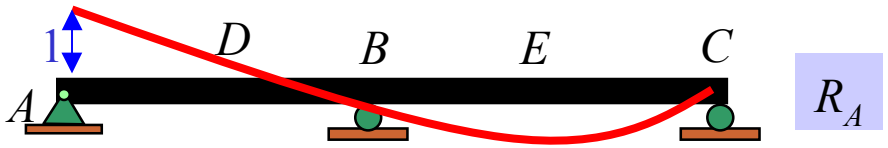
INFLUENCE LINES FOR STATICALLY INDETERMINATE BEAMS

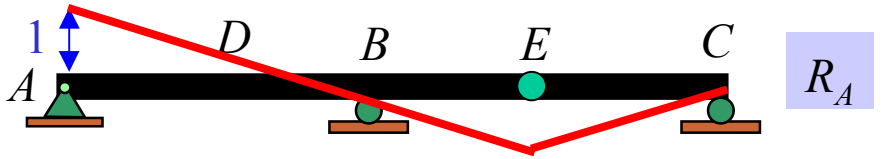
- Comparison Between Indeterminate and Determinate
- Influence line for Statically Indeterminate Beams
- Qualitative Influence Lines for Frames

Comparison between Indeterminate and Determinate

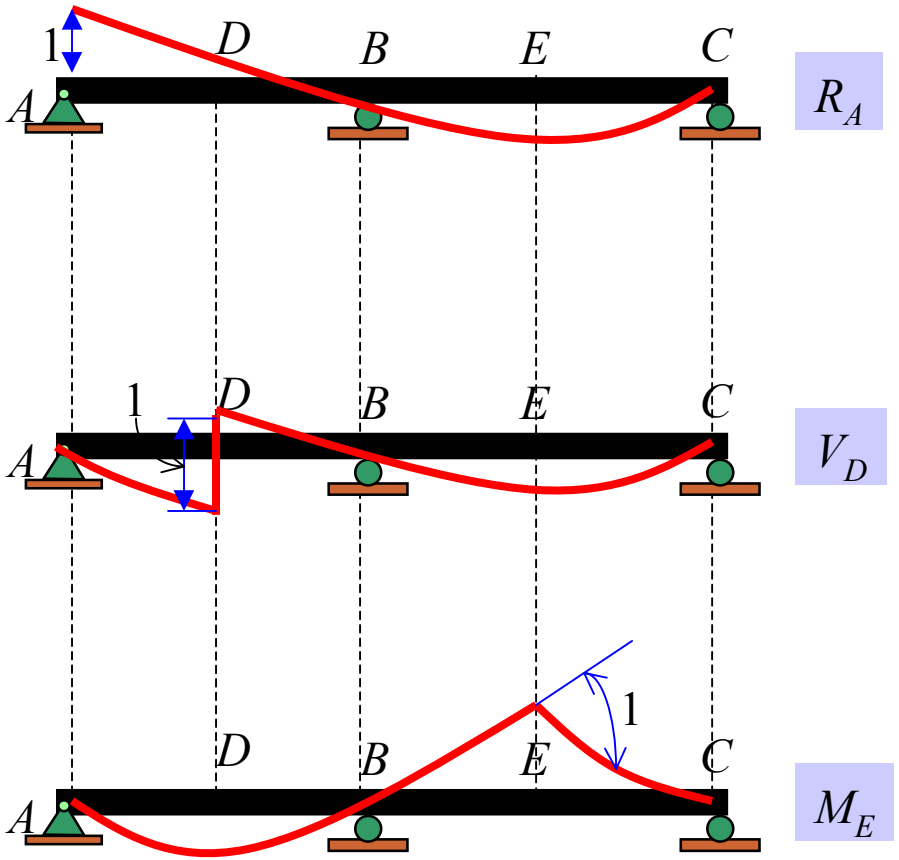
Indeterminate



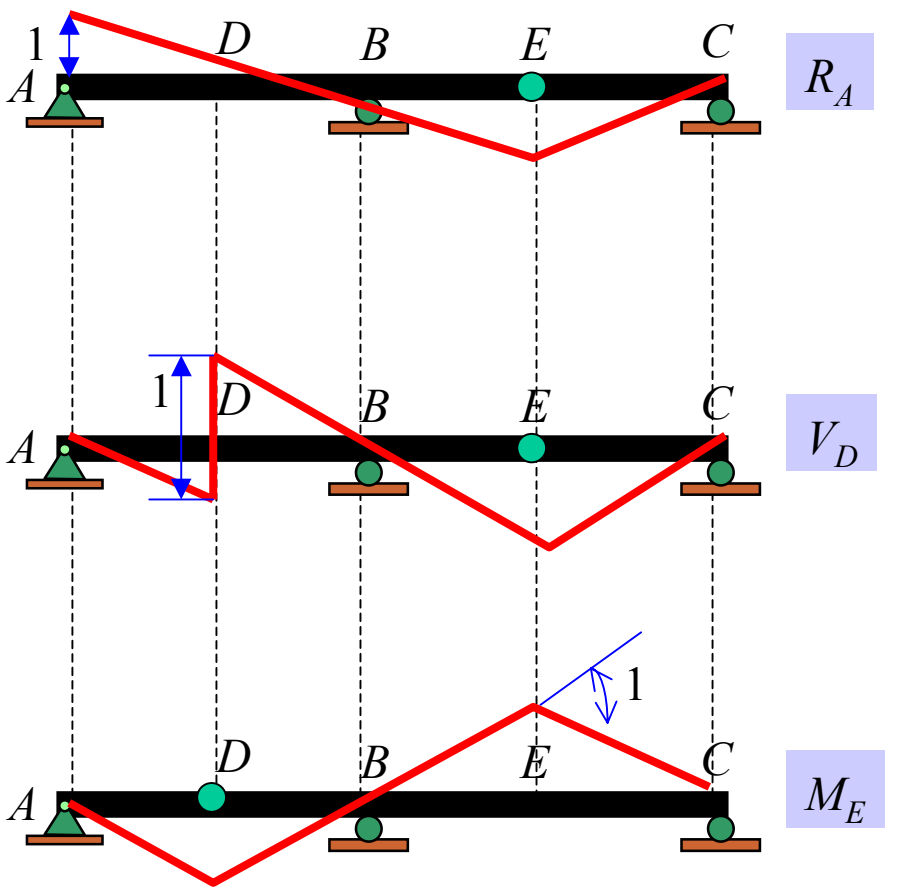
Determinate



Indeterminate

