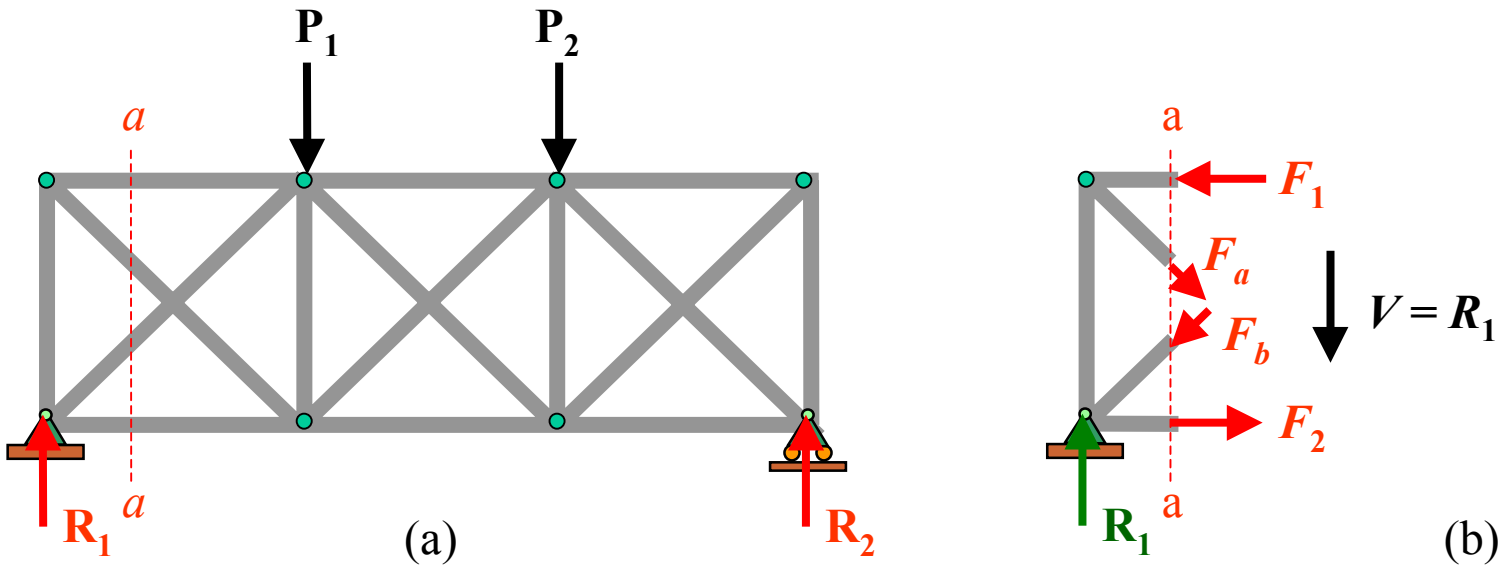
APPROXIMATE ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES

- Trusses
- Vertical Loads on Building Frames
- Lateral Loads on Building Frames: Portal Method
- Lateral Loads on Building Frames: Cantilever Method Problems

Trusses

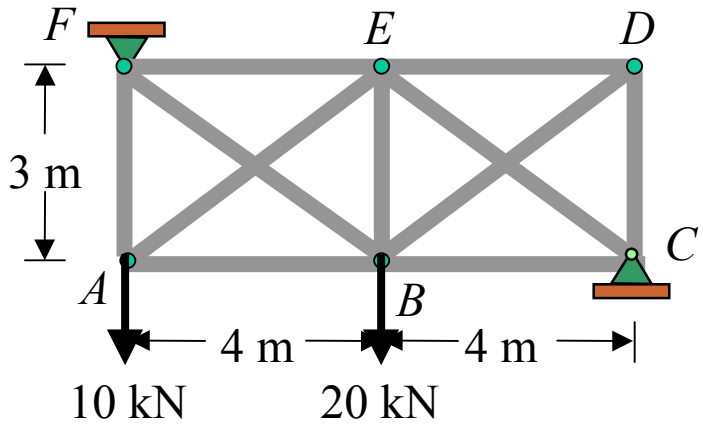


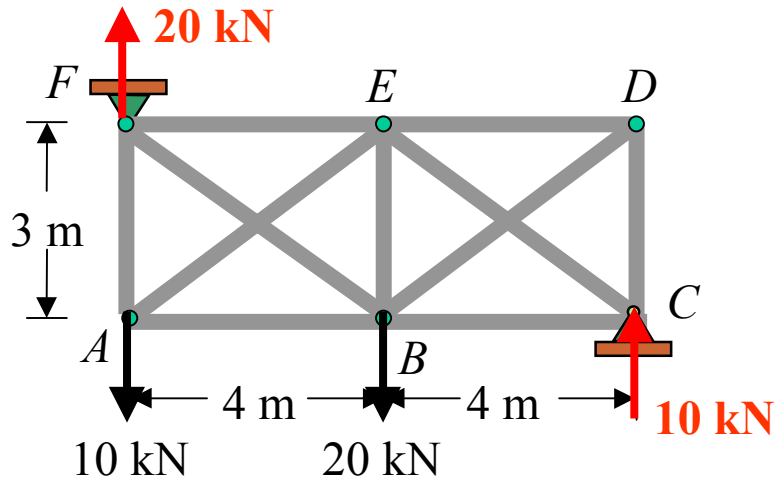
Method 1 *:* If the diagonals are intentionally designed to be *long and slender*, it is reasonable to assume that the panel shear is resisted entirely by the tension *diagonal*, whereas the *compressive diagonal is assumed to be a zero-force member.*

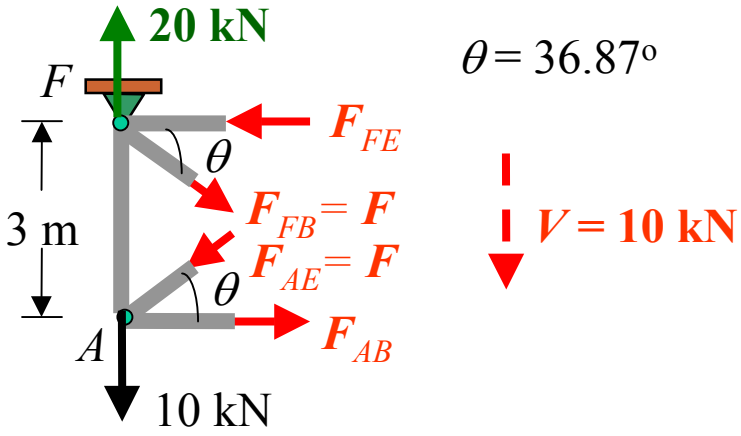
Method 2 *:* If the diagonal members are intended to be constructed from large rolled sections such as angles or channels, we will assume that the tension and compression diagonals each carry *half* the panel shear.

Example 1

Determine (approximately) the forces in the members of the truss shown in Figure. The diagonals are to be designed to support both tensile and compressive forces, and therefore each is assumed to carry half the panel shear. The support reactions have been computed.







$\theta = 36.87^o$

$V = 10$ kN

$+\uparrow \Sigma F_y = 0$: $\quad 20 - 10 - 2F\sin(36.87^o) = 0$

$F = 8.33$ kN

$F_{FB} = 8.33$ kN (T)

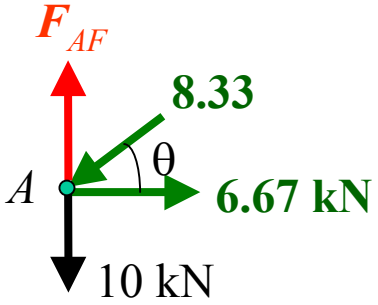
$F_{AE} = 8.33$ kN (C)

$+\circlearrowleft \Sigma M_A = 0$: $\quad F_{FE}(3) - 8.33\cos 36.87^o(3) = 0$

$F_{FE} = 6.67$ kN (C)

$+\circlearrowleft \Sigma M_F = 0$: $\quad F_{AB}(3) - 8.33\cos 36.87^o(3) = 0$

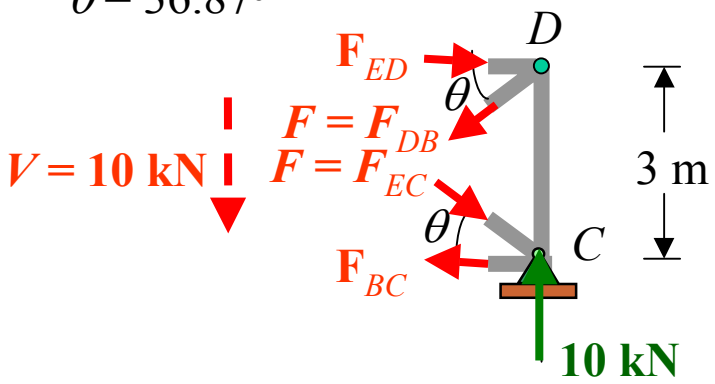
$F_{AB} = 6.67$ kN (T)



$+\uparrow \Sigma F_y = 0$: $\quad F_{AF} - 10 - 8.33\sin(36.87^o) = 0$

$F_{AF} = 15$ kN (T)

$\theta = 36.87^o$



$+\uparrow \Sigma F_y = 0$: $\quad 10 - 2F\sin(36.87^o) = 0$

$F = 8.33$ kN

$F_{DB} = 8.33$ kN (T)

$F_{EC} = 8.33$ kN (C)

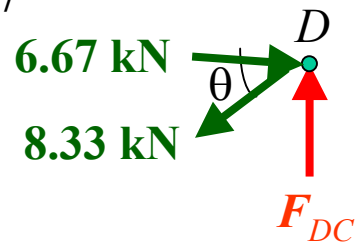
$+\circlearrowleft \Sigma M_C = 0$: $\quad F_{ED}(3) - 8.33\cos 36.87^o(3) = 0$

$F_{ED} = 6.67$ kN (C)

$+\circlearrowleft \Sigma M_D = 0$: $\quad F_{BC}(3) - 8.33\cos 36.87^o(3) = 0$

$F_{BC} = 6.67$ kN (T)

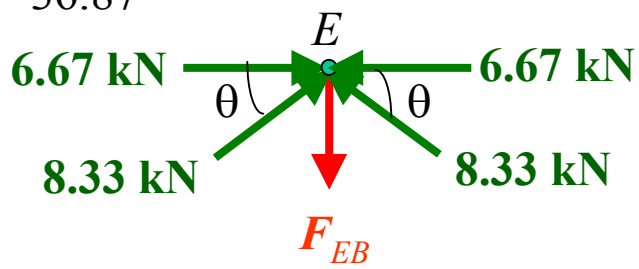
$\theta = 36.87^o$



$+\uparrow \Sigma F_y = 0$: $\quad F_{DC} - 8.33\sin(36.87^o) = 0$

$F_{DC} = 5$ kN (C)

$\theta = 36.87^o$

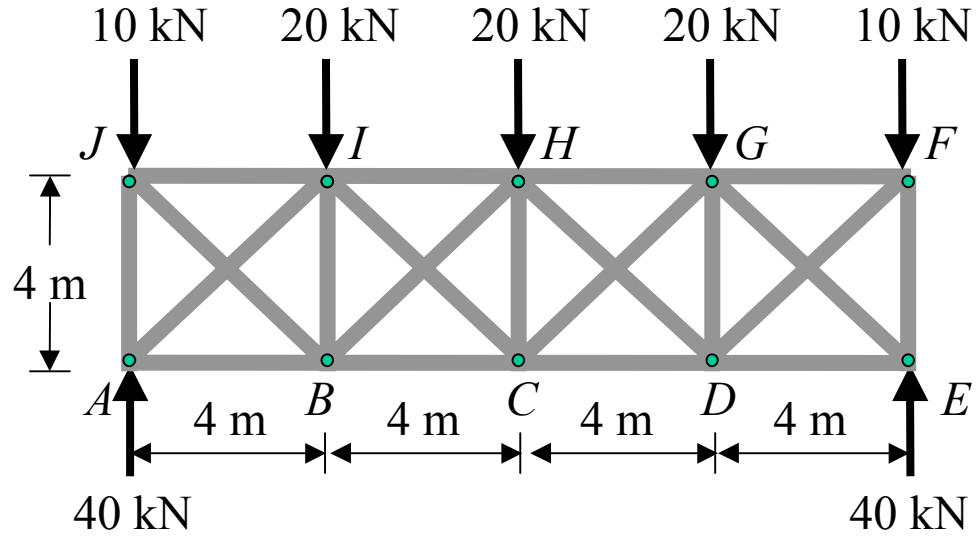


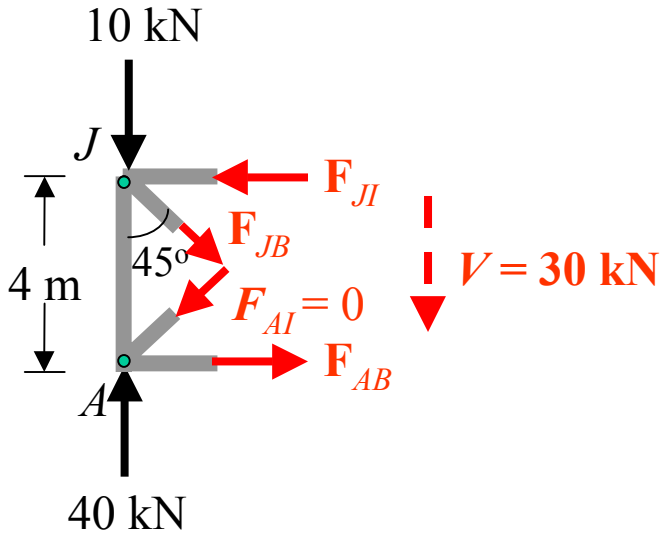
$+\uparrow \Sigma F_y = 0$:

$F_{EB} = 2(8.33\sin 36.87^o) = 10$ kN (T)

Example 2

Determine (approximately) the forces in the members of the truss shown in Figure. The diagonals are slender and therefore will not support a compressive force. The support reactions have been computed.





