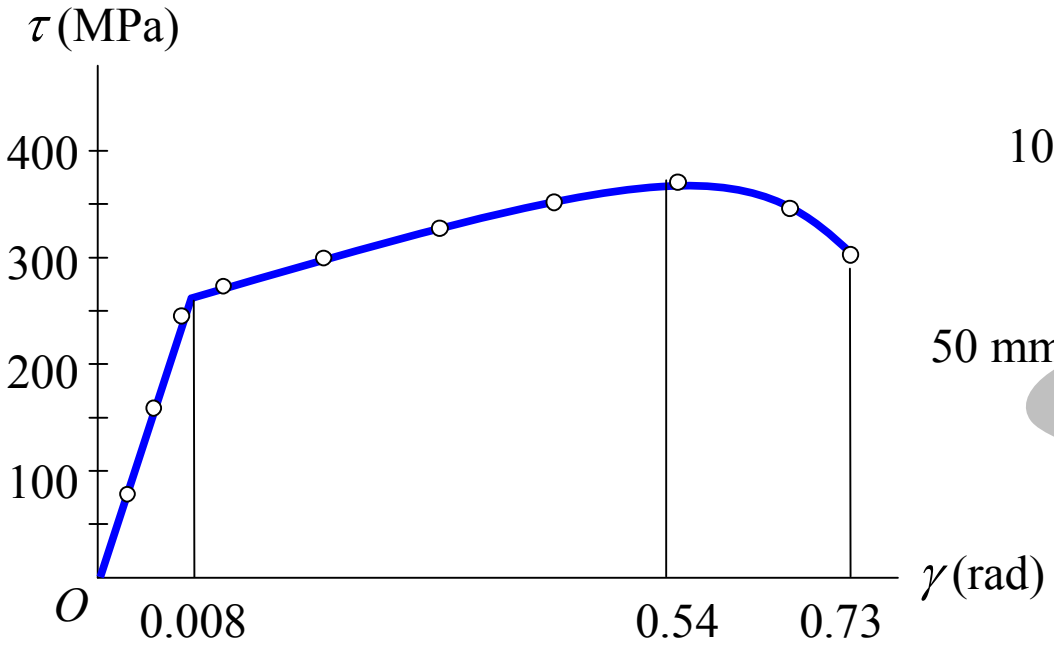


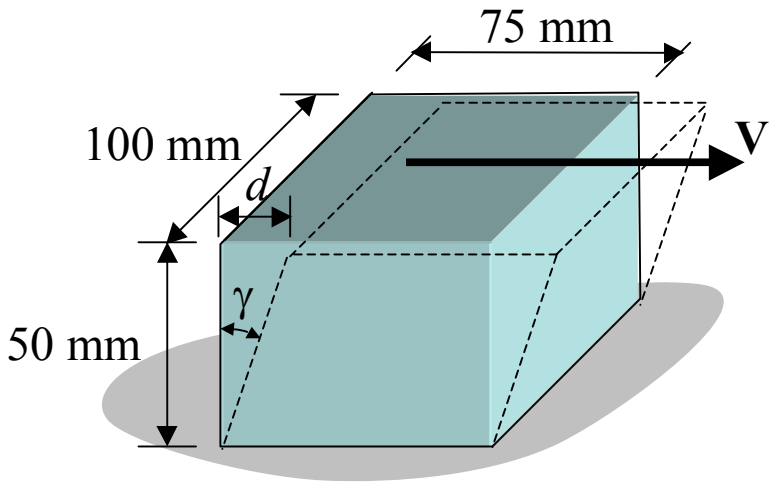
$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