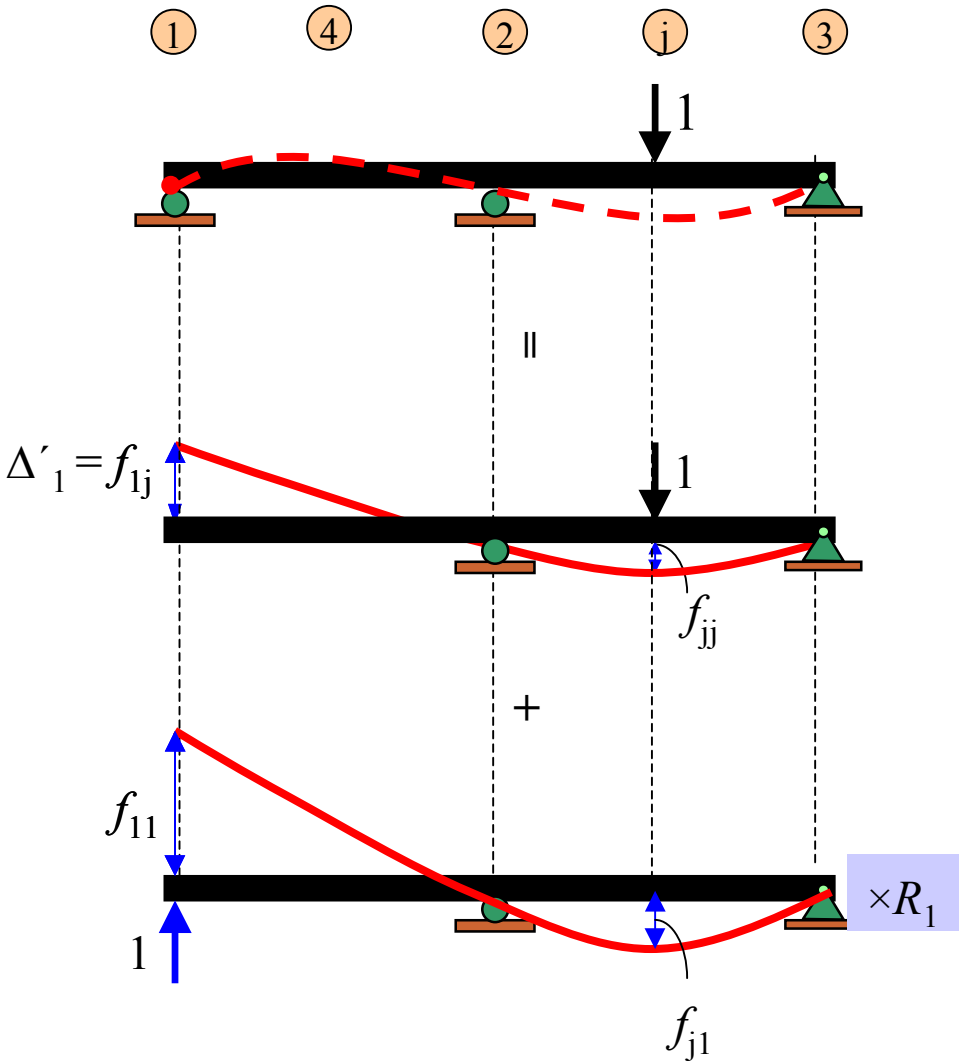


Determinate



Influence Lines for Reaction

▶ **Redundant R_1 applied**



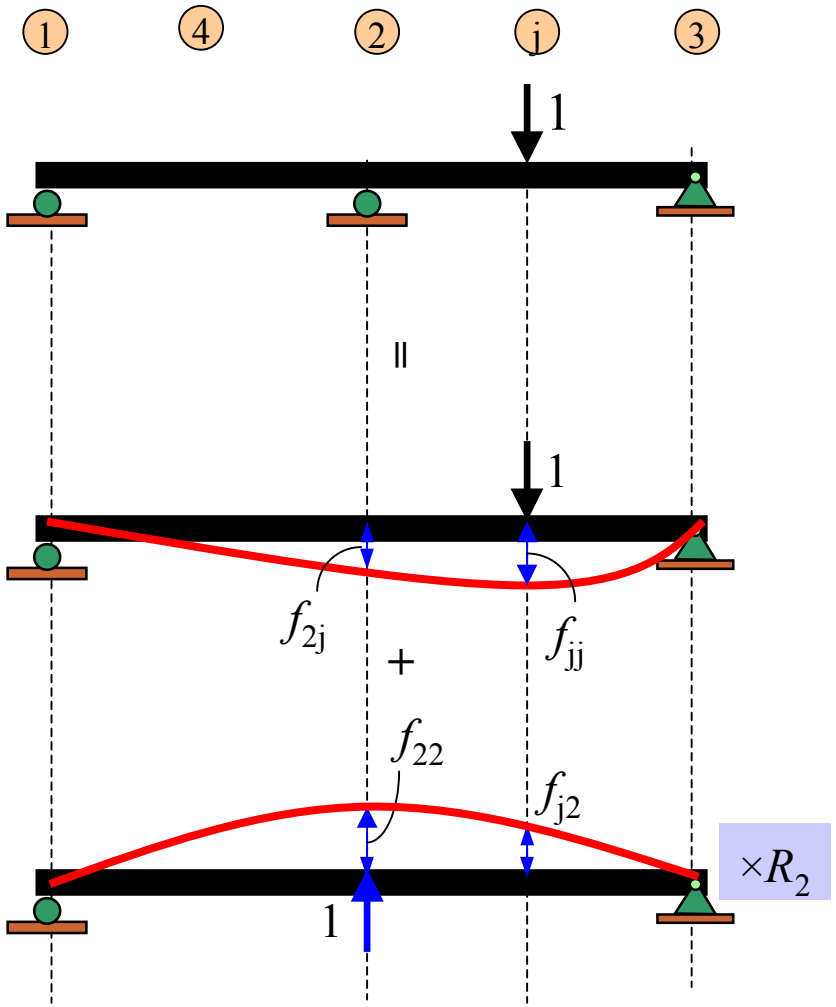
Compatibility equation:

$$f_{1j} + f_{11}R_1 = \Delta_1 = 0$$

$$R_1 = -f_{1j}\left(\frac{1}{f_{11}}\right)$$

$$R_1 = \left(\frac{f_{j1}}{f_{11}}\right)$$

▶ **Redundant R_2 applied**



Compatibility equation.

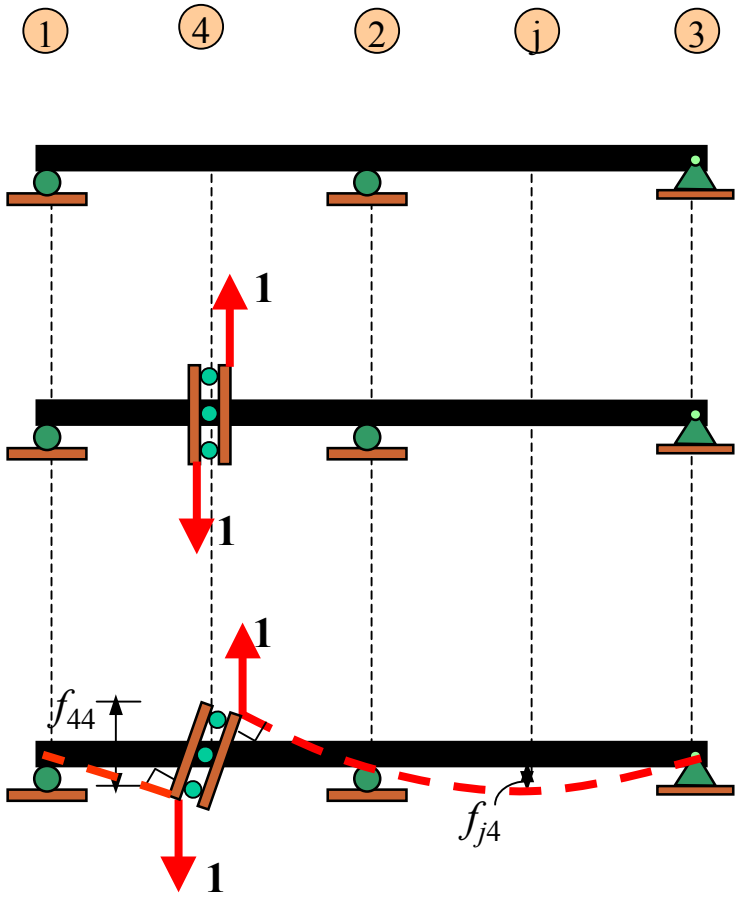
$$\Delta'_2 + f_{22}R_2 = \Delta_2 = 0$$