$\boldsymbol{F_{AI} = 0}$

$+\uparrow \Sigma F_y = 0$: $\quad 40 - 10 - F_{JB}\cos 45^o = 0$

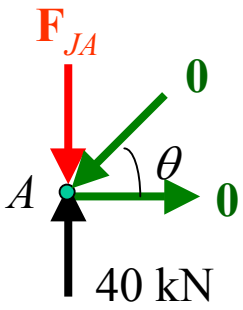
$F_{JB} = 42.43$ kN (T)

$+\circlearrowleft \Sigma M_A = 0$: $\quad F_{JI}(4) - 42.43\sin 45^o(4) = 0$

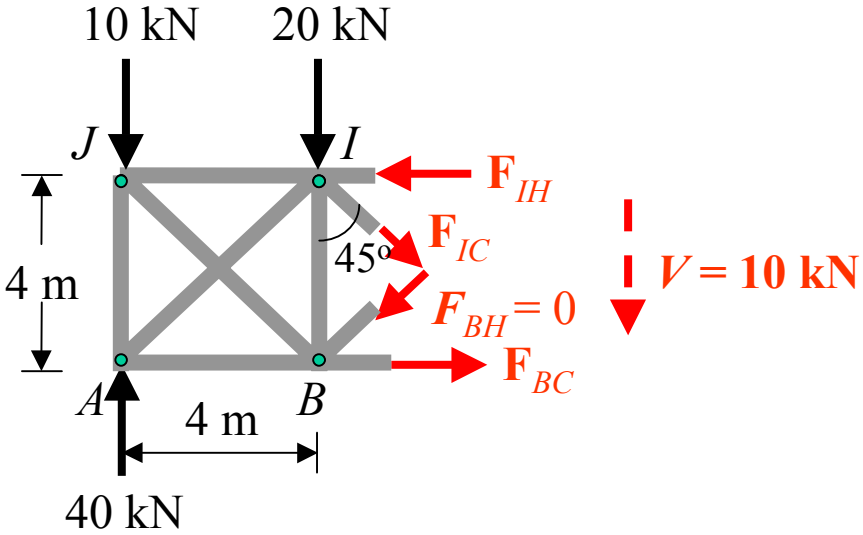
$F_{JI} = 30$ kN (C)

$+\circlearrowleft \Sigma M_J = 0$: $\quad F_{AB}(4) = 0$

$F_{AB} = 0$



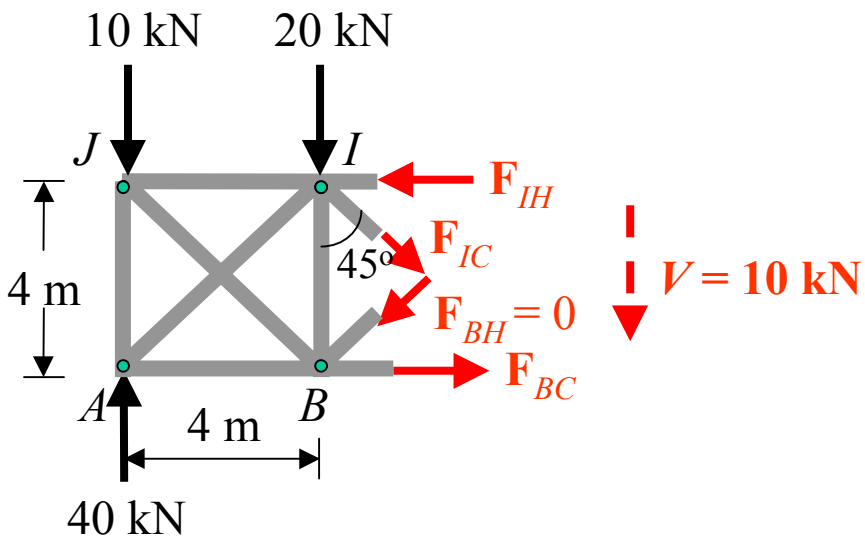
$+\uparrow \Sigma F_y = 0$: $\quad F_{JA} = 40$ kN (C)



$\boldsymbol{F_{BH} = 0}$

$+\uparrow \Sigma F_y = 0$: $\quad 40 - 10 - 20 - F_{IC}\cos 45^o = 0$

$F_{IC} = 14.14$ kN (T)



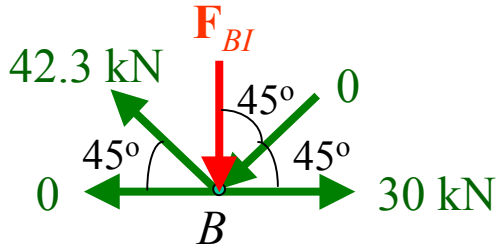
$+\circlearrowleft \Sigma M_B = 0$:

$F_{IH}(4) - 14.14\sin 45^o(4) + 10(4) - 40(4) = 0$

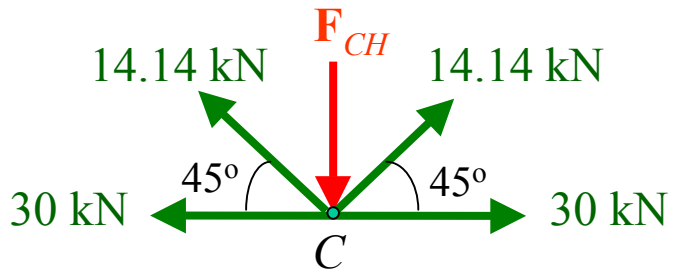
$F_{JH} = 40$ kN (C)

$+\circlearrowleft \Sigma M_I = 0$: $F_{BC}(4) - 40(4) + 10(4) = 0$

$F_{BC} = 30$ kN (T)



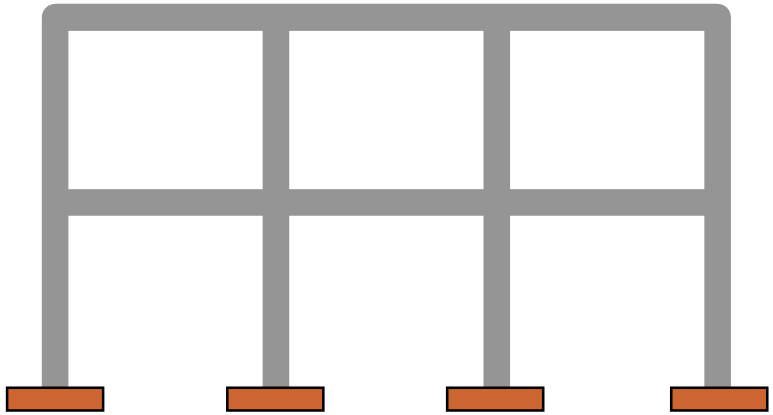
$+\uparrow \Sigma F_y = 0$: $F_{BI} = 42.3 \sin 45^o = 30$ kN (T)



$+\uparrow \Sigma F_y = 0$:

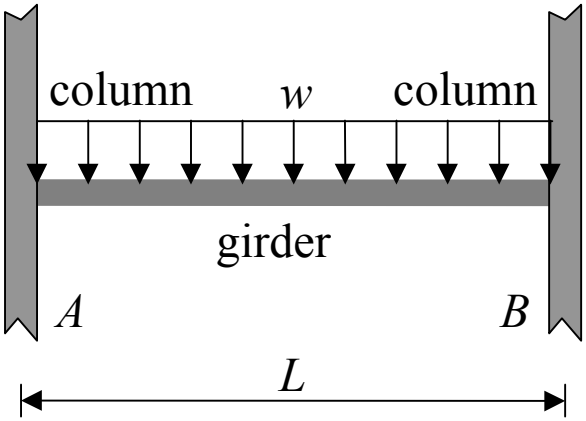
$F_{BI} = 2(14.1442.3 \sin 45^o) = 20$ kN (C)

Vertical Loads on Building Frames

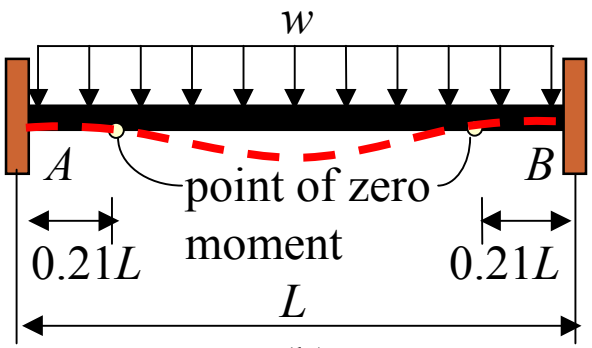


typical building frame

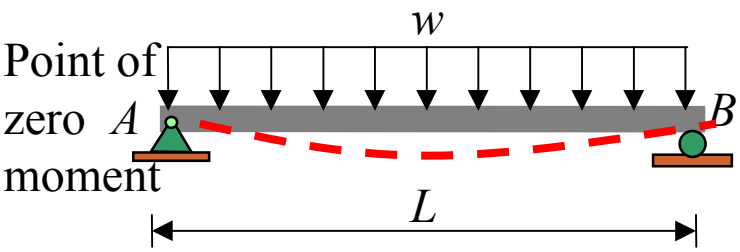
• **Assumptions for Approximate Analysis**



(a)

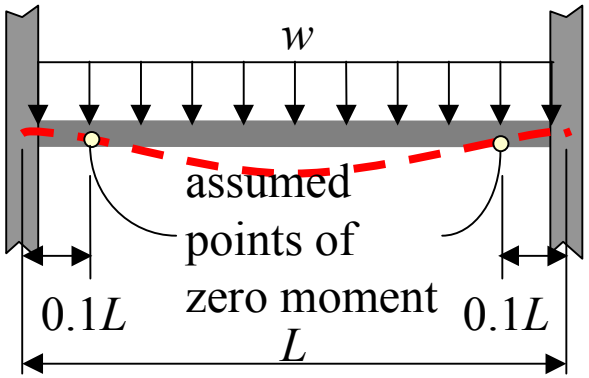


(b)



Simply supported

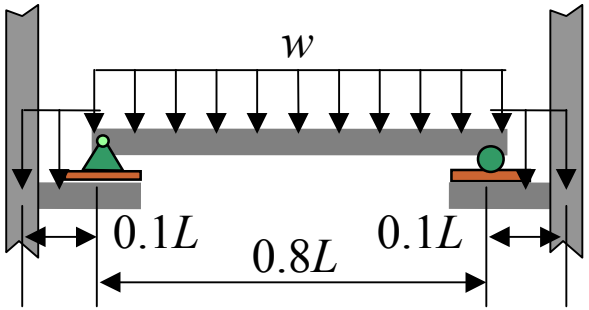
(c)



approximate case

(d)

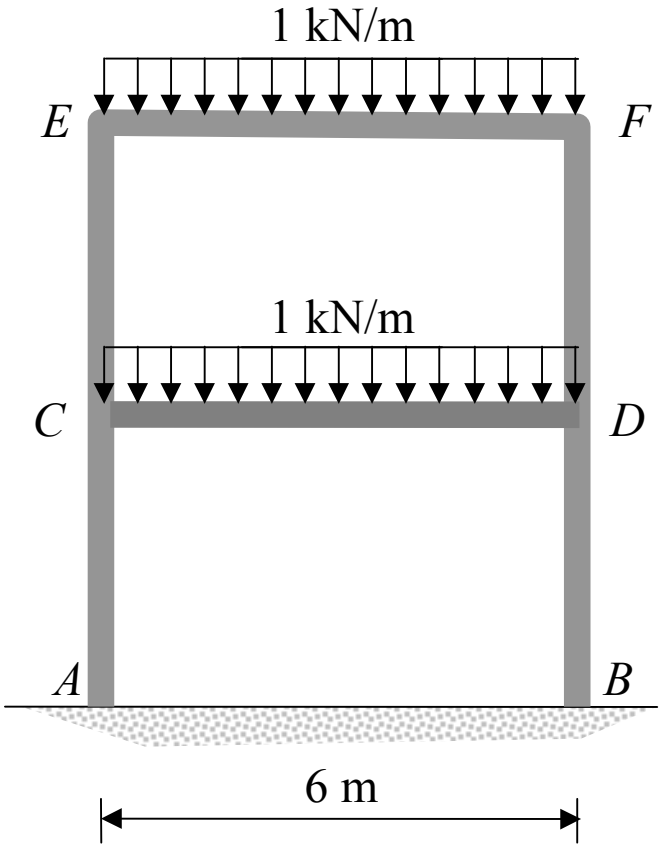
Point of zero moment

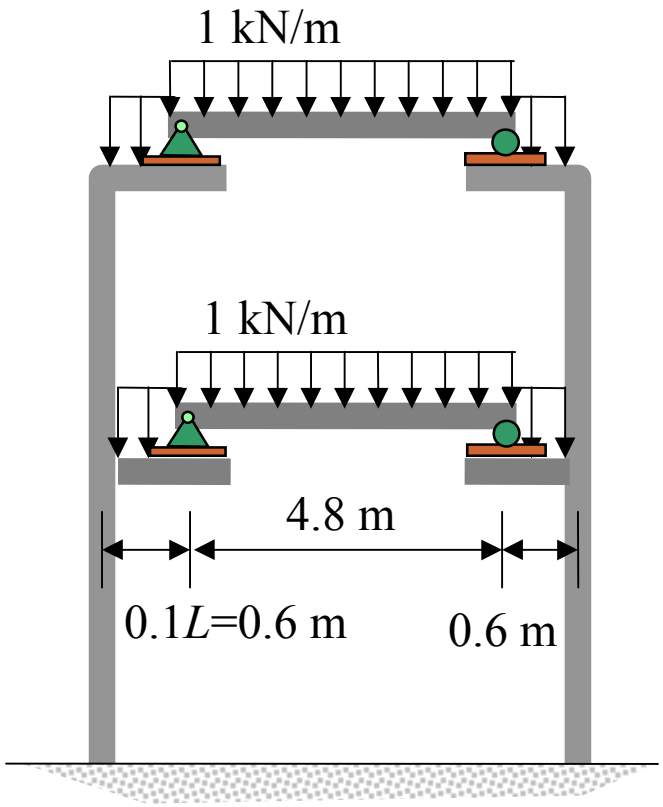


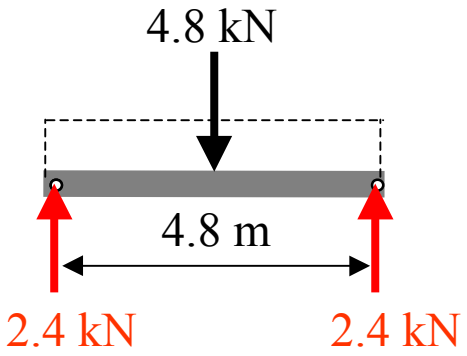
model (e)

Example 3

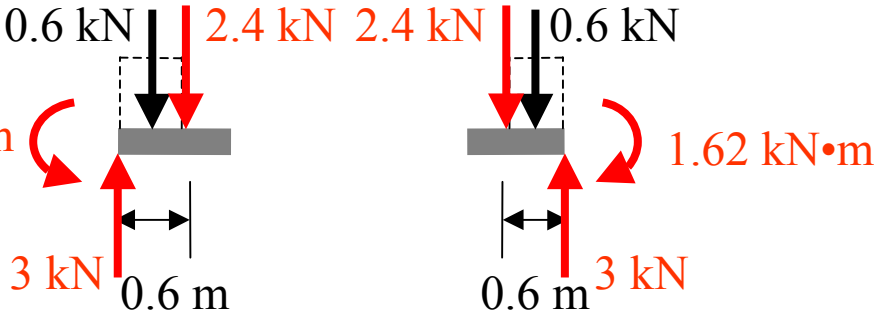
Determine (approximately) the moment at the joints *E* and *C* caused by members *EF* and *CD* of the building bent in the figure.