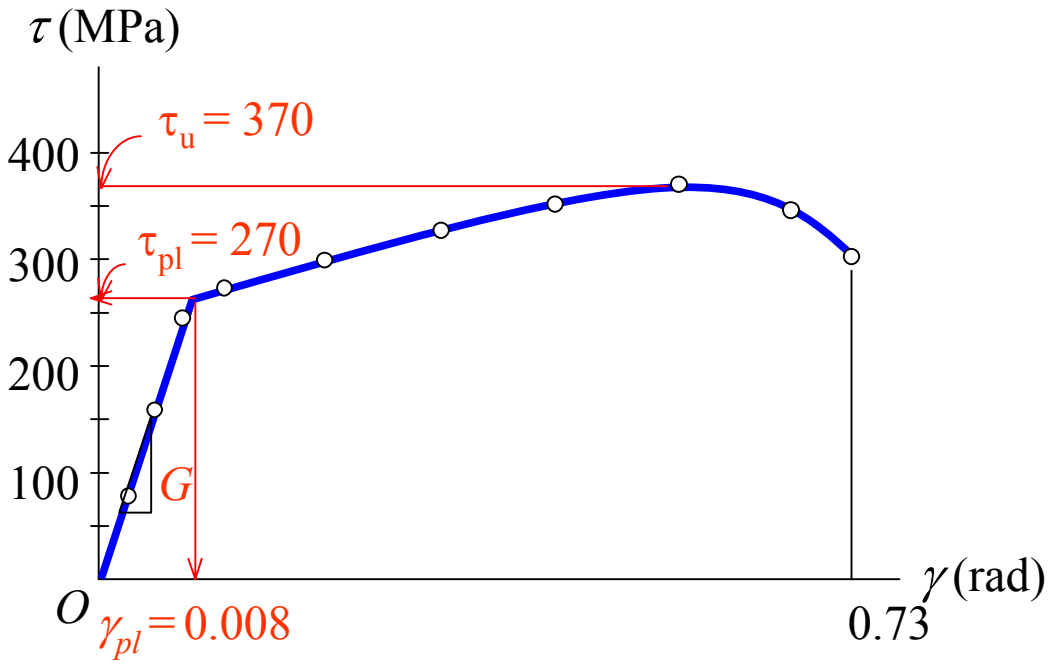
Example 5

A specimen of titanium alloy is tested in torsion and the shear stress-strain diagram is shown.
(a) Determine the shear modulus *G*, the proportional limit, and the ultimate shear stress.
(b) Determine the distance *d* that the top of a block of this material, shown, could be displaced horizontally by a shear force **V** of 135 MN .





(a) The shear modulus, proportional limit, and the ultimate shear stress.