$$f_{2j} + f_{22}R_2 = \Delta_2 = 0$$

$$R_2 = -f_{2j}\left(\frac{1}{f_{22}}\right)$$

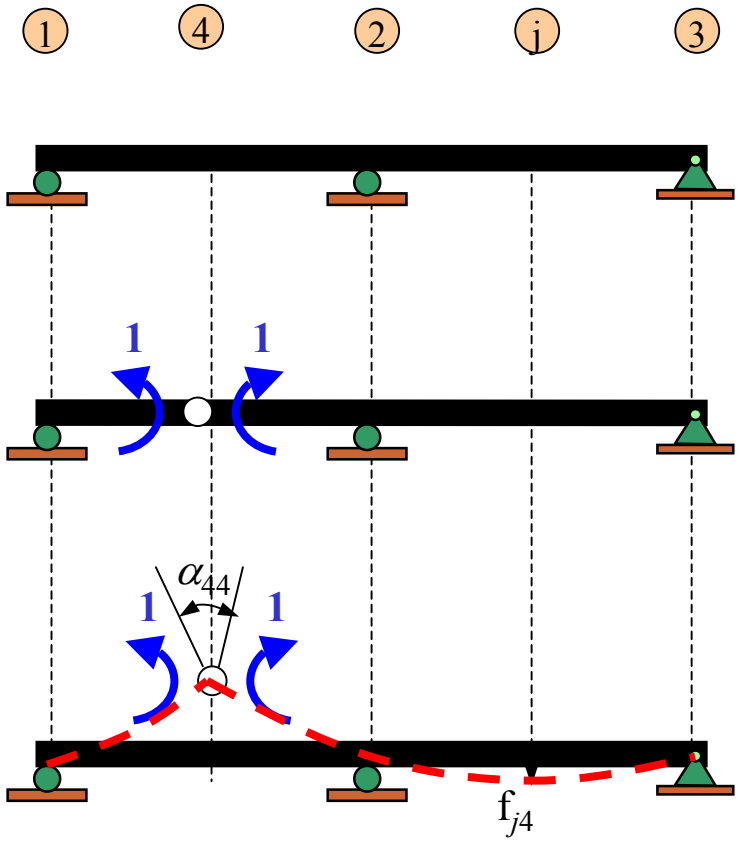
$$R_2 = \left(\frac{f_{j2}}{f_{22}}\right)$$