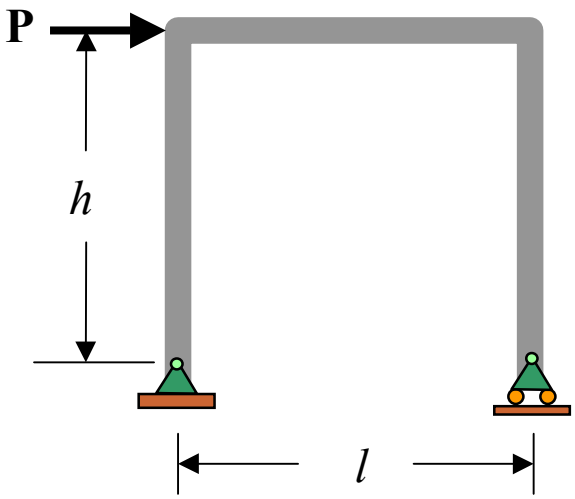


0.6(0.3) + 2.4(0.6) = 1.62 kN•m

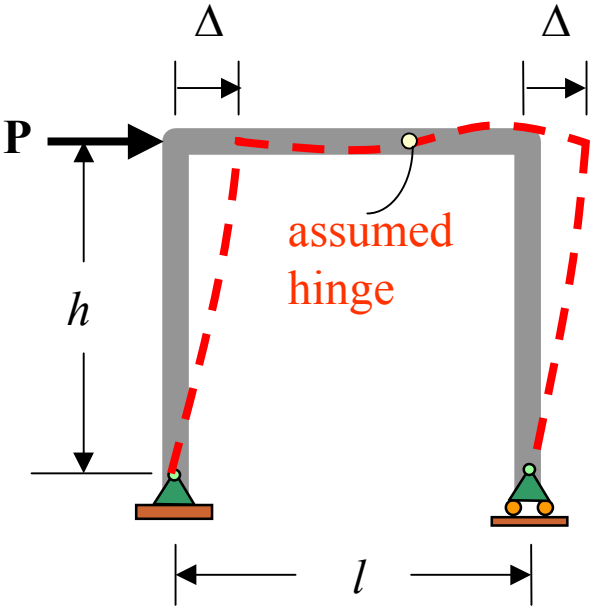


Portal Frames and Trusses

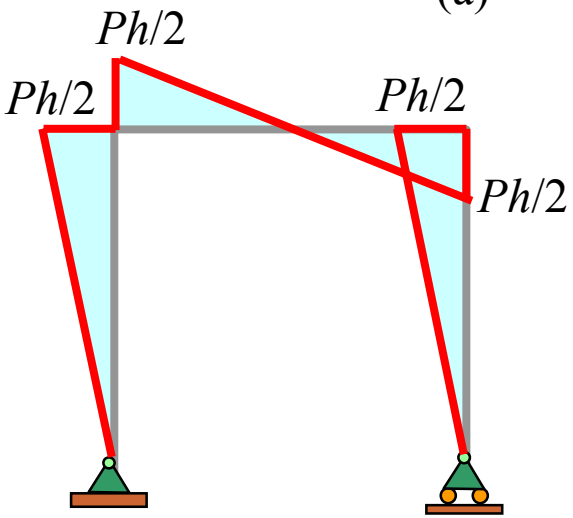
- **Frames: Pin-Supported**



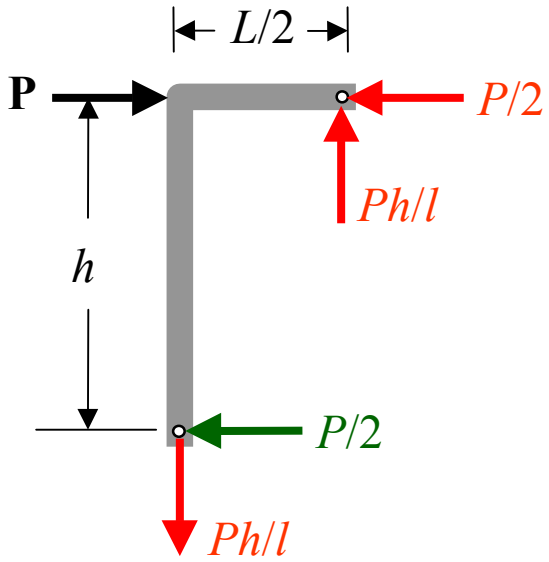
(a)

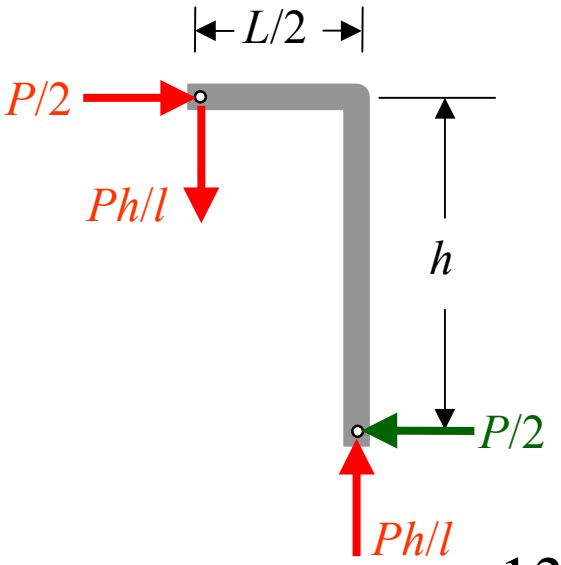


(b)



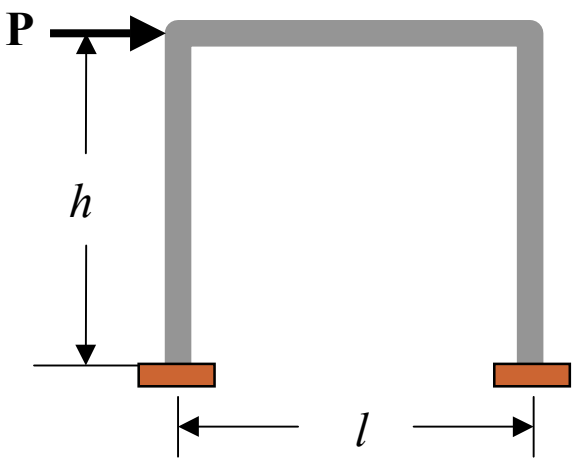
(d)



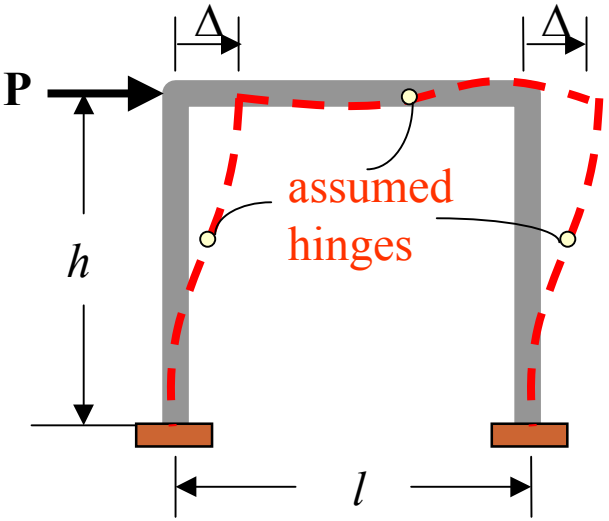


(c)

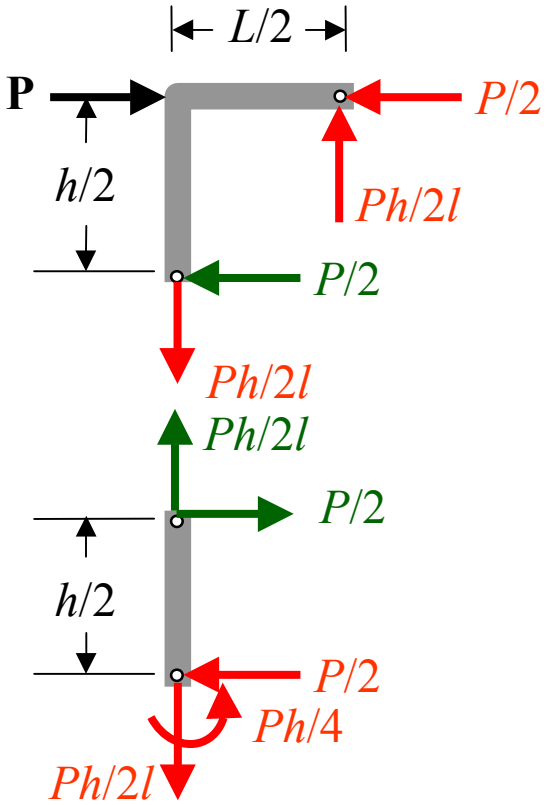
• **Frames : Fixed-Supported**



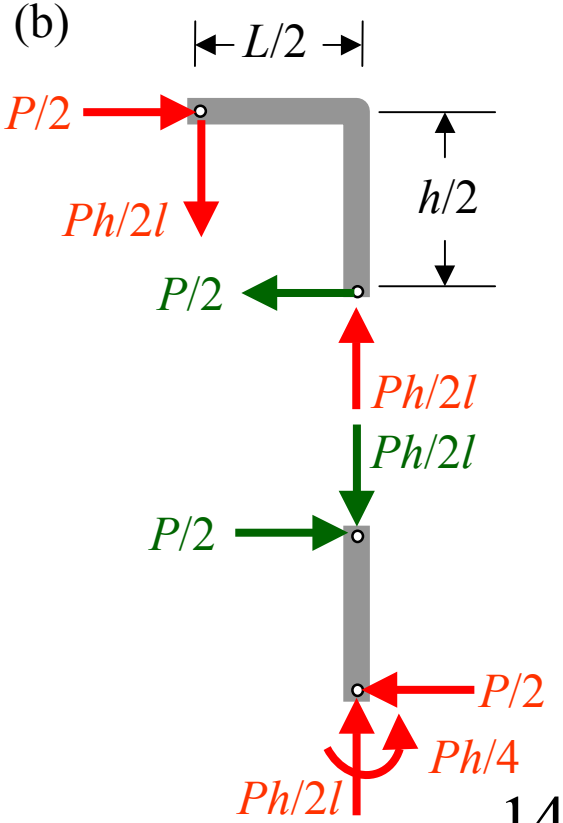
(a)

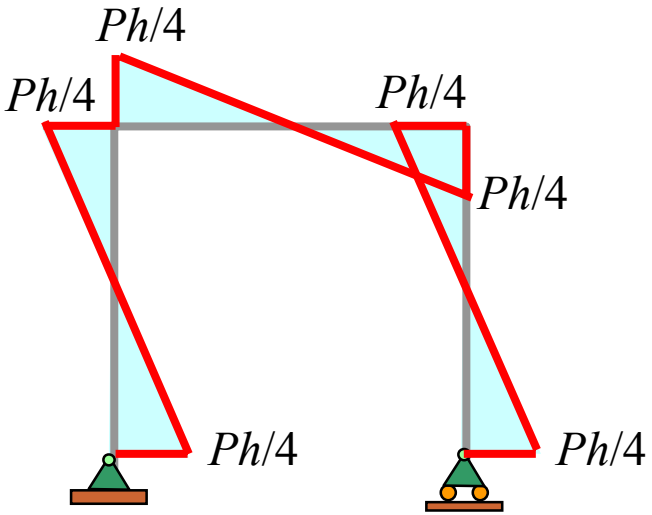


(b)



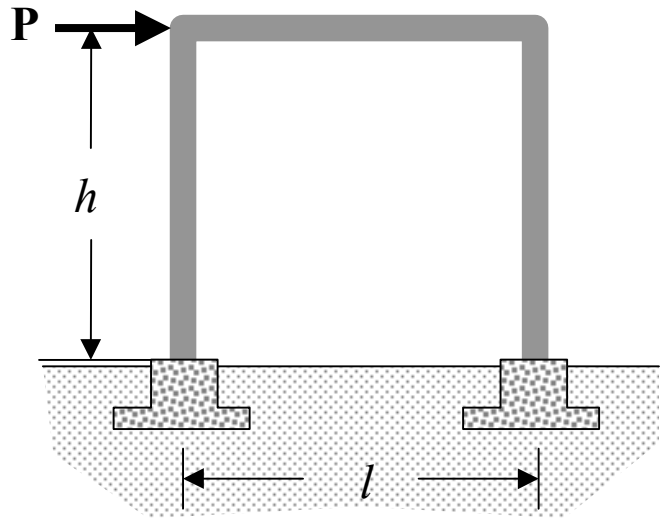
(c)



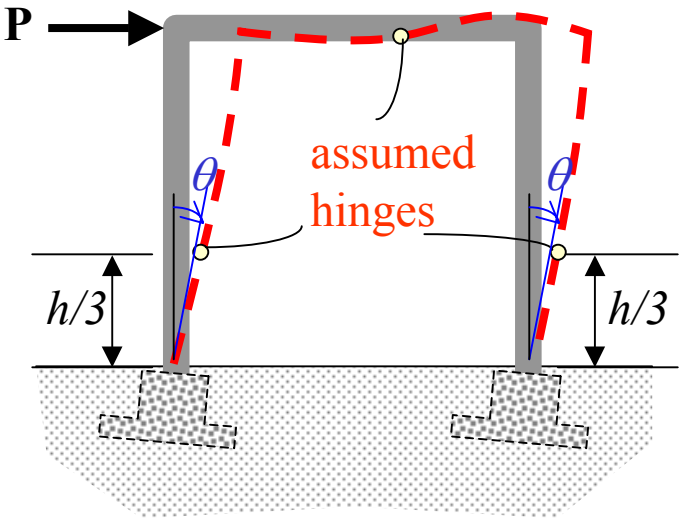


(d)

• **Frames : Partial Fixity**

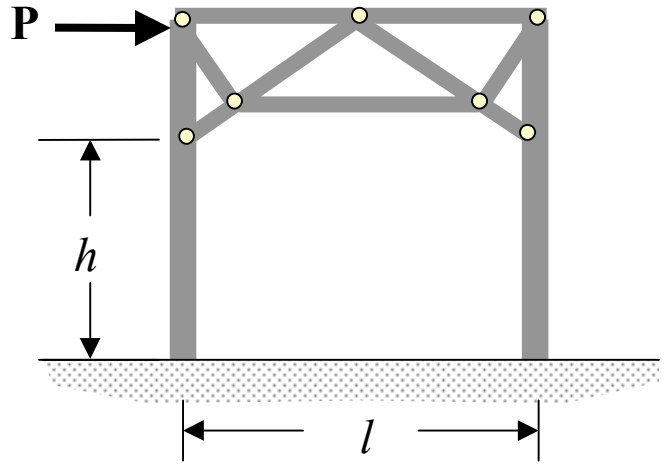


(a)

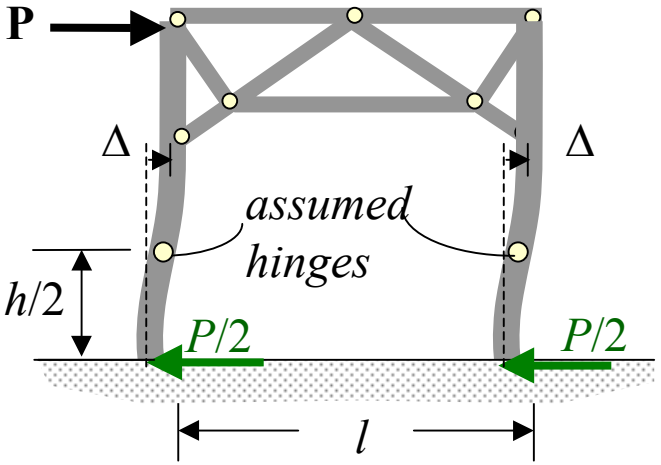


(b)