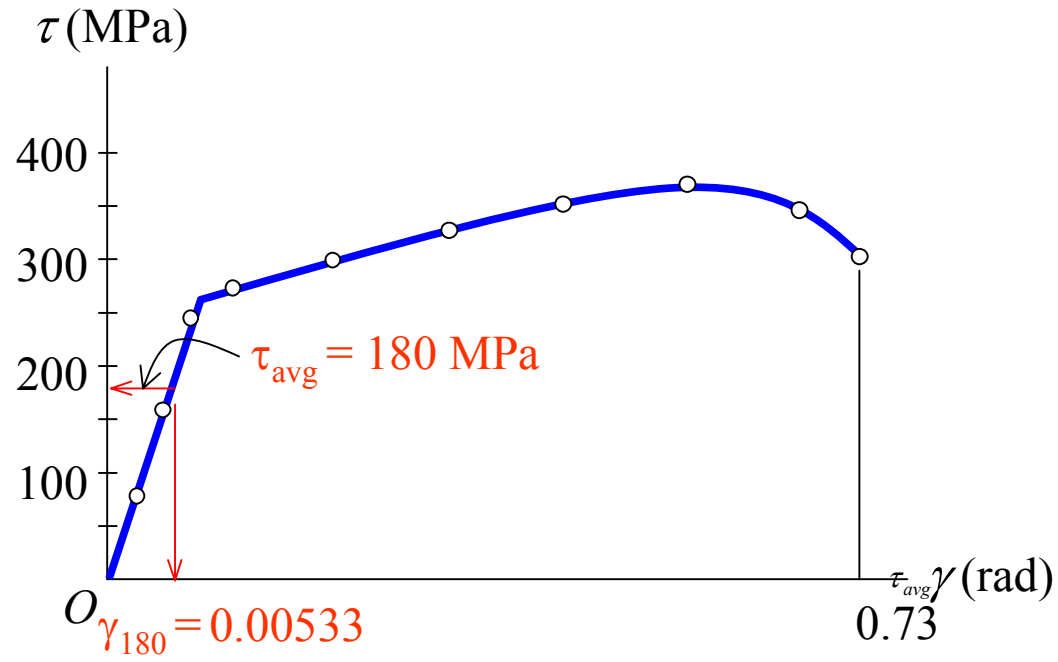


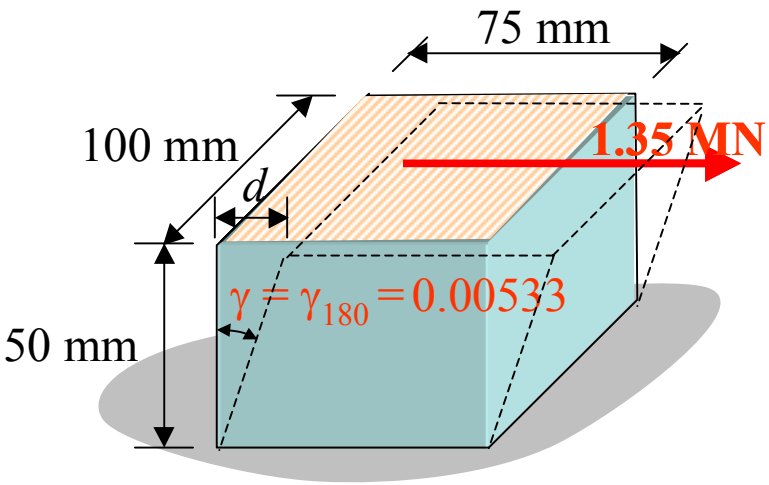
- Proportional limit ; $\tau_{pl} = 270$ MPa

- Shear Modulus ; $G = \dfrac{270\ MPa}{0.008\ rad} = 33.75\ GPa$

- Ultimate shear stress; $\tau_u = 370$ MPa

(b) The maximum distance *d* and the magnitude of V if the material behaves elastically