Influence Lines for Shear



$$V_E = \left(\frac{1}{f_{44}}\right) f_{j4}$$

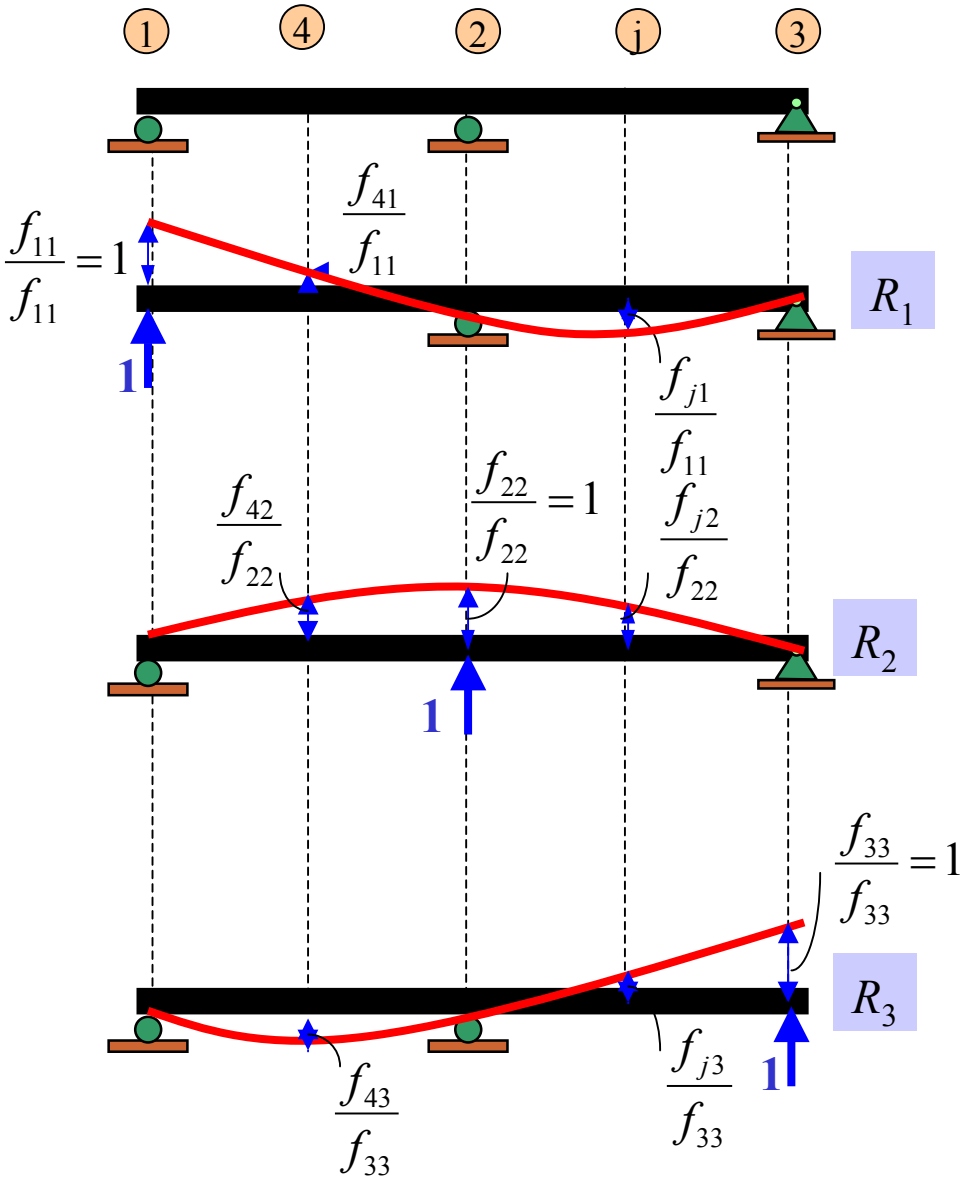
Influence Lines for Bending Moment



$$M_E = (\frac{1}{\alpha_{44}}) f_{j4}$$

Using Equilibrium Condition for Shear and Bending Moment

- **Influence line of *Reaction***

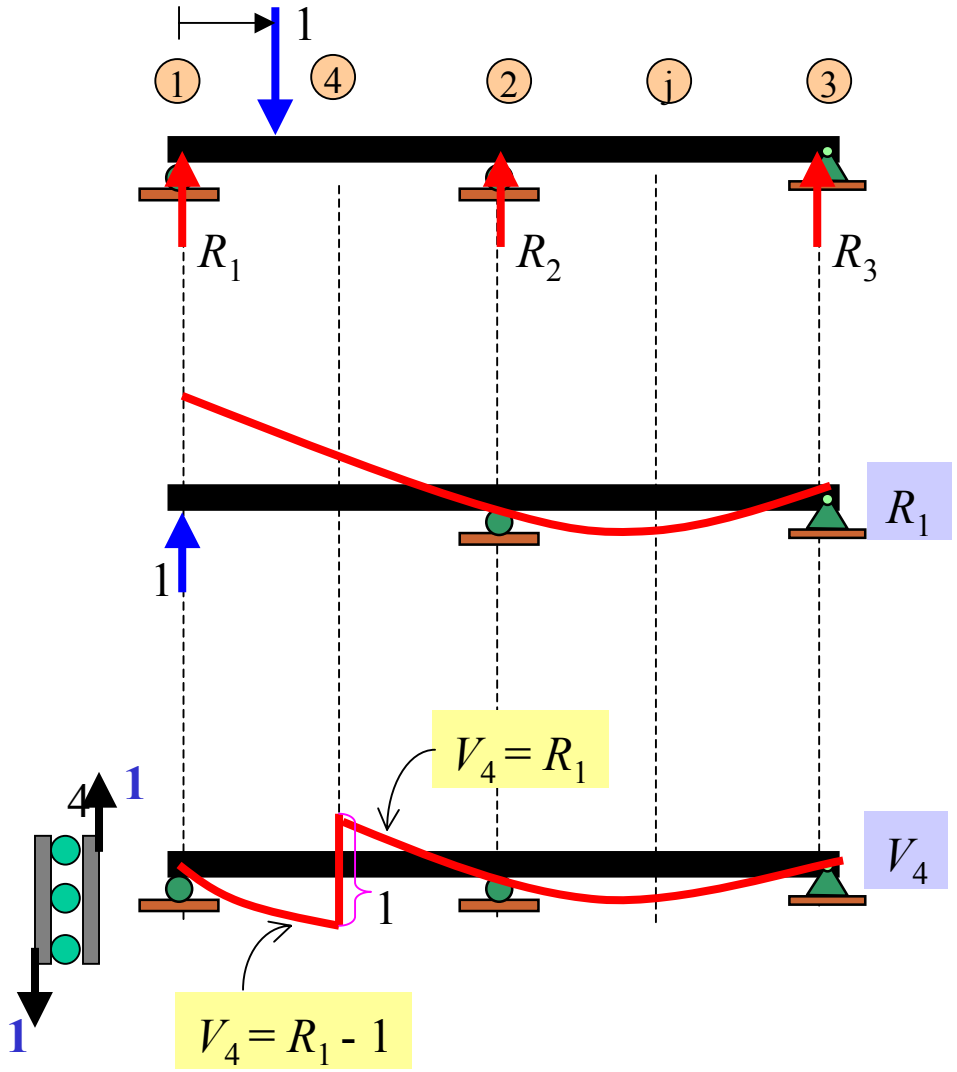


$$R_1=\left(\frac{f_{j1}}{f_{11}}\right)$$

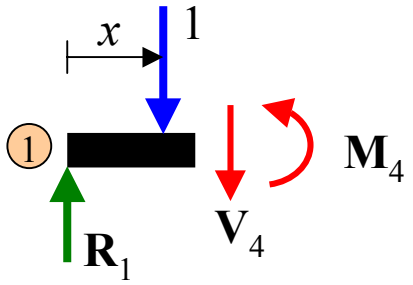
$$R_2=\left(\frac{f_{j2}}{f_{22}}\right)$$