• **Trusses**



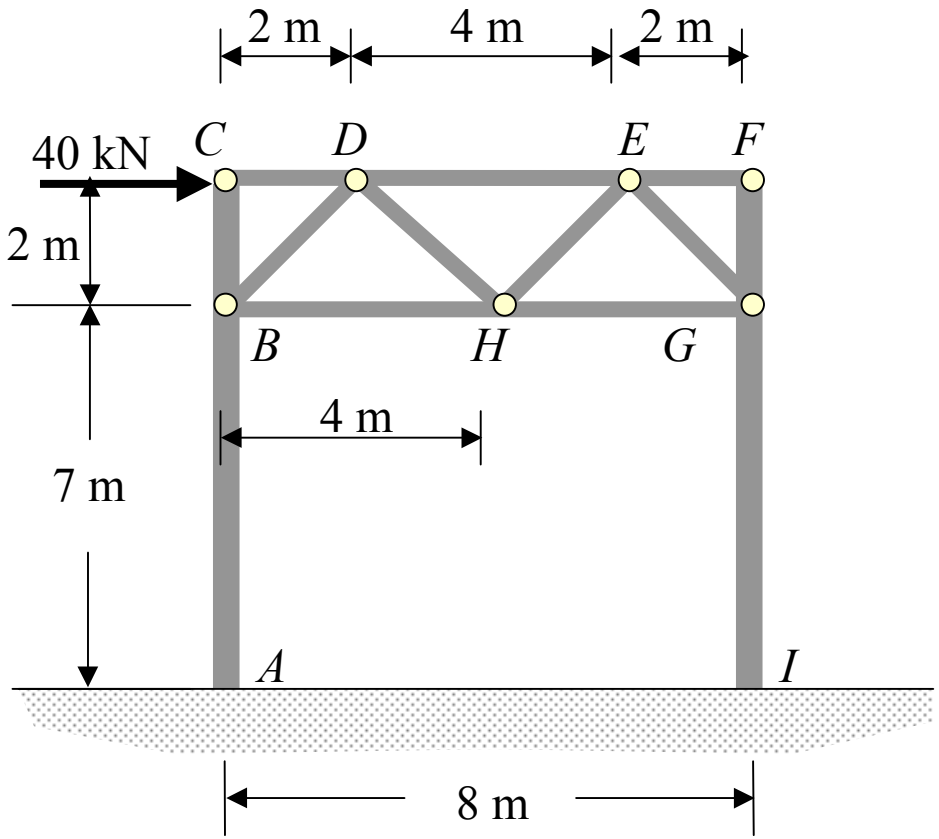
(a)

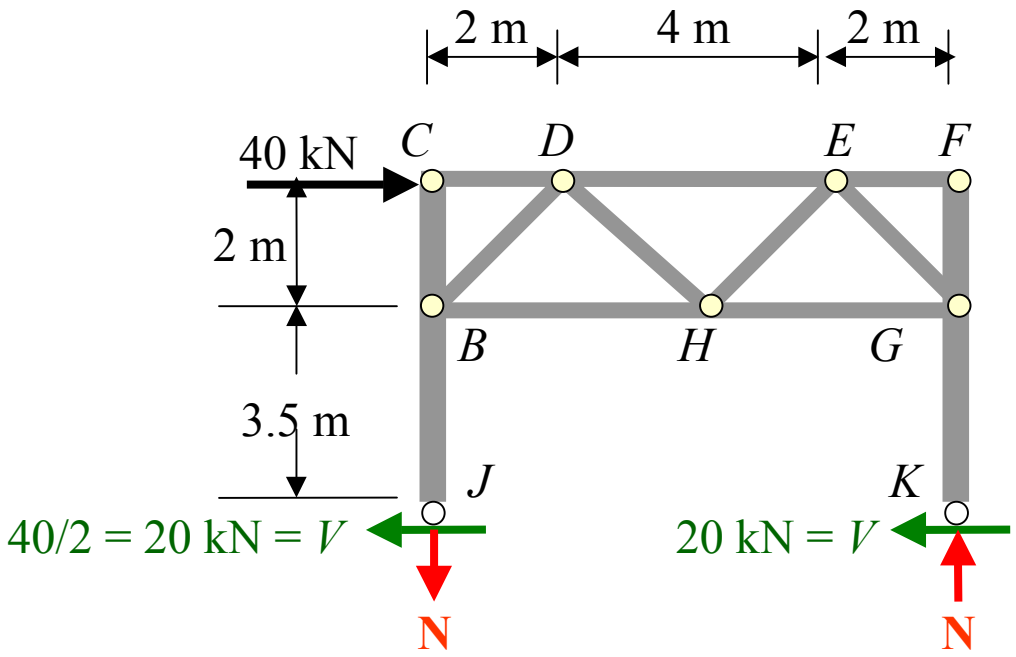


(b)

Example 4

Determine by approximate methods the forces acting in the members of the Warren portal shown in the figure.

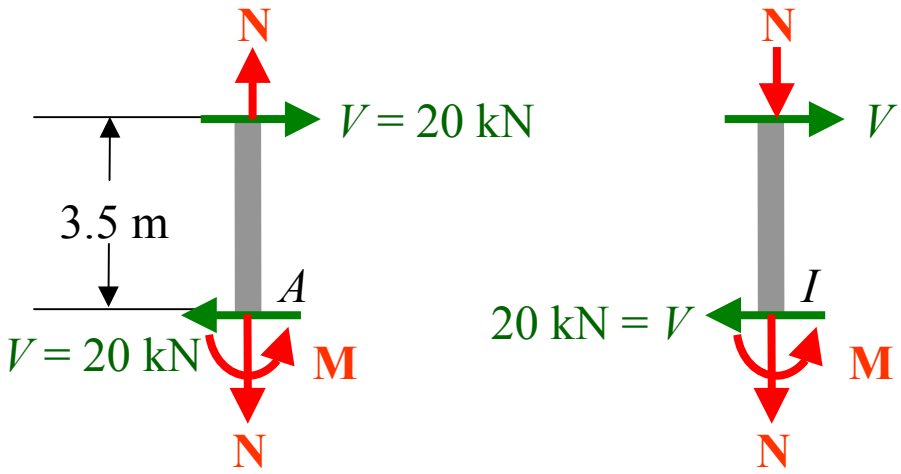




$+\circlearrowleft \Sigma M_J = 0$:

$N(8) - 40(5.5) = 0$

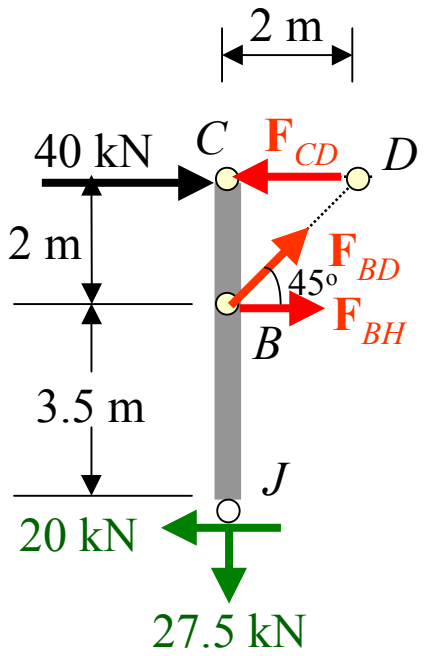
$N = 27.5$ kN



$+\circlearrowleft \Sigma M_A = 0$:

$M - 20(3.5) = 0$

$M = 70$ kN•m



$+\uparrow \Sigma F_y = 0$: $\quad -27.5 + F_{BD}\cos 45^o = 0$

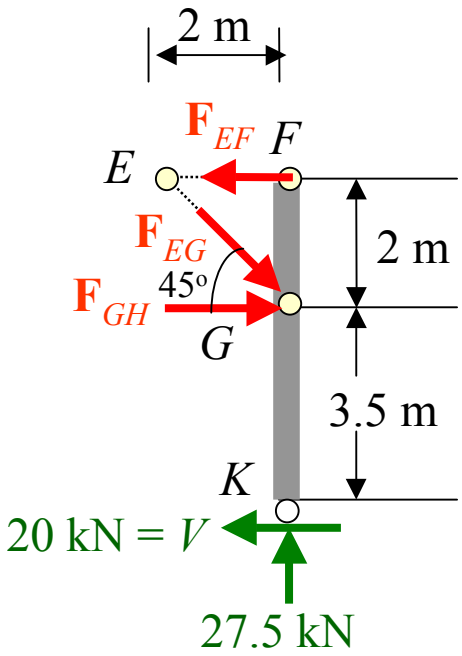
$F_{BD} = 38.9$ kN (T)

$+\circlearrowleft \Sigma M_B = 0$: $\quad F_{CD}(2) - 40(2) - 20(3.5) = 0$

$F_{CD} = 75$ kN (C)

$+\circlearrowleft \Sigma M_D = 0$: $F_{BH}(2) + 27.5(2) - 20(5.5) = 0$

$F_{BH} = 27.5$ kN (T)



$+\uparrow \Sigma F_y = 0$: $\quad 27.5 - F_{EG}\cos 45^o = 0$

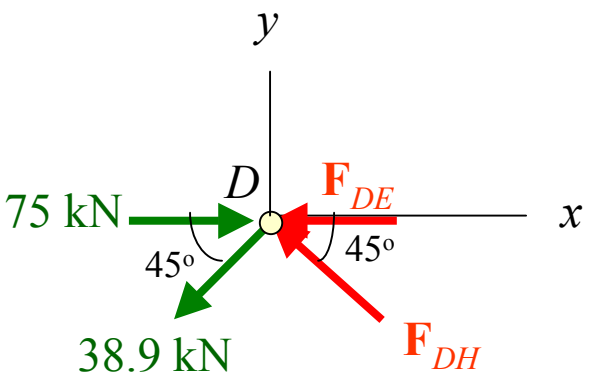
$F_{EG} = 38.9$ kN (C)

$+\circlearrowleft \Sigma M_G = 0$: $\quad F_{EF}(2) - 20(3.5) = 0$

$F_{EF} = 35$ kN (T)

$+\circlearrowleft \Sigma M_E = 0$: $F_{GH}(2) + 27.5(2) - 20(5.5) = 0$

$F_{GH} = 27.5$ kN (C)

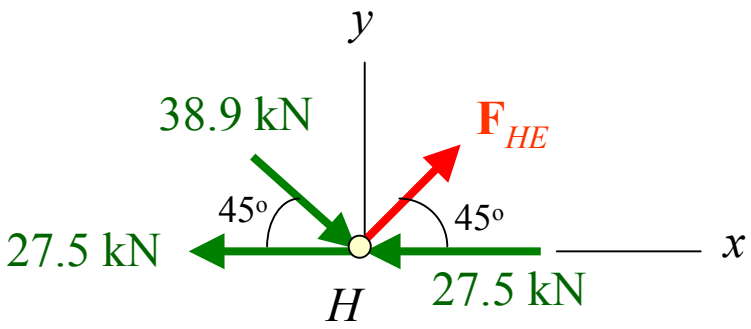


$+\uparrow \Sigma F_y = 0$: $F_{DH}\sin 45^o - 38.9\sin 45^o = 0$

$F_{DH} = 38.9$ kN (C)

$\pm\rightarrow \Sigma F_x = 0$: $75 - 2(38.9 \cos 45^o) - F_{DE} = 0$

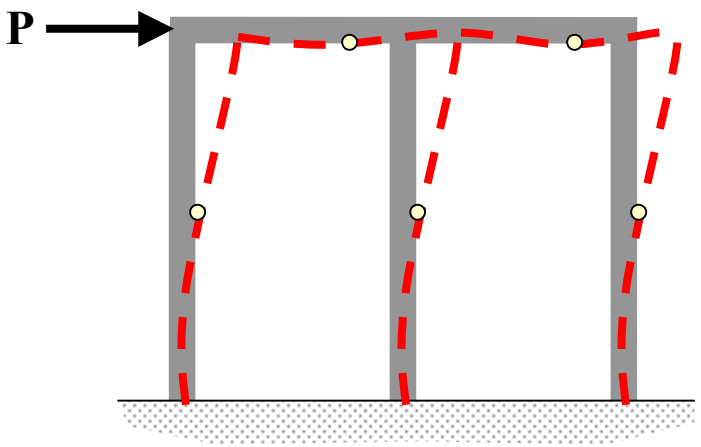
$F_{DE} = 20$ kN (C)



$+\uparrow \Sigma F_y = 0$: $F_{HE}\sin 45^o - 38.9\sin 45^o = 0$

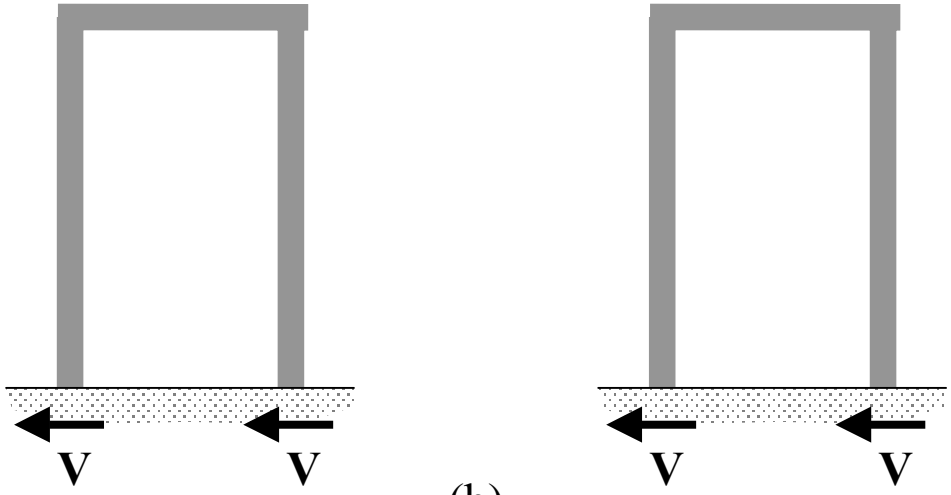
$F_{HE} = 38.9$ kN (T)

Lateral Loads on Building Frames: Portal Method



○ = inflection point

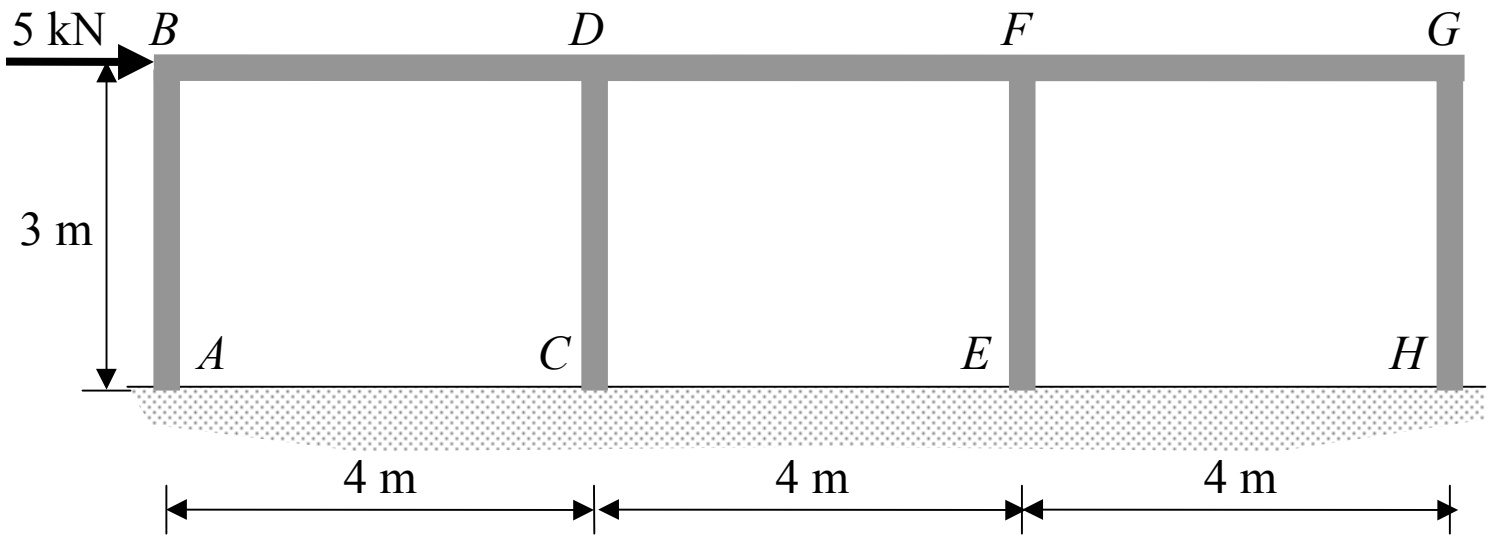
(a)