- Shear stress τ

$$\tau_{avg} = \frac{V}{A}$$

$$\tau_{avg} = \frac{1.35MN}{(0.1\,m \times 0.075\,m)}$$

$$= 180 \text{ MPa}$$

- Shear strain γ

$$\gamma = \frac{\tau_{avg}}{G}$$

$$\gamma = \frac{180MPa}{(33.75GPa)}$$

$$= 0.005333$$

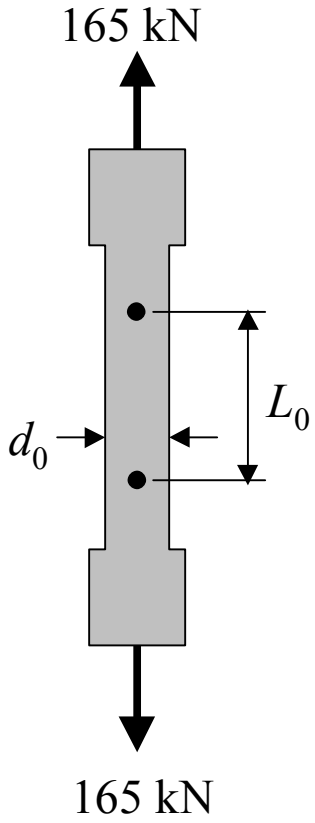
- The distance *d*

$$\tan(0.00533\,rad) \approx 0.00533\,rad = \frac{d}{50\,mm}$$

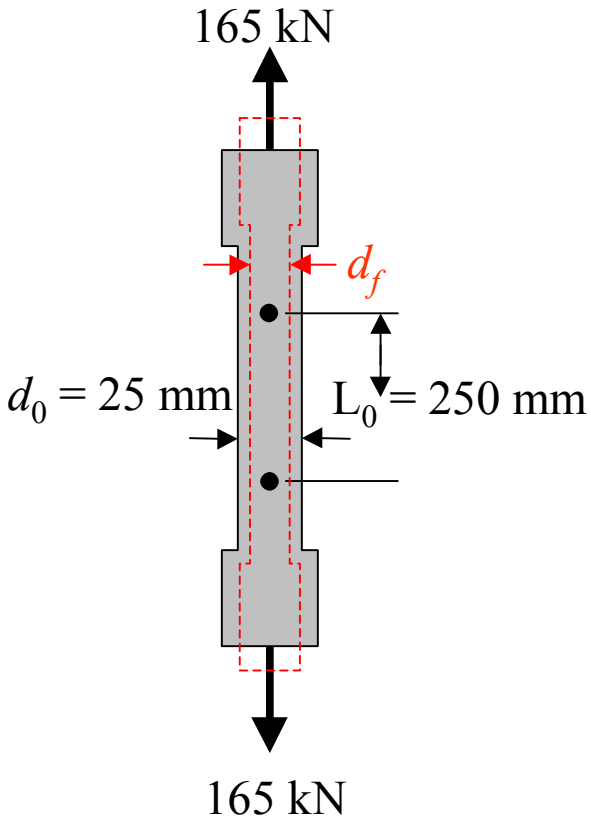
$$d = 0.267 \text{ mm}$$

Example 6

An aluminum specimen shown has a diameter of d_0 = 25 mm and a gauge length of L_0 = 250 mm. If a force of 165 kN applies to the specimen shown, determine the diameter of the specimen. Take E = 70 GPa ,G_{al} = 26 GPa and σ_Y = 440 MPa.



- **The diameter of the specimen (d_f) if a force 165 kN applies**



$$d_f = d_0 + \delta = d_0 + e_{lat} d_0 \qquad \text{----------(1)}$$

- Stress and Strain Relation ship

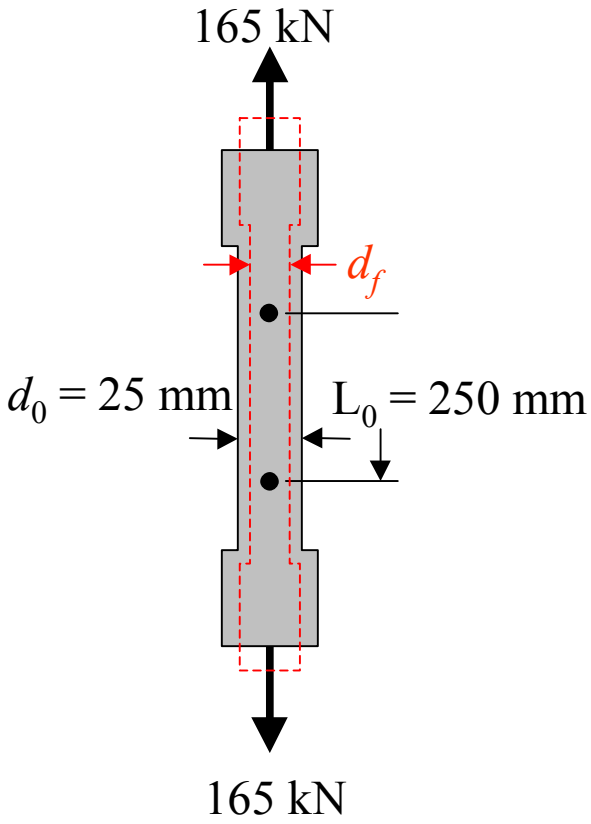
$$\sigma = \frac{P}{A} = \frac{165\ kN}{(\pi/4)(0.025\ m)^2} = 336.1\ MPa$$

Since $\sigma < \sigma_Y = 440$ MPa, the material behaves elastically. The modulus of elasticity is