$$R_3=\left(\frac{f_{j3}}{f_{33}}\right)$$

• **Influence line of *Shear***



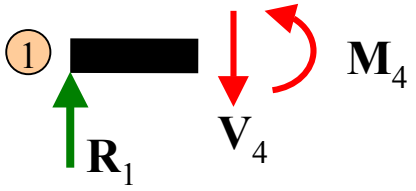
• Unit load **to the left of** 4



$$+\uparrow \Sigma F_y = 0; \quad R_1 - 1 - V_4 = 0$$

$$V_4 = R_1 - 1$$

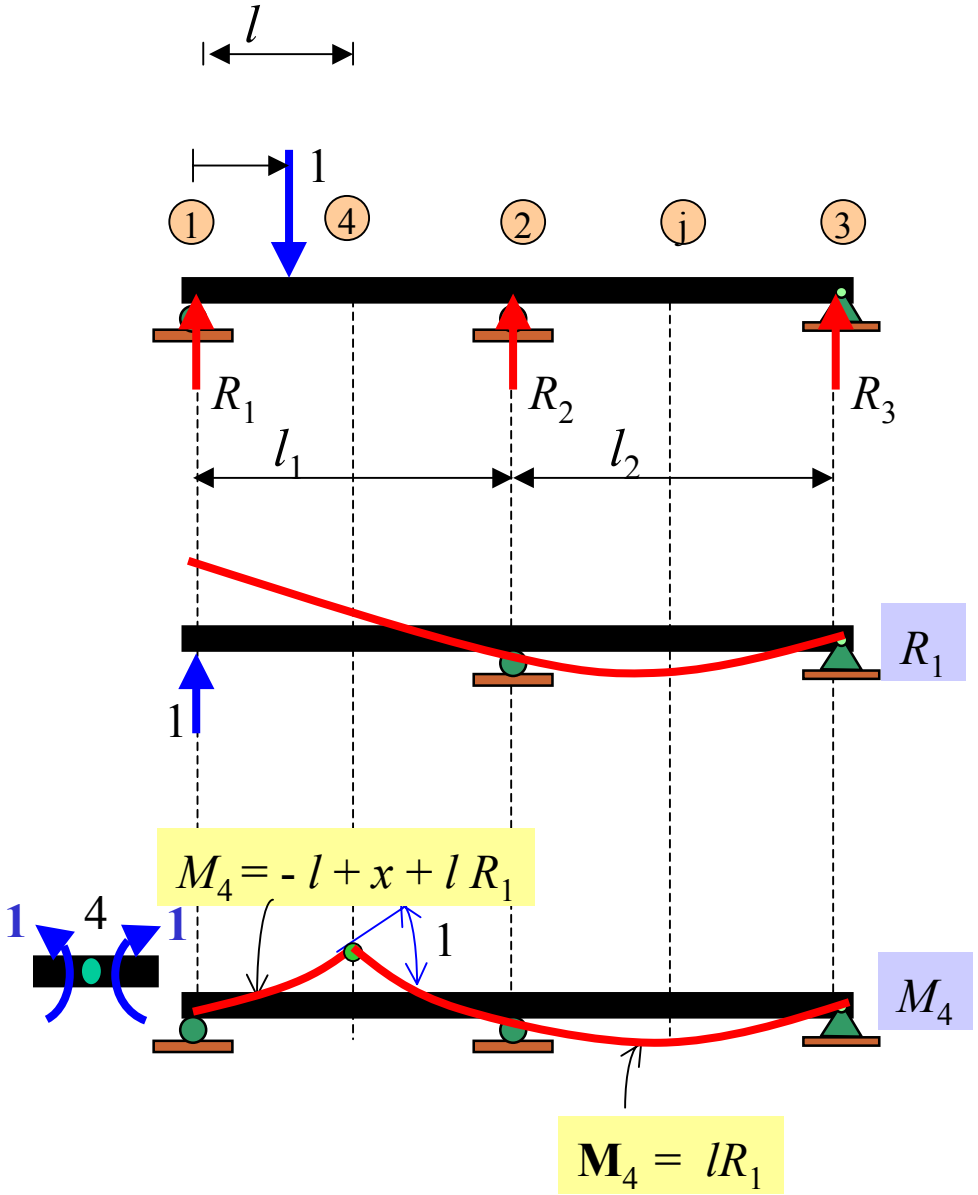
• Unit load **to the right of** 4



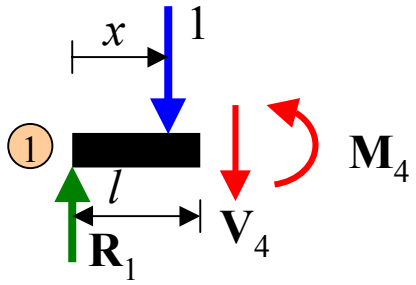
$$+\uparrow \Sigma F_y = 0; \quad R_1 - V_4 = 0$$

$$V_4 = R_1$$

• **Influence line of *Bending moment***



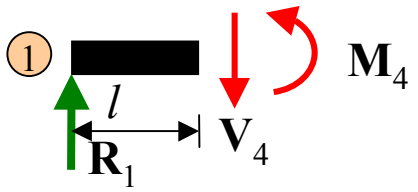
• Unit load **to the left** of ④



$\circlearrowleft + \Sigma M_4 = 0$: $M_4 - 1\,(l\text{-}x) - l\,R_1 = 0$

$M_4 = -\,l + x + l\,R_1$

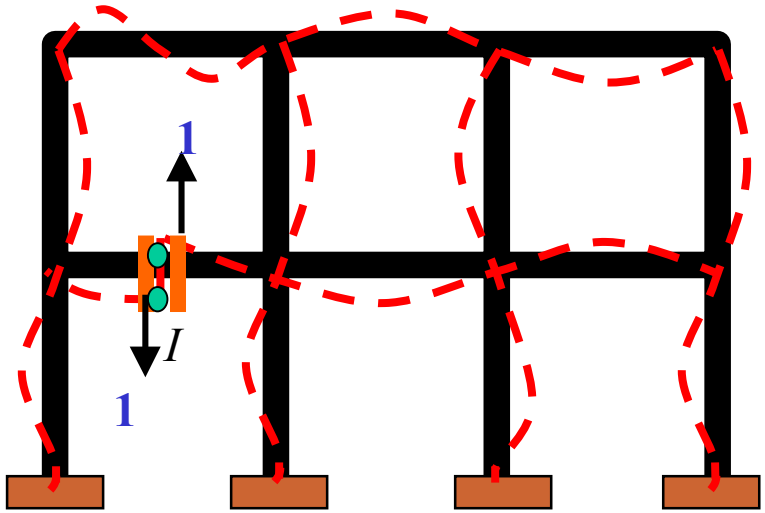
• Unit load **to right** of ④



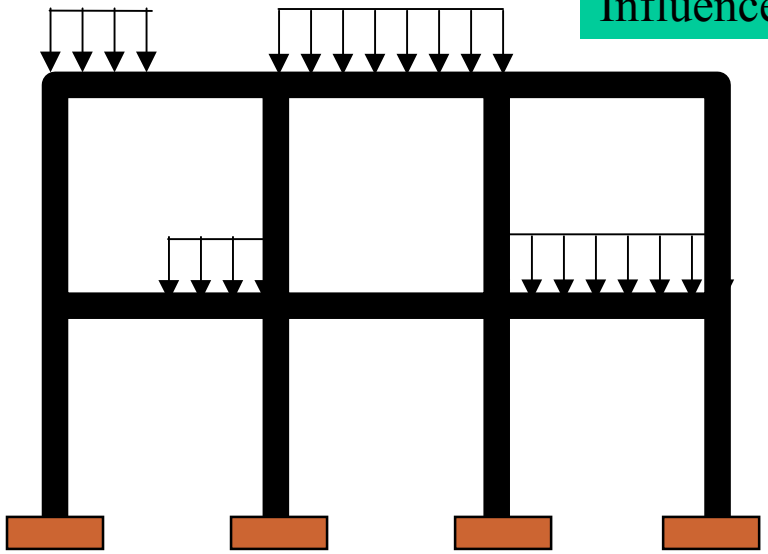
$\circlearrowleft + \Sigma M_4 = 0$: $M_4 - l\,R_1 = 0$