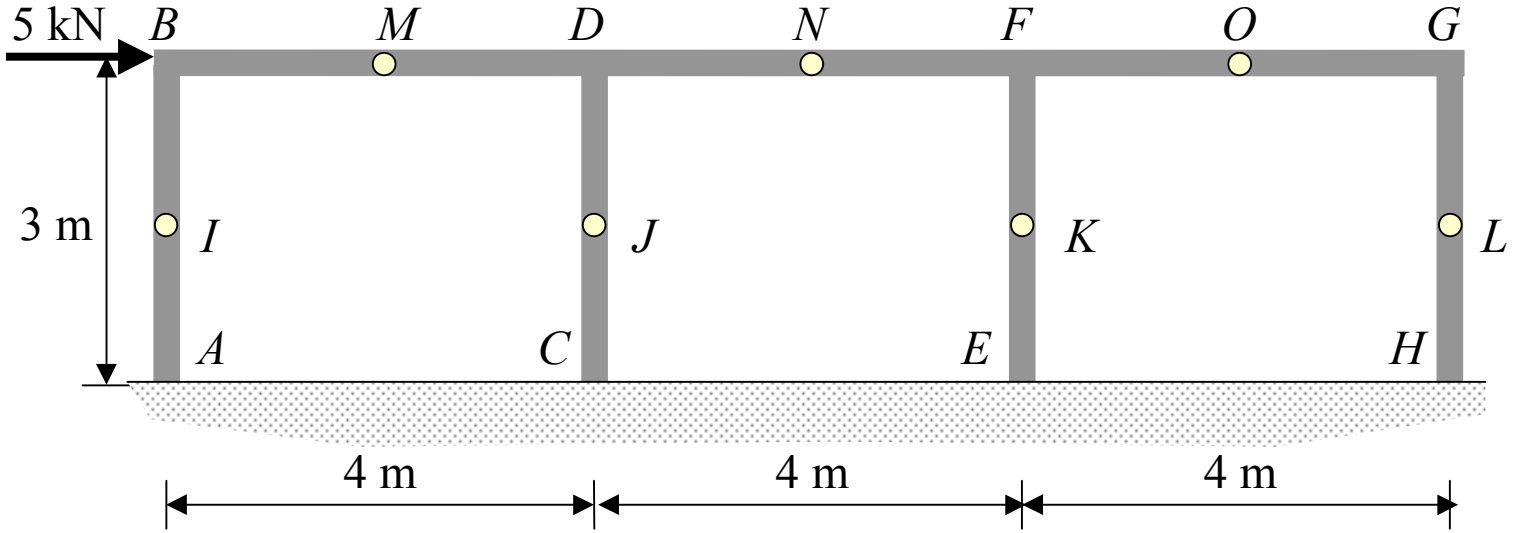


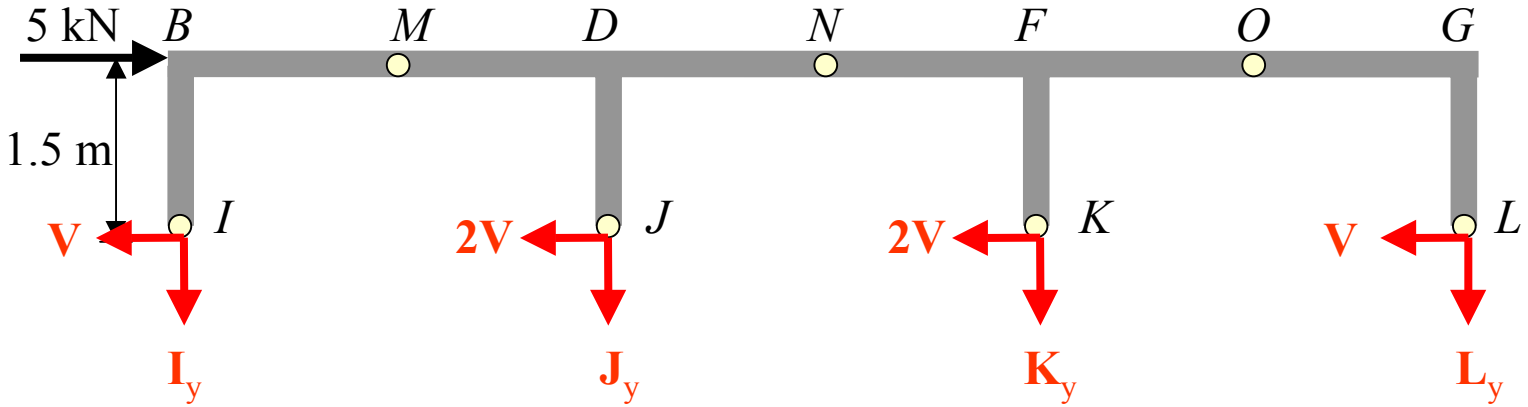
(b)

Example 5

Determine by approximate methods the forces acting in the members of the Warren portal shown in the figure.

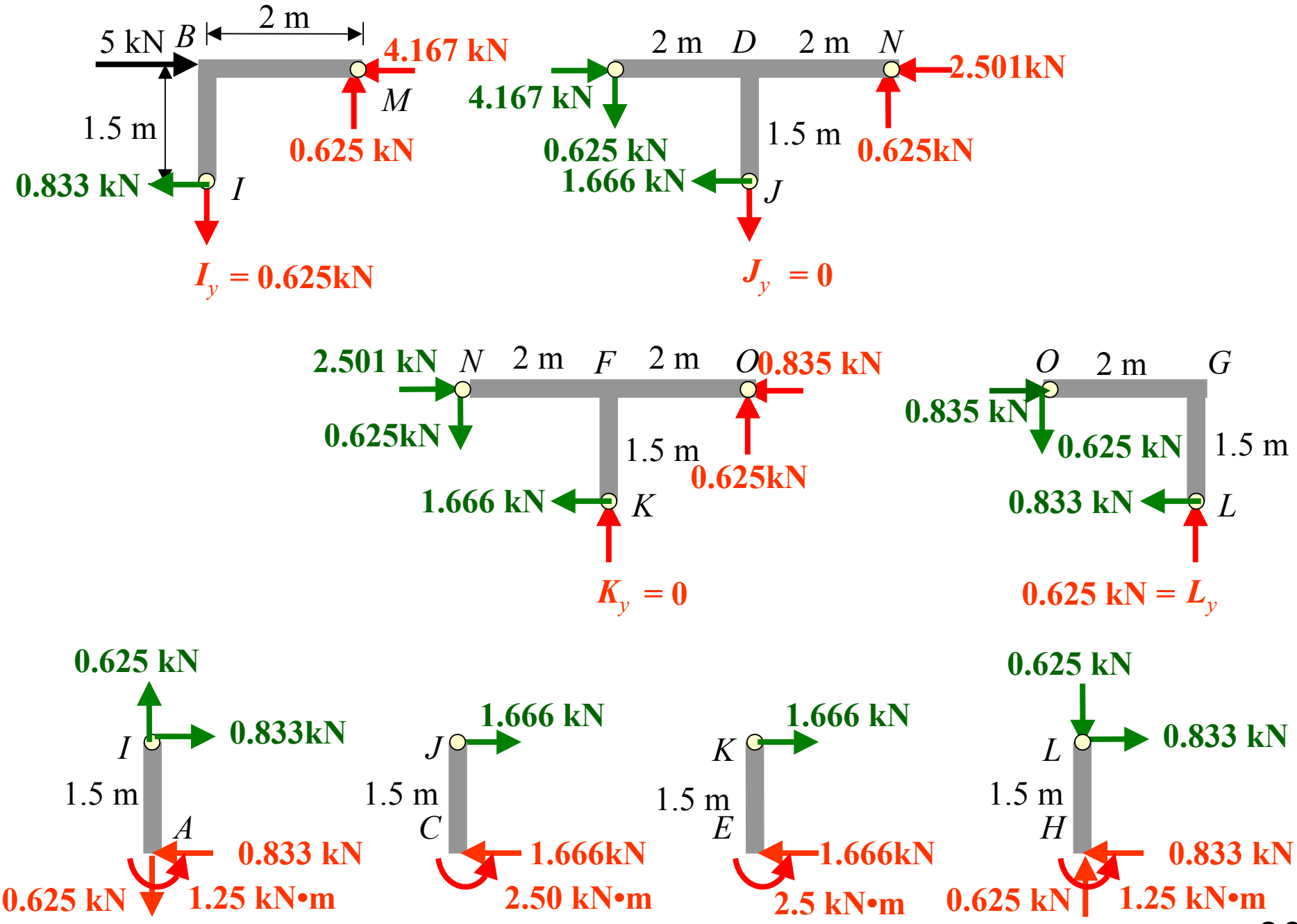






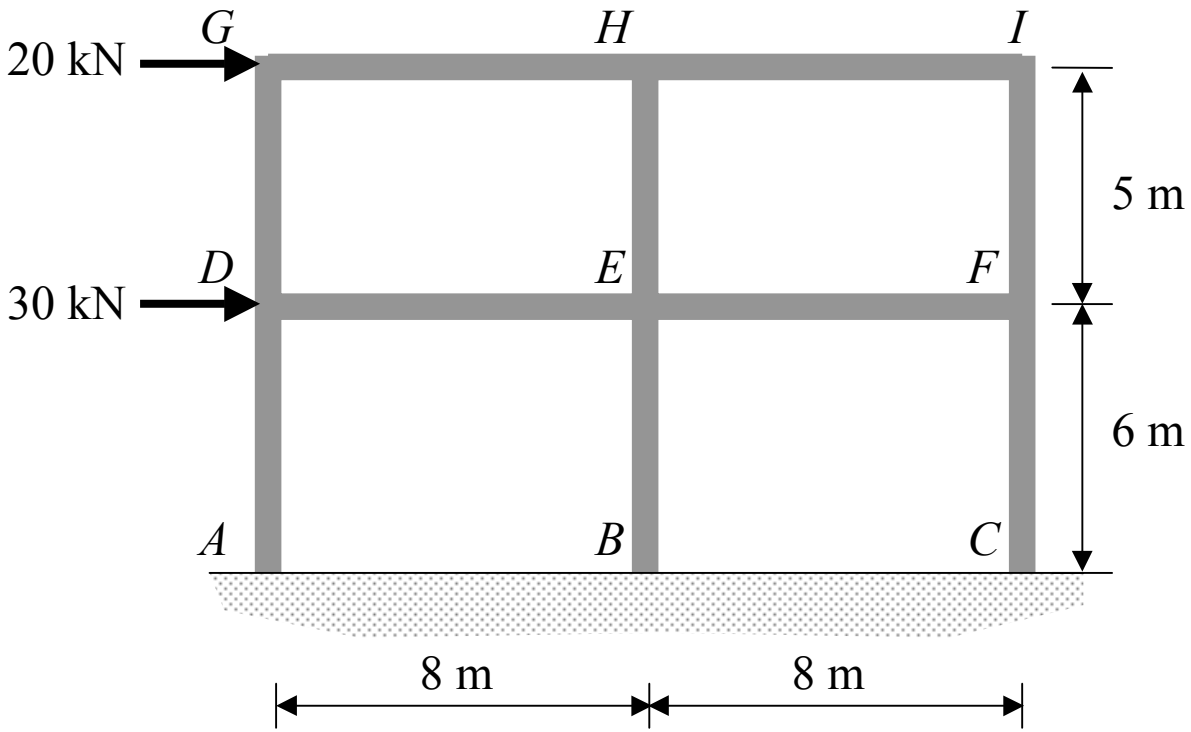
$\pm\rightarrow \Sigma F_x = 0$: $\quad 5 - 6V = 0$

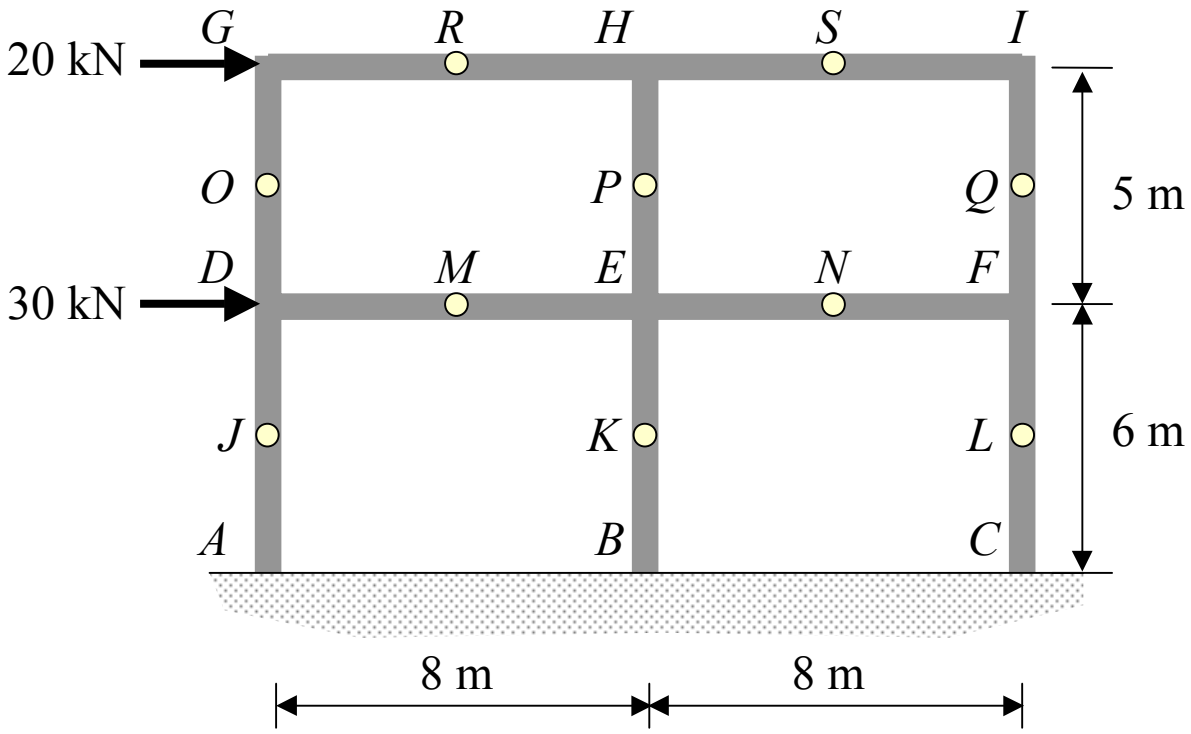
$V = 0.833$ kN

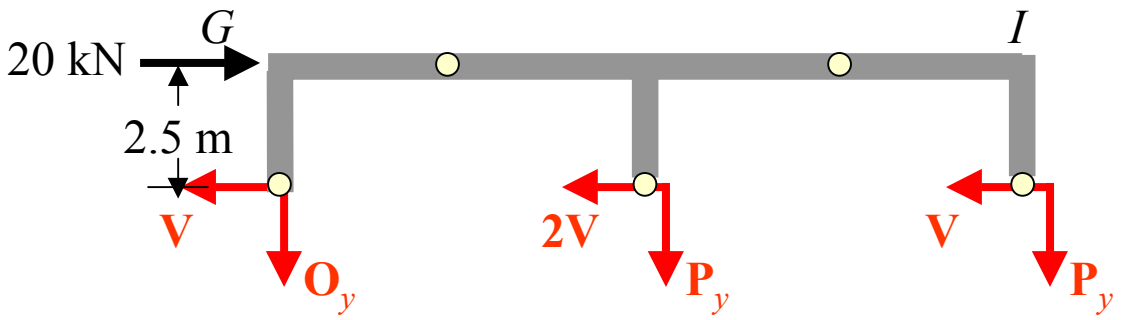

5 kN
B
2 m
4.167 kN
M
1.5 m
0.625 kN
0.833 kN
I
I_y = 0.625kN
2 m
D
2 m
N
2.501kN
4.167 kN
0.625 kN
1.5 m
0.625kN
1.666 kN
J
J_y = 0
2.501 kN
N
2 m
F
2 m
O
0.835 kN
0.625kN
1.5 m
0.625kN
1.666 kN
K
K_y = 0
O
2 m
G
0.835 kN
0.625 kN
1.5 m
0.833 kN
L
0.625 kN = L_y
0.625 kN
0.833kN
I
1.5 m
A
0.833 kN
0.625 kN
1.25 kN•m
1.666 kN
J
1.5 m
C
1.666kN
2.50 kN•m
1.666 kN
K
1.5 m
E
1.666kN
2.5 kN•m
0.625 kN
0.833 kN
L
1.5 m
H
0.833 kN
0.625 kN
1.25 kN•m