$$E_{al} = \frac{\sigma}{\varepsilon_{long}}$$

$$70.0 \times 10^9\ Pa = \frac{336.1 \times 10^6\ Pa}{\varepsilon_{long}}$$

$$\varepsilon_{long} = 0.0048$$



- *G* and *E* Relationship

$$G = \frac{E}{2(1+\nu)}$$

$$26\ GPa = \frac{70\ GPa}{2(1+\nu)}$$

$$\nu = 0.346$$

- Poisson's Ratio

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

$$0.346 = -\frac{\varepsilon_{lat}}{0.00480\ mm/mm}$$

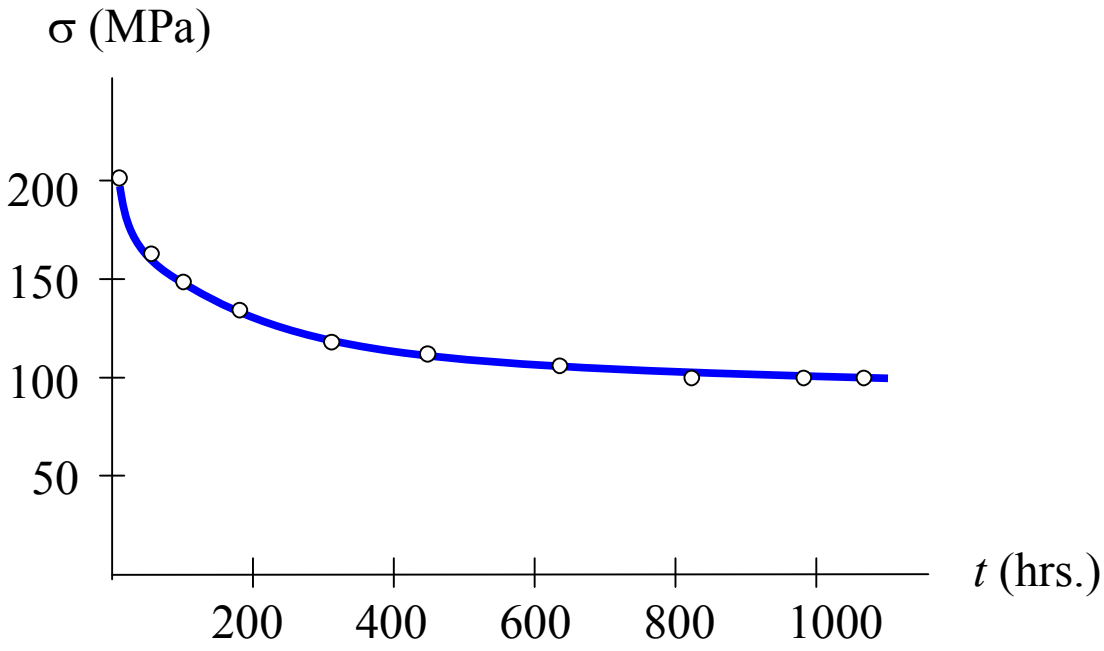
$$\varepsilon_{lat} = -0.00166 \text{ mm/mm}$$

$$\delta' = (0.00166)(25 \text{ mm}) = 0.0415 \text{ mm}$$

$$From \quad eq.(1): \qquad d_f = d_0 + \varepsilon_{lat} d_0 = 25\ mm + (-0.00166)(25\ mm) = 24.96\ mm$$

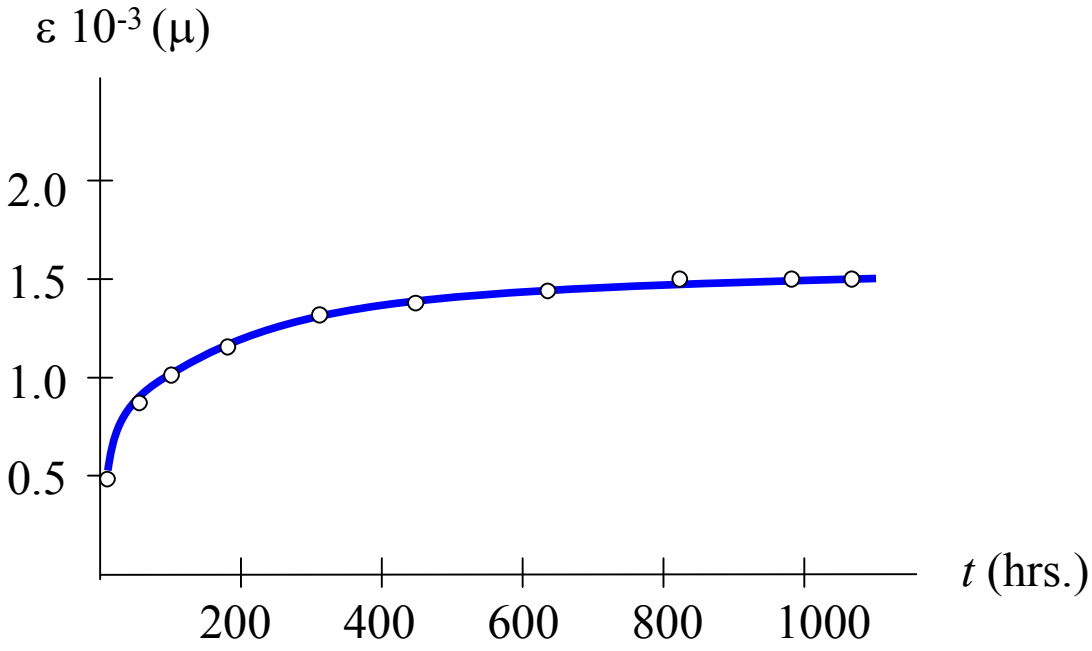
Failure of Materials due to Stress Relaxation, Creep, and Fatigue