$M_4 = l\,R_1$

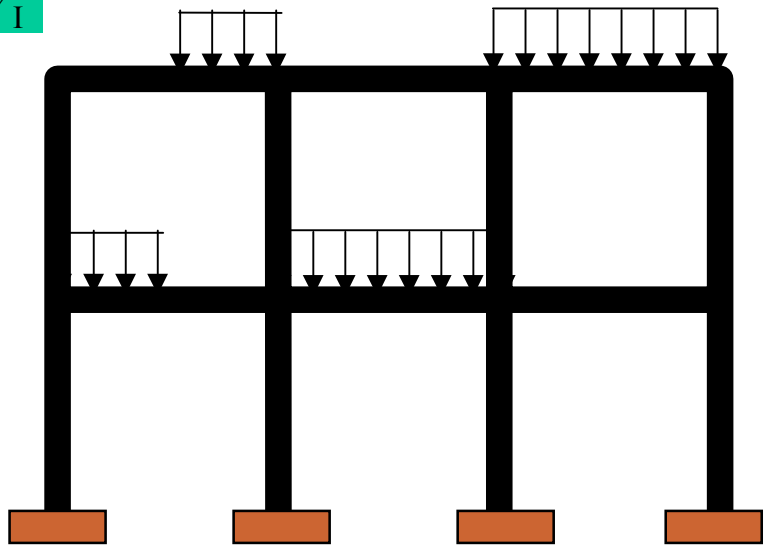
Qualitative Influence Lines for Frames



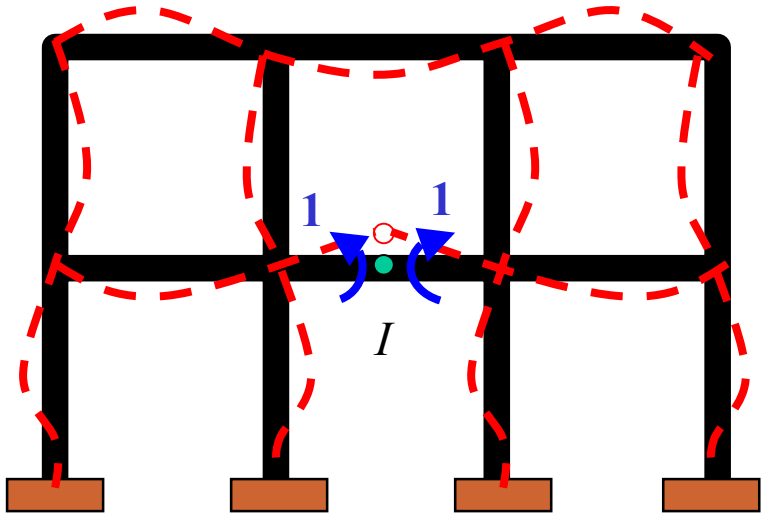
Influence Line of V_I



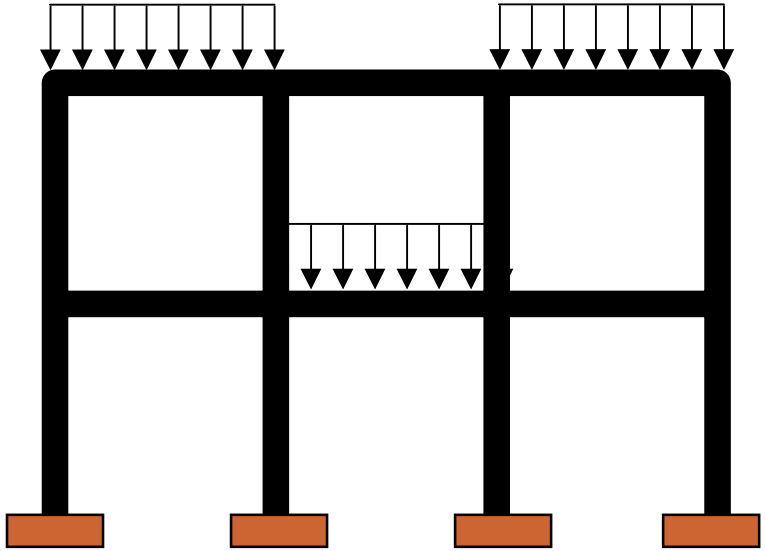
Maximum **positive** shear