Example 6

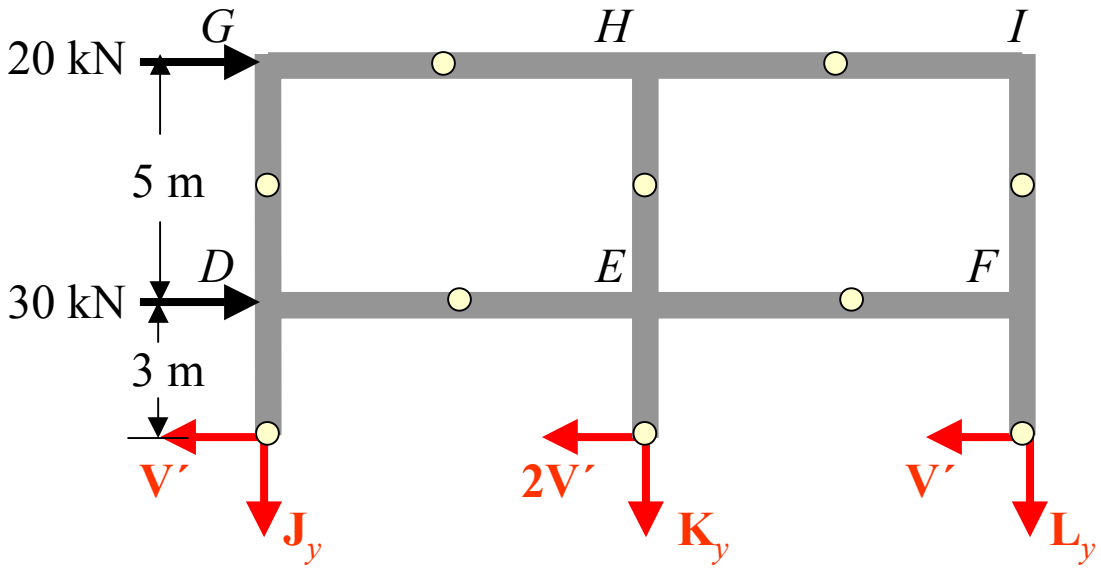
Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7-14a. Use the portal method of analysis.



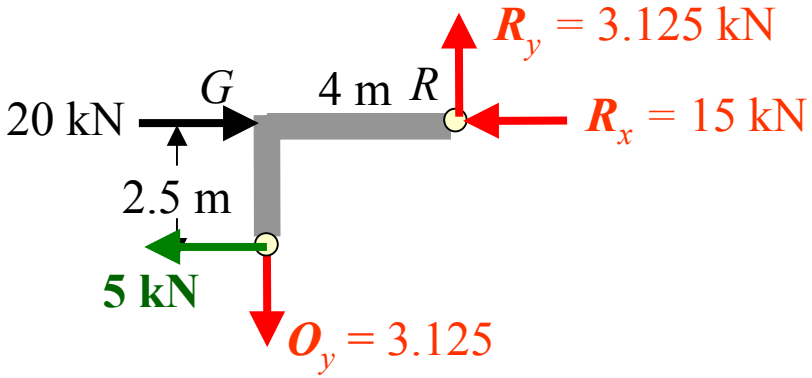

G
R
H
S
I
20 kN
O
P
Q
5 m
D
M
E
N
F
30 kN
J
K
L
6 m
A
B
C
8 m
8 m

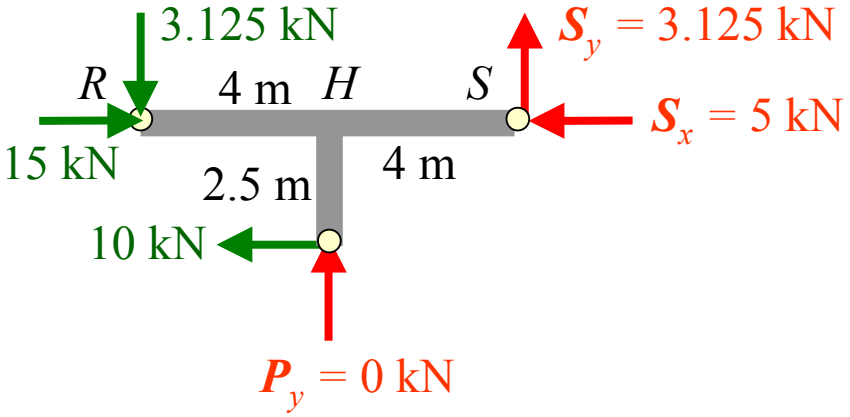


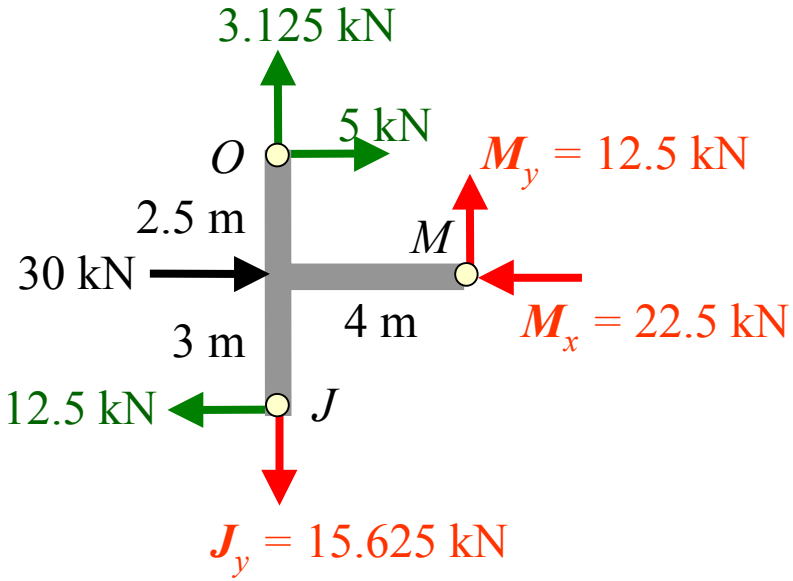
$\pm\rightarrow \Sigma F_x = 0$: $\quad 20 - 4V = 0 \quad V = 5$ kN

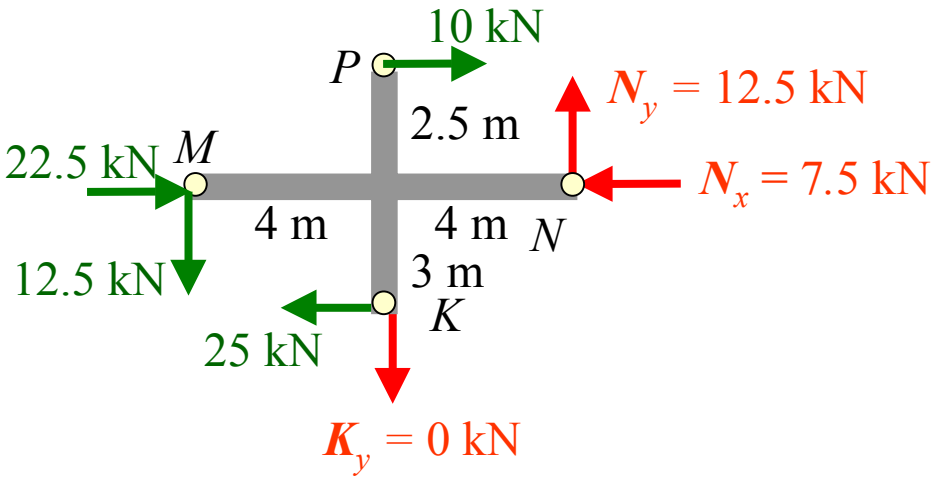


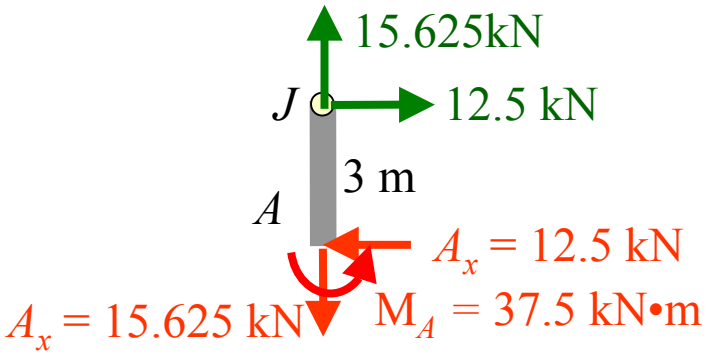
$\pm\rightarrow \Sigma F_x = 0$: $\quad 20 + 30 - 4V' = 0 \quad V' = 12.5$ kN

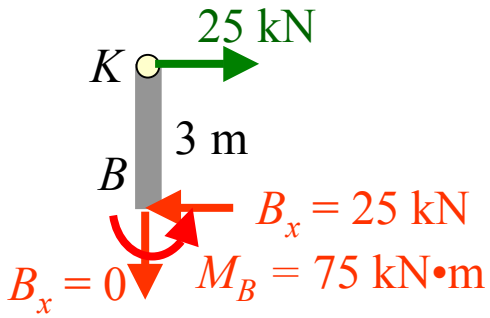

R_y = 3.125 kN
20 kN
G
4 m
R
R_x = 15 kN
2.5 m
5 kN
O_y = 3.125


3.125 kN
S_y = 3.125 kN
R
4 m
H
S
15 kN
S_x = 5 kN
2.5 m
4 m
10 kN
P_y = 0 kN

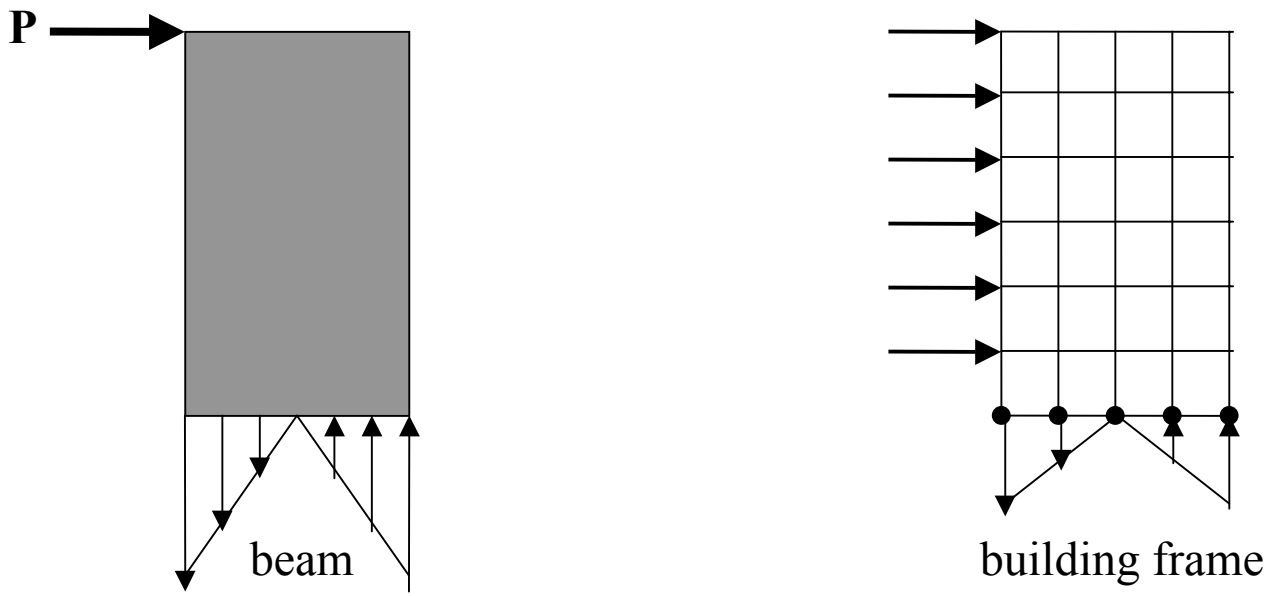

3.125 kN
5 kN
O
M_y = 12.5 kN
2.5 m
M
30 kN
4 m
M_x = 22.5 kN
3 m
12.5 kN
J
J_y = 15.625 kN


10 kN
P
N_y = 12.5 kN
2.5 m
22.5 kN
M
N_x = 7.5 kN
4 m
4 m
N
12.5 kN
3 m
K
25 kN
K_y = 0 kN


15.625kN
J
12.5 kN
3 m
A
A_x = 12.5 kN
A_x = 15.625 kN
M_A = 37.5 kN•m


25 kN
K
3 m
B
B_x = 25 kN
B_x = 0
M_B = 75 kN•m

Lateral Loads on Building Frames: Cantilever Method



In summary, using the cantilever method, the following assumptions apply to a fixed-supported frame.

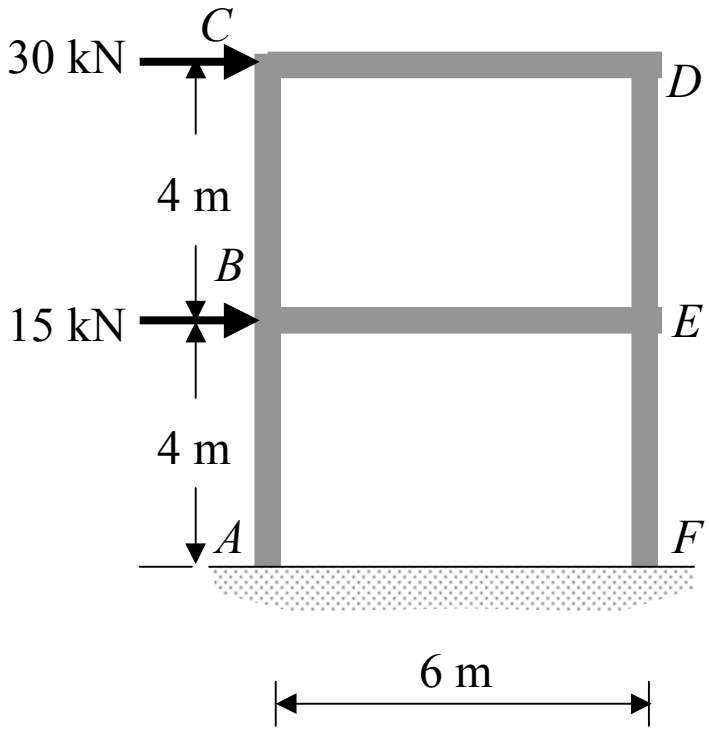
1. A hinge is place at the center of each girder, since this is assumed to be point of zero moment.