- **Stress Relaxation**



σ–τ diagram for stainless steel
at 1200°F and creep strain at 1%

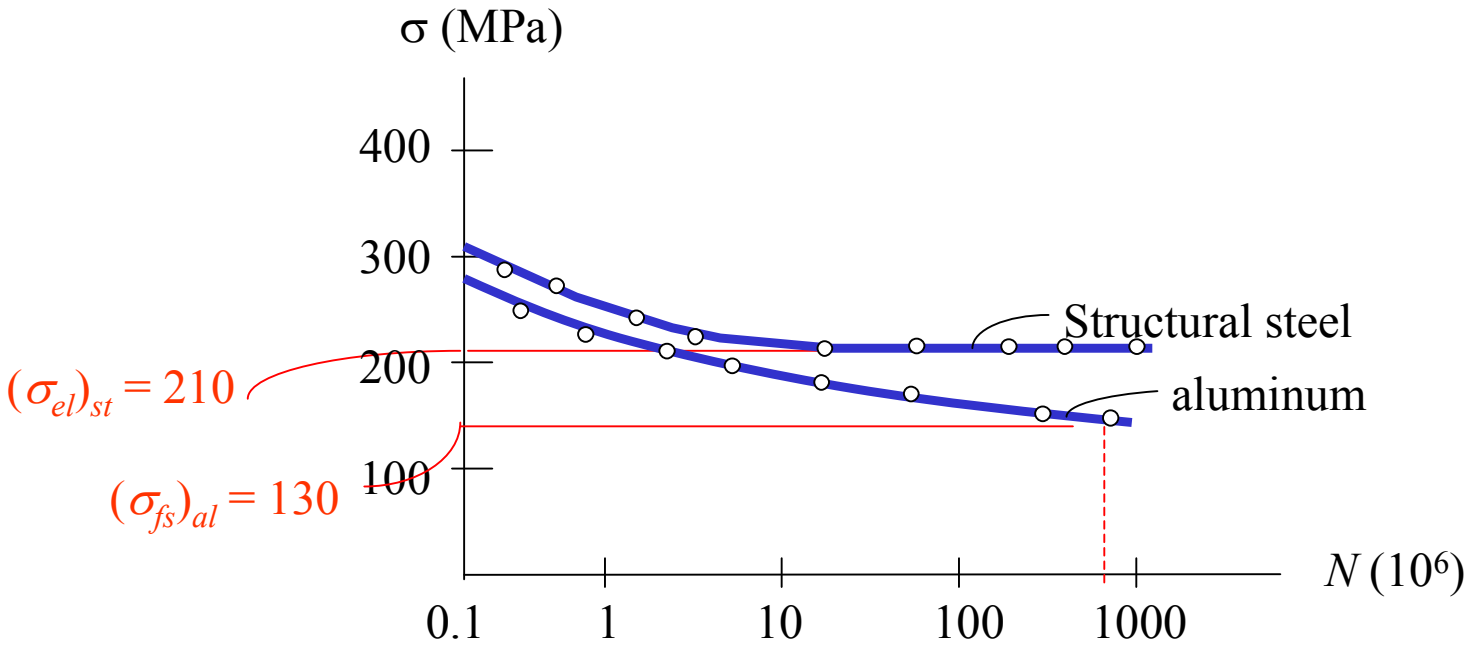
• **Creep**



σ–τ diagram, typical aluminum

• **Fatigue**

Fatigue limit (endurance limit)



S-N diagram for steel and aluminum alloys
(*N* axis has a logarithmic scale)

(σ_{el}), Endurance limit

(σ_{fs}), Fatigue strength @ 500(10^6) cycles