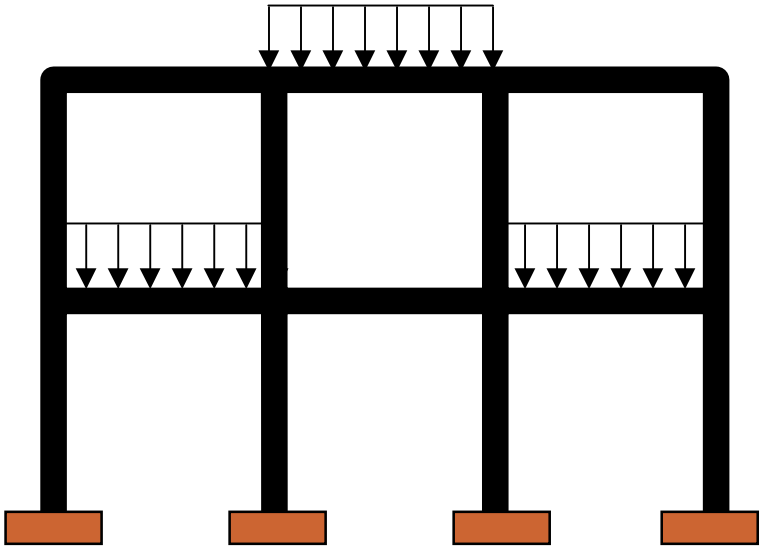
Maximum **negative** shear



Influence Line of M_I

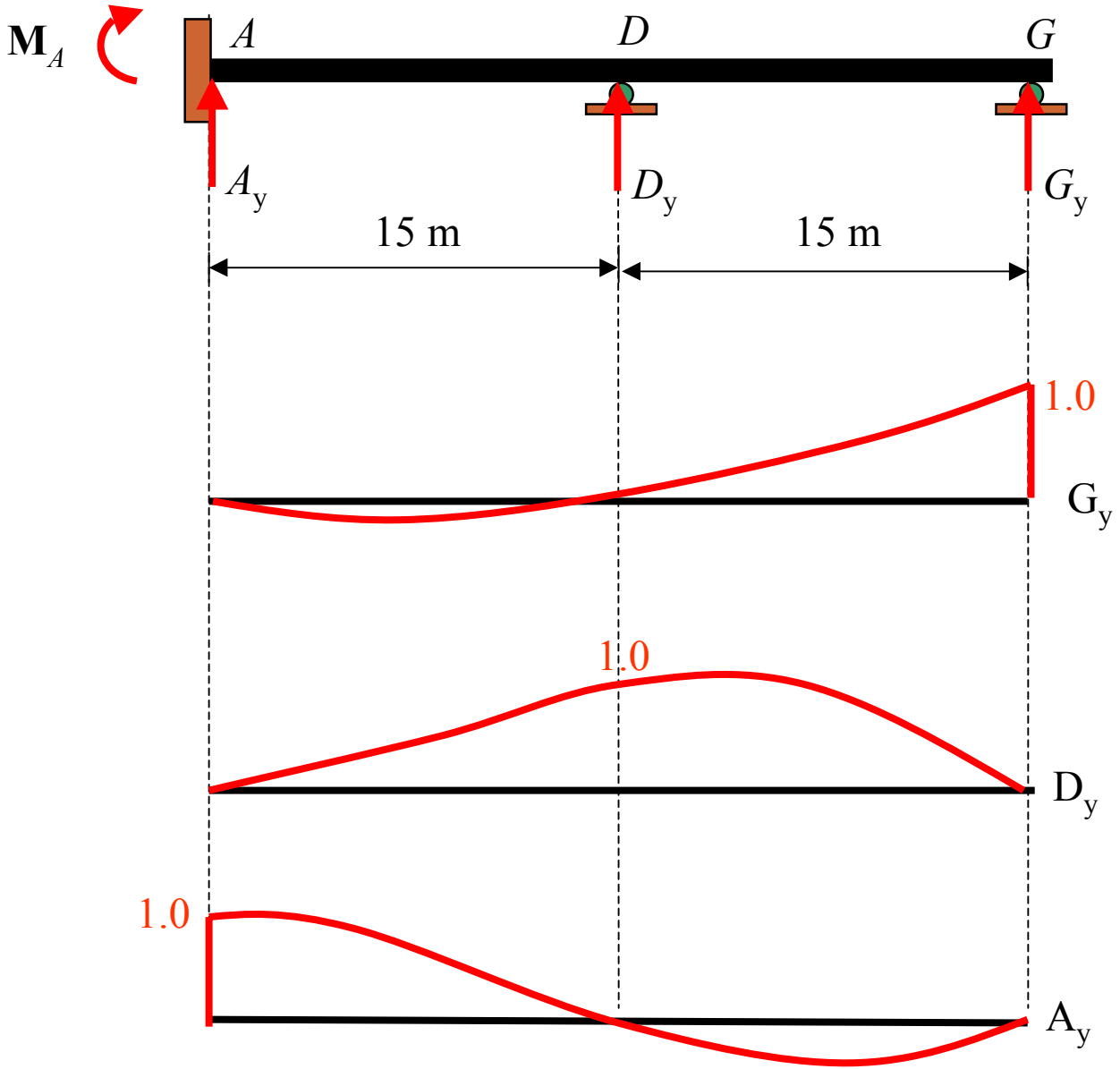


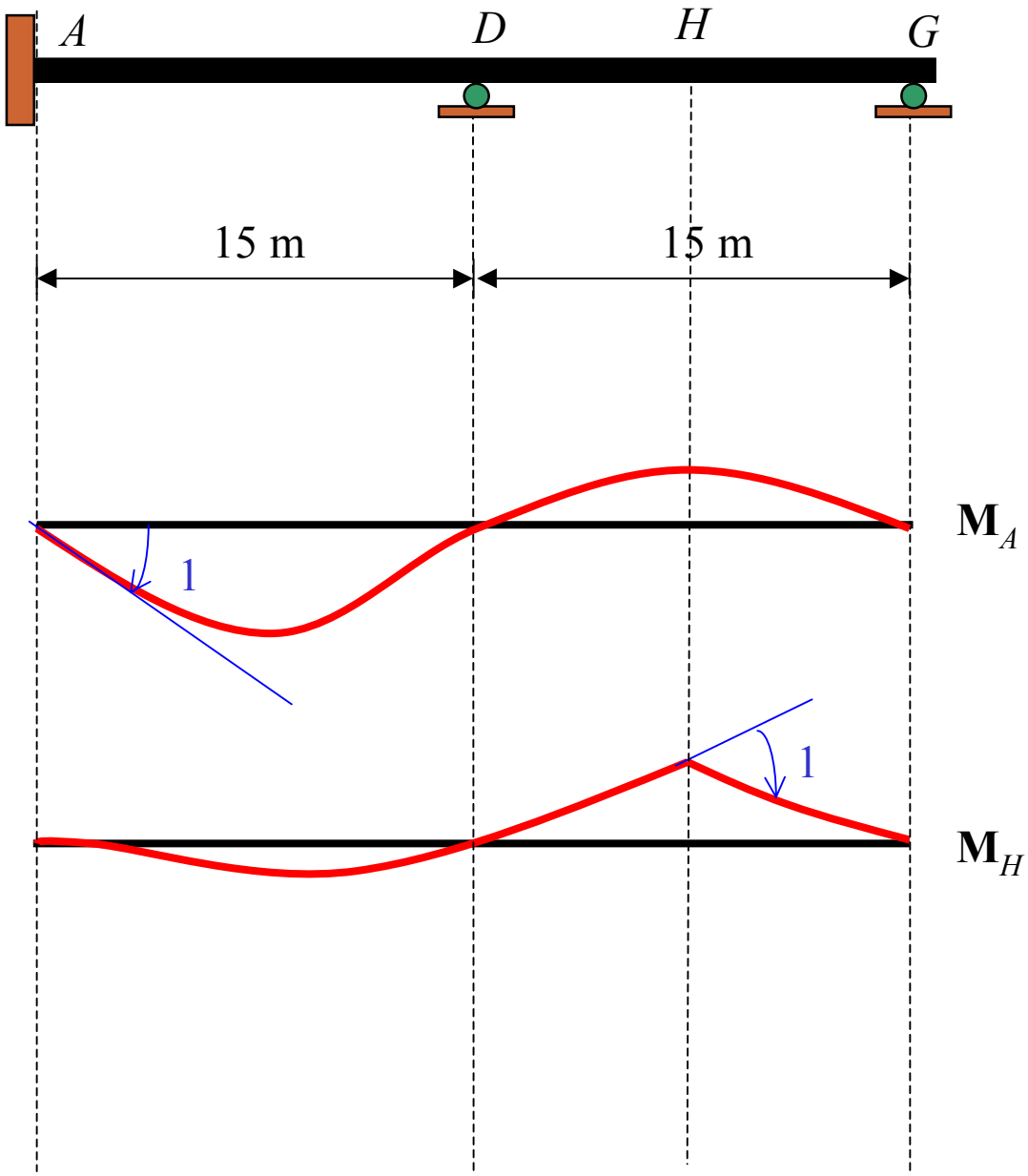
Maximum **positive** moment

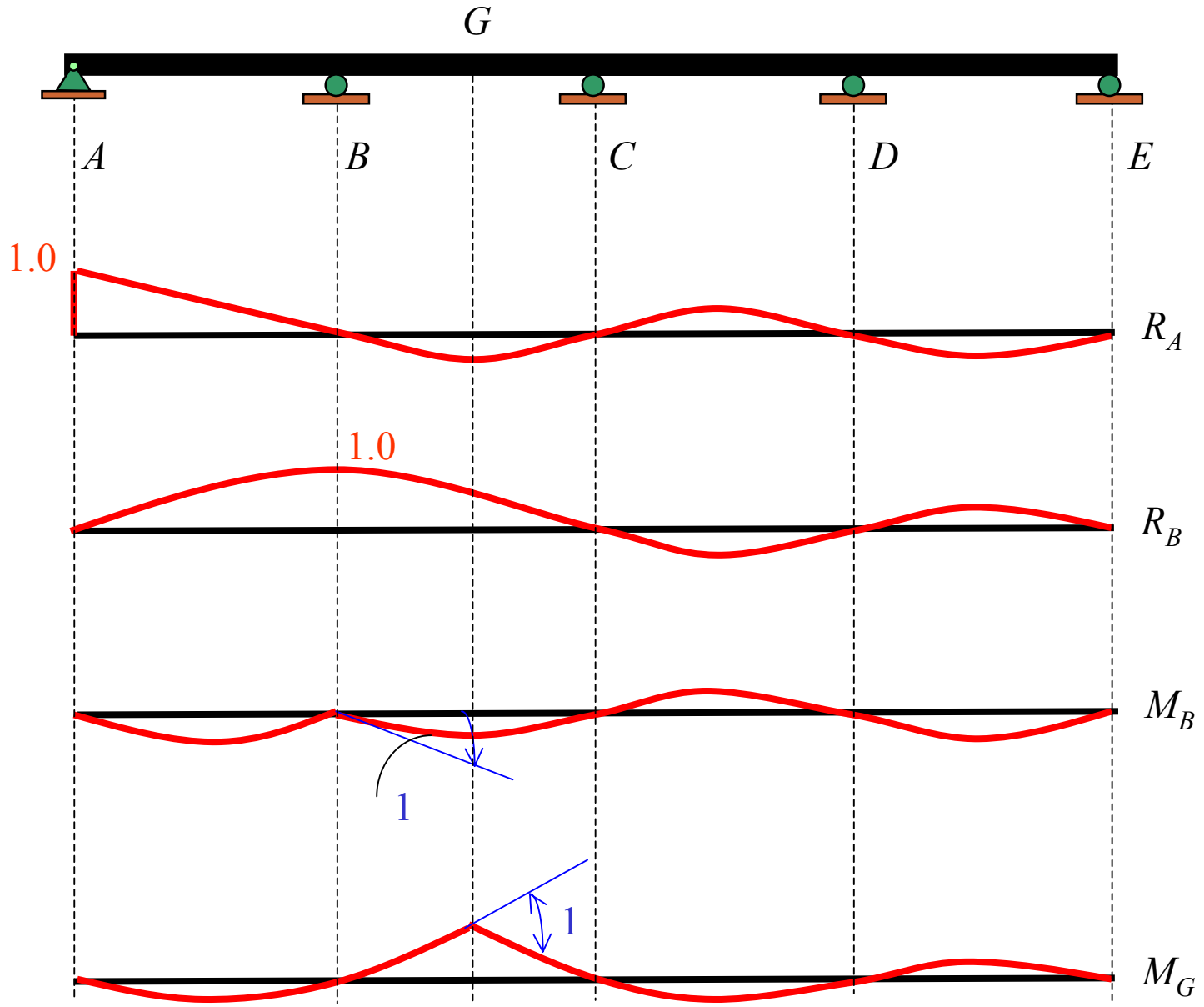