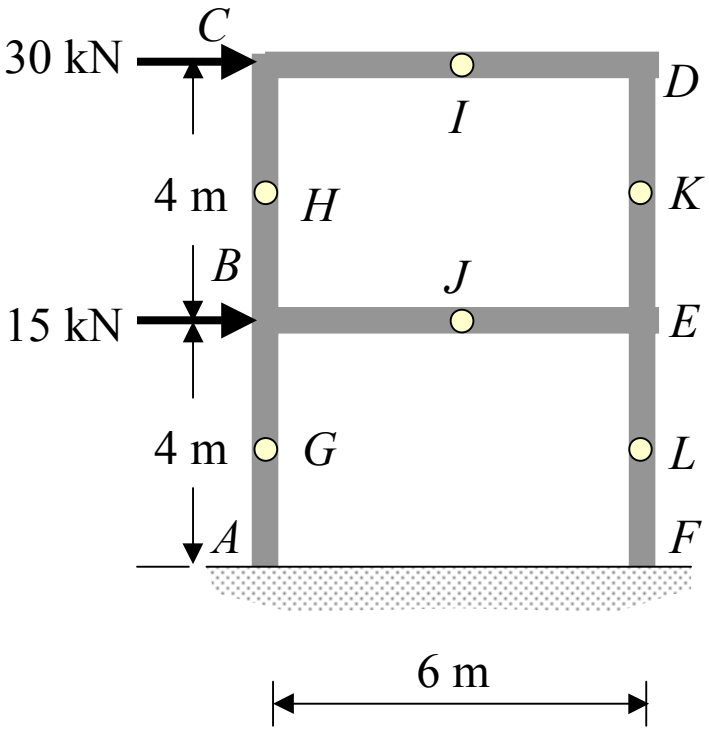
2. A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.

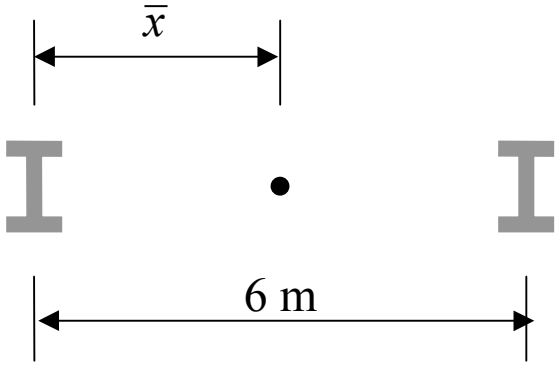
3. The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level. Since stress equals force per area, then in the special case of the columns having equal cross-sectional areas, the force in a column is also proportional to its distance from the centroid of the column areas.

Example 7

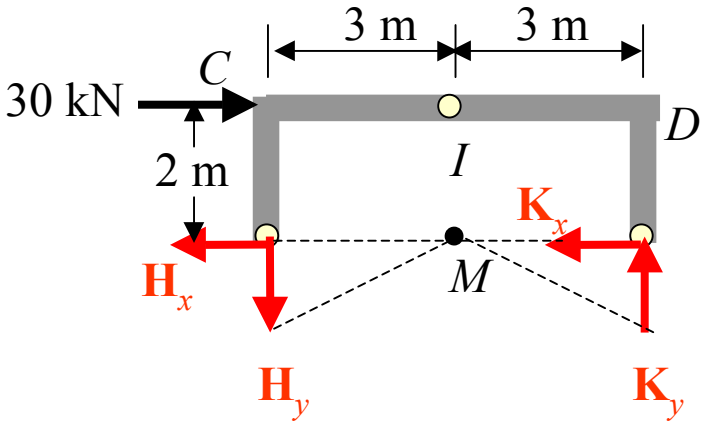
Determine (approximately) the reactions at the base of the columns of the frame shown. The columns are assumed to have equal crossectional areas. Use the cantilever method of analysis.







$$\bar{x} = \frac{\sum \tilde{x} A}{\sum A} = \frac{0(A) + 6(A)}{A + A} = 3$$

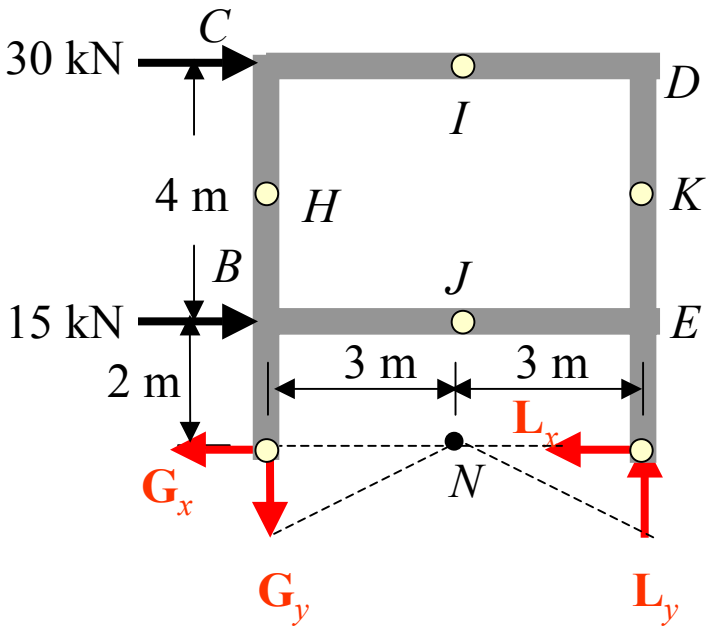


$+\circlearrowleft \Sigma M_M = 0$: $\quad -30(2) + 3H_y + 3K_y = 0$

The unknowns can be related by proportional triangles, that is

$$\frac{H_y}{3} = \frac{K_y}{3} \quad or \quad H_y = K_y$$

$$H_y = K_y = 10\,kN$$

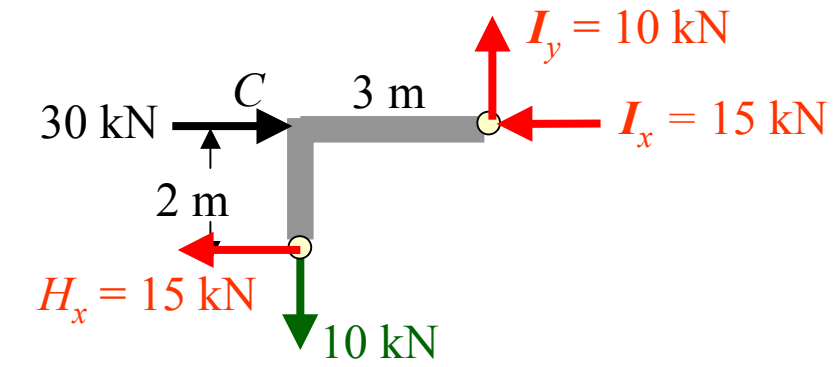


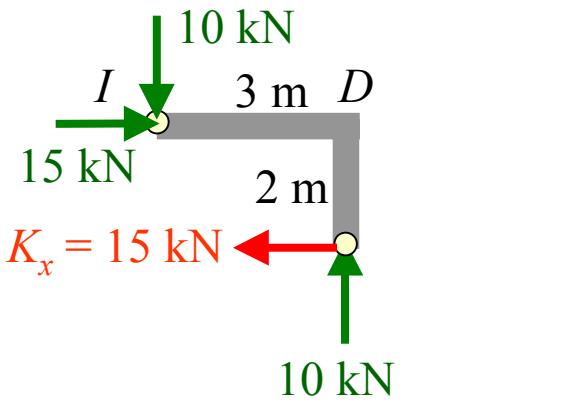
$+\circlearrowleft \Sigma M_N = 0$: $\quad -30(6) - 15(2) + 3G_y + 3L_y = 0$

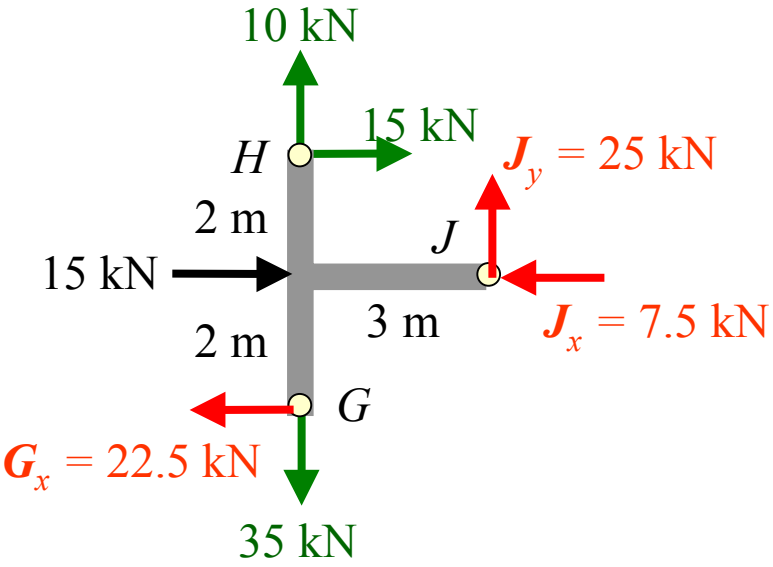
The unknowns can be related by proportional triangles, that is

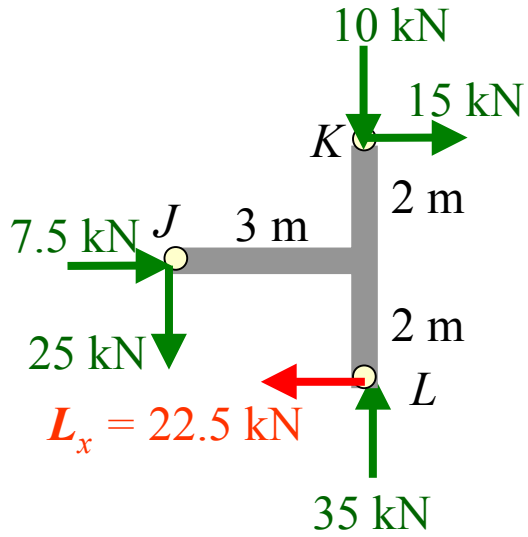
$$\frac{G_y}{3} = \frac{G_y}{3} \quad or \quad G_y = L_y$$

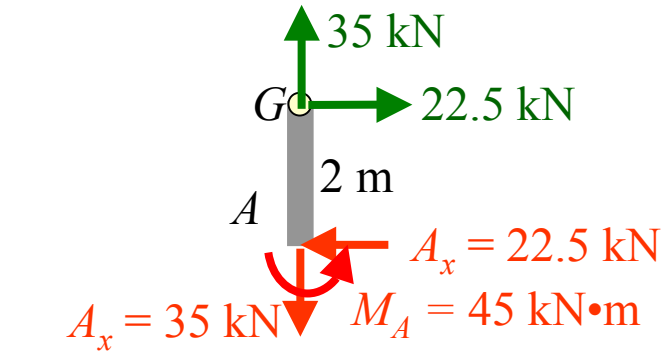
$$G_y = L_y = 35\,kN$$

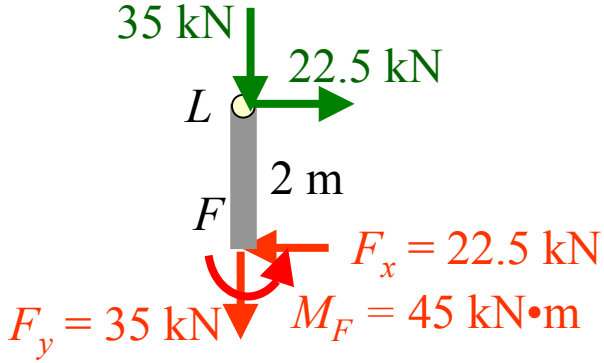

I_y = 10 kN
C
3 m
30 kN
I_x = 15 kN
2 m
H_x = 15 kN
10 kN


10 kN
I
3 m
D
15 kN
2 m
K_x = 15 kN
10 kN


10 kN
H
15 kN
J_y = 25 kN
2 m
J
15 kN
3 m
J_x = 7.5 kN
2 m
G
G_x = 22.5 kN
35 kN

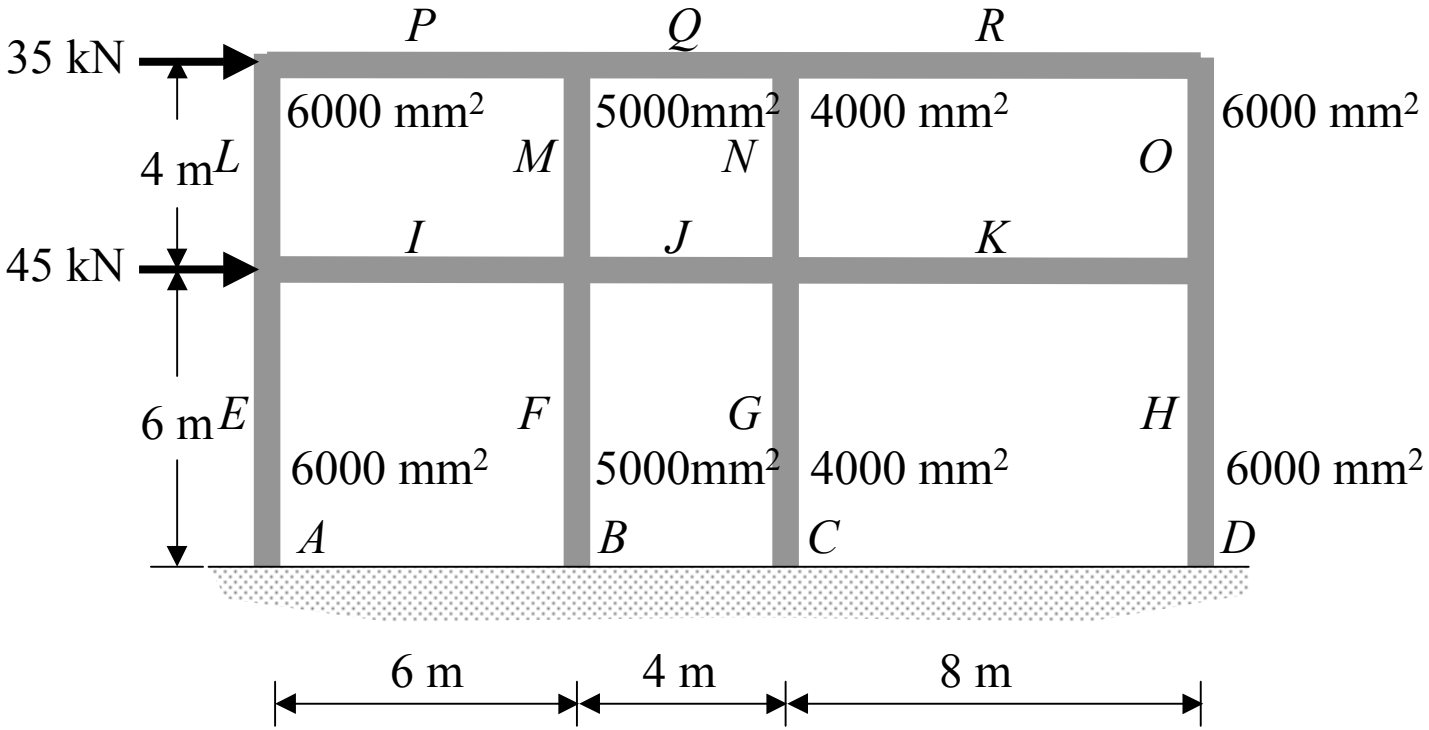

10 kN
15 kN
K
2 m
7.5 kN
J
3 m
2 m
25 kN
L
L_x = 22.5 kN
35 kN

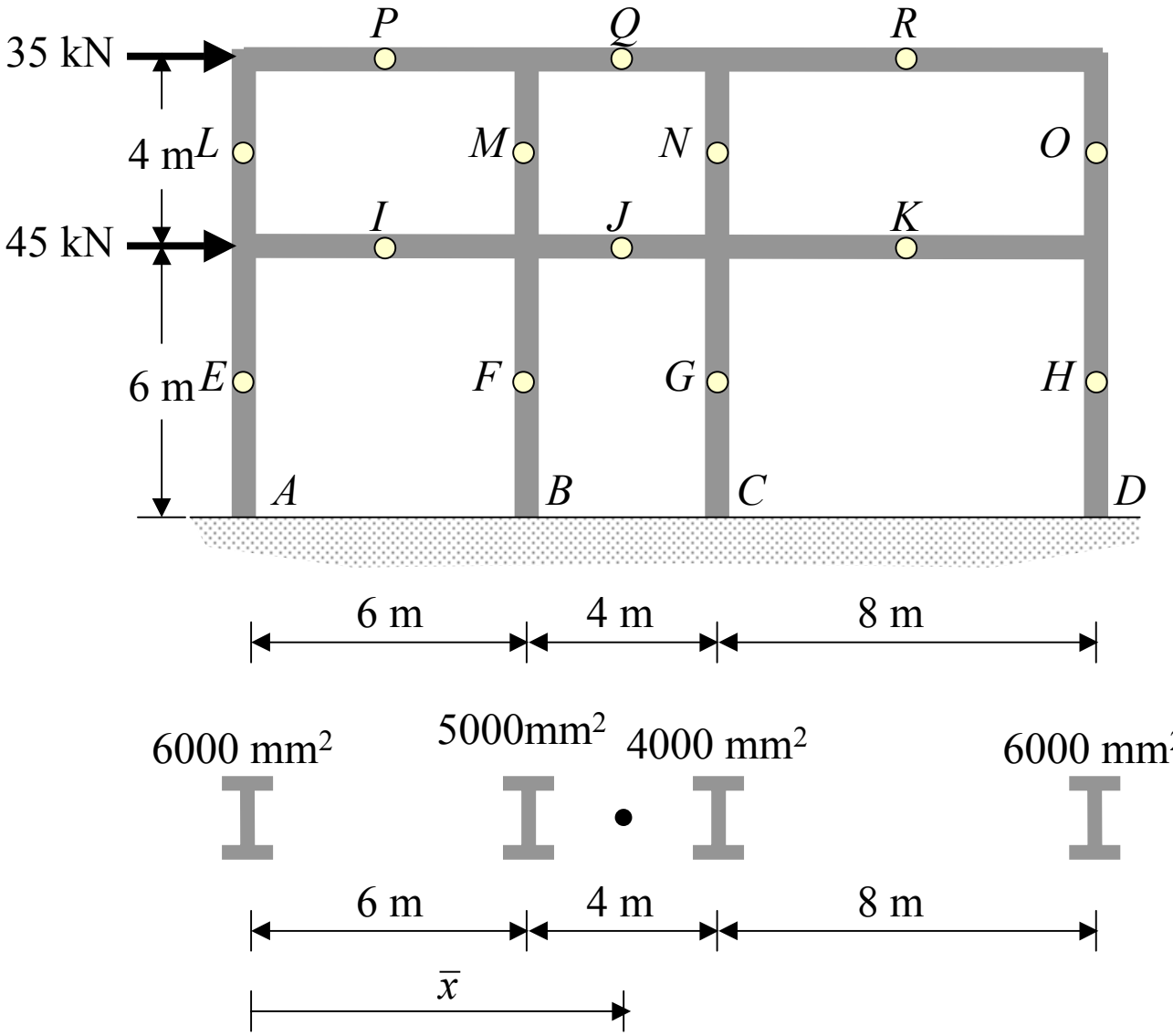

35 kN
G
22.5 kN
2 m
A
A_x = 22.5 kN
A_x = 35 kN
M_A = 45 kN•m


35 kN
22.5 kN
L
2 m
F
F_x = 22.5 kN
F_y = 35 kN
M_F = 45 kN•m

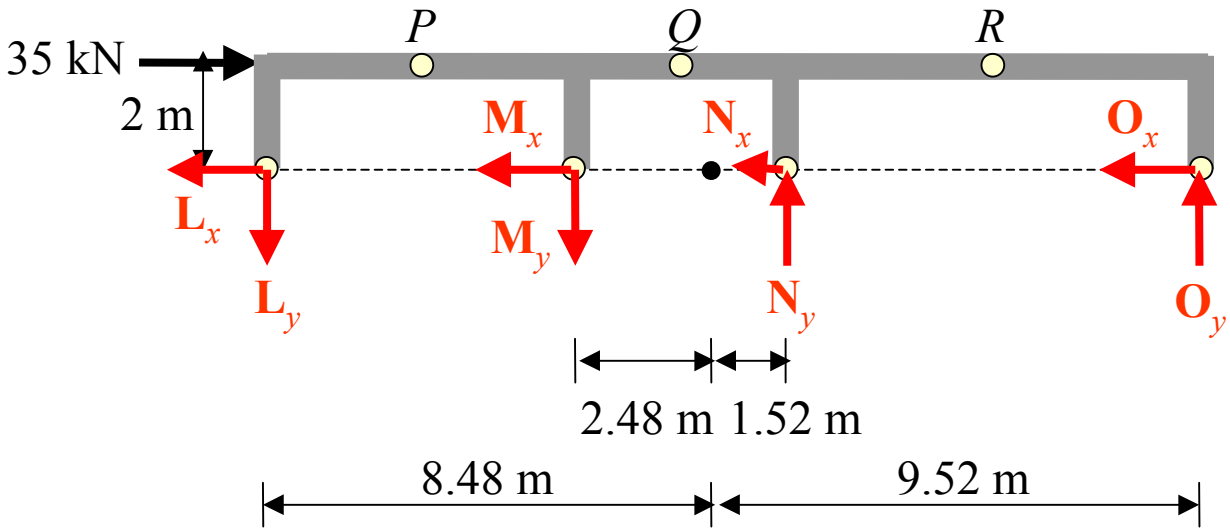
Example 8

Show how to determine (approximately) the reactions at the base of the columns of the frame shown. The columns have the crossectional areas show. Use the cantilever of analysis.





$$\bar{x} = \frac{\sum \tilde{x} A}{\sum A} = \frac{6000(0) + 5000(6) + 4000(10) + 6000(18)}{6000 + 5000 + 4000 + 6000} = 8.48\,m$$



$+\circlearrowleft \Sigma M_{NA} = 0$: $\quad -35(2) + L_y(8.48) + M_y(2.48) + N_y(1.52) + O_y(9.52) = 0 \quad$ -----(1)

Since any column stress σ is proportional to its distance from the neutral axis

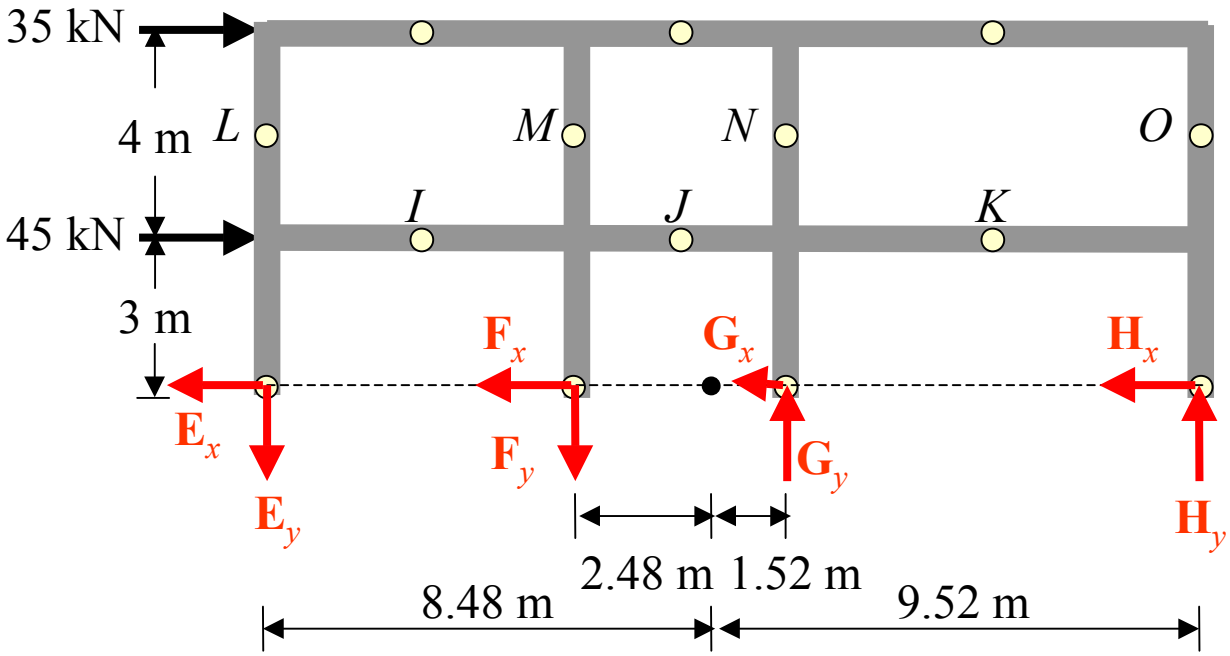
$$\frac{\sigma_M}{2.48} = \frac{\sigma_L}{8.48}; \quad \sigma_M = \frac{2.48}{8.48}\sigma_L; \quad \frac{M_y}{5000(10^{-6})} = \frac{2.48}{8.48}\left(\frac{L_y}{6000(10^{-6})}\right) \quad -----(2)$$

$$\frac{\sigma_N}{1.52} = \frac{\sigma_L}{8.48}; \quad \sigma_N = \frac{1.52}{8.48}\sigma_L; \quad \frac{N_y}{4000(10^{-6})} = \frac{1.52}{8.48}\left(\frac{L_y}{6000(10^{-6})}\right) \quad -----(3)$$

$$\frac{\sigma_O}{9.52} = \frac{\sigma_L}{8.48}; \quad \sigma_O = \frac{9.52}{8.48}\sigma_L; \quad \frac{O_y}{6000(10^{-6})} = \frac{9.52}{8.48}\left(\frac{L_y}{6000(10^{-6})}\right) \quad -----(4)$$

Solving Eqs. (1) - (4) yields $\quad L_y = 3.508$ kN $\quad M_y = 0.855$ kN

$N_y = 0.419$ kN $\quad O_y = 3.938$ kN



$+\circlearrowleft \Sigma M_{NA} = 0$: $\quad -45(3) - 35(7) + E_y(8.48) + F_y(2.48) + G_y(1.52) + H_y(9.52) = 0 \quad$ -----(5)

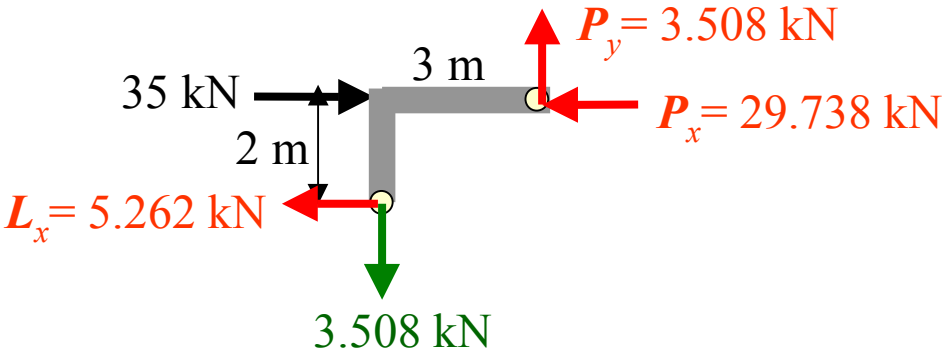
Since any column stress σ is proportional to its distance from the neutral axis ;

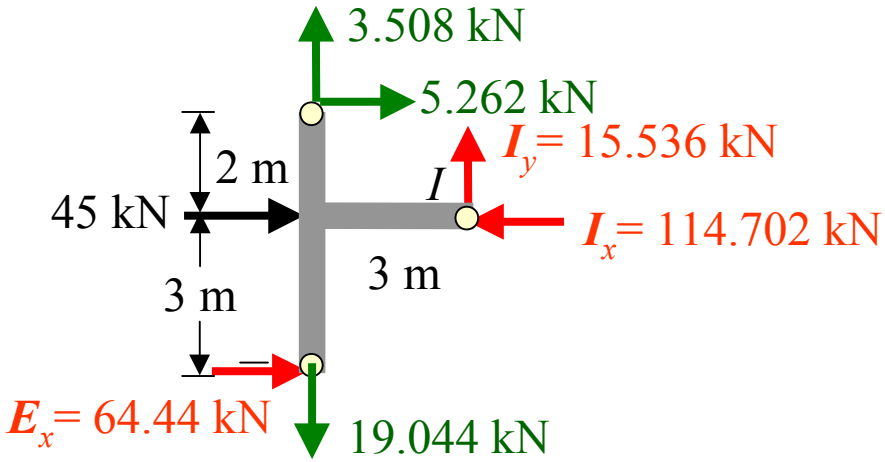
$$\frac{\sigma_F}{2.48} = \frac{\sigma_E}{8.48}; \quad \sigma_F = \frac{2.48}{8.48}\sigma_E; \quad \frac{F_y}{5000(10^{-6})} = \frac{2.48}{8.48}\left(\frac{E_y}{6000(10^{-6})}\right) \qquad -----(6)$$

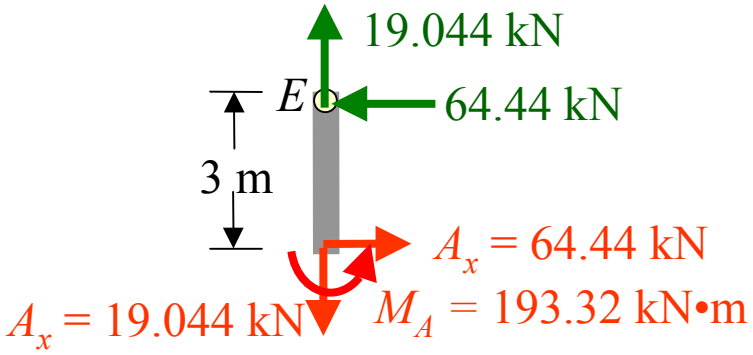
$$\frac{\sigma_G}{1.52} = \frac{\sigma_E}{8.48}; \quad \sigma_G = \frac{1.52}{8.48}\sigma_E; \quad \frac{G_y}{4000(10^{-6})} = \frac{1.52}{8.48}\left(\frac{E_y}{6000(10^{-6})}\right) \qquad -----(7)$$

$$\frac{\sigma_H}{9.52} = \frac{\sigma_E}{8.48}; \quad \sigma_H = \frac{9.52}{8.48}\sigma_E; \quad \frac{H_y}{6000(10^{-6})} = \frac{9.52}{8.48}\left(\frac{E_y}{6000(10^{-6})}\right) \qquad -----(8)$$

Solving Eqs. (1) - (4) yields $E_y = 19.044$ kN $F_y = 4.641$ kN $G_y = 2.276$ kN $H_y = 21.38$ kN







One can continue to analyze the other segments in sequence, i.e., *PQM*, then *MJFI*, then *FB*, and so on.