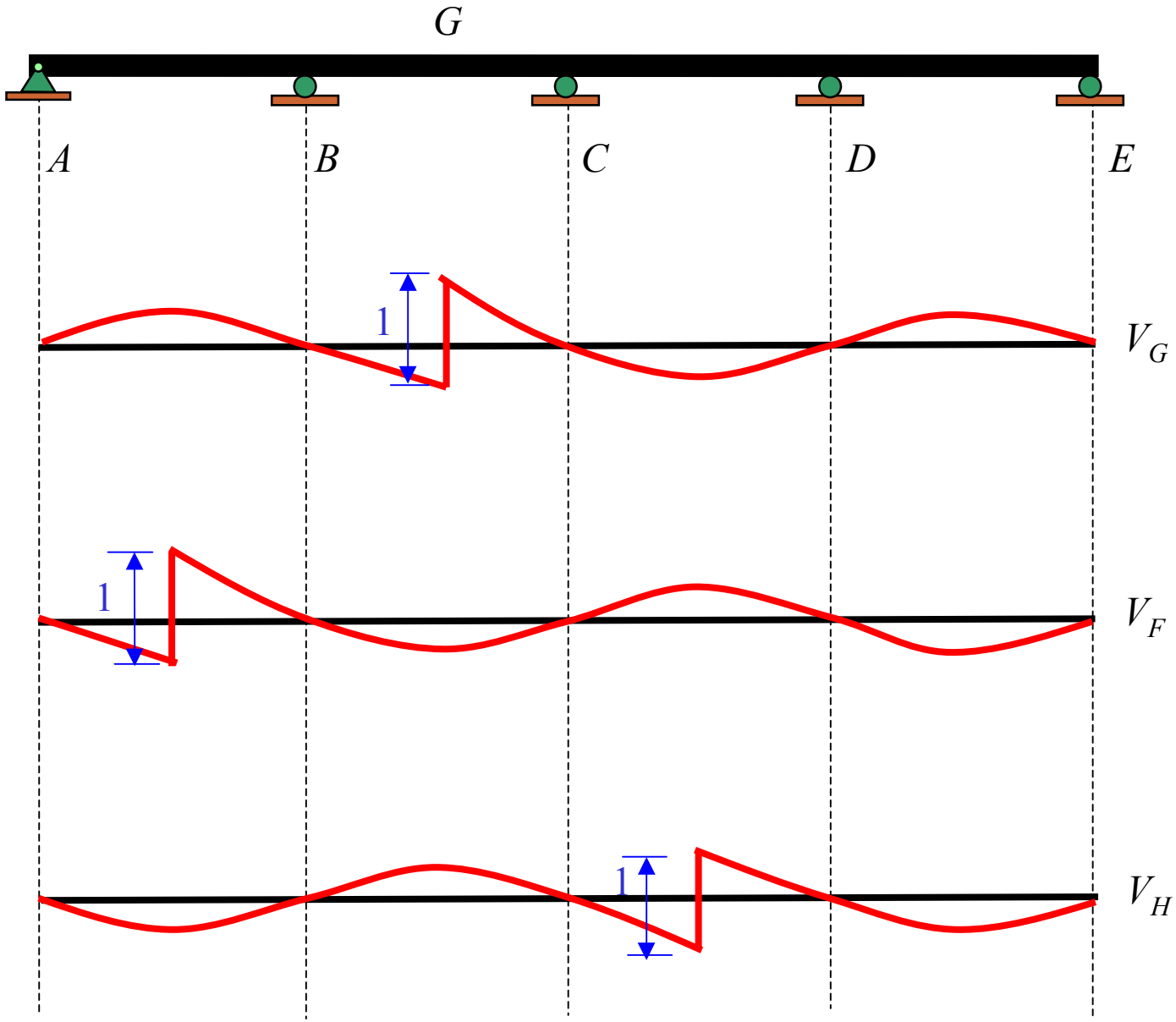
Maximum **negative** moment

▶ Influence Line for MOF




A
D
H
G
15 m
15 m
1
1
M_A
M_H


G
A
B
C
D
E
1.0
R_A
1.0
R_B
M_B
1
1
M_G

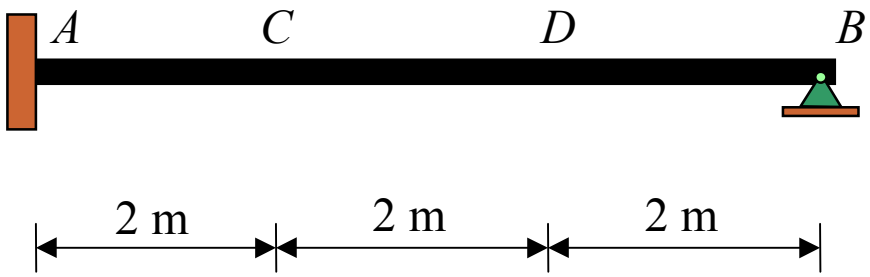

G
A
B
C
D
E
1
V_G
1
V_F
1
V_H

Example 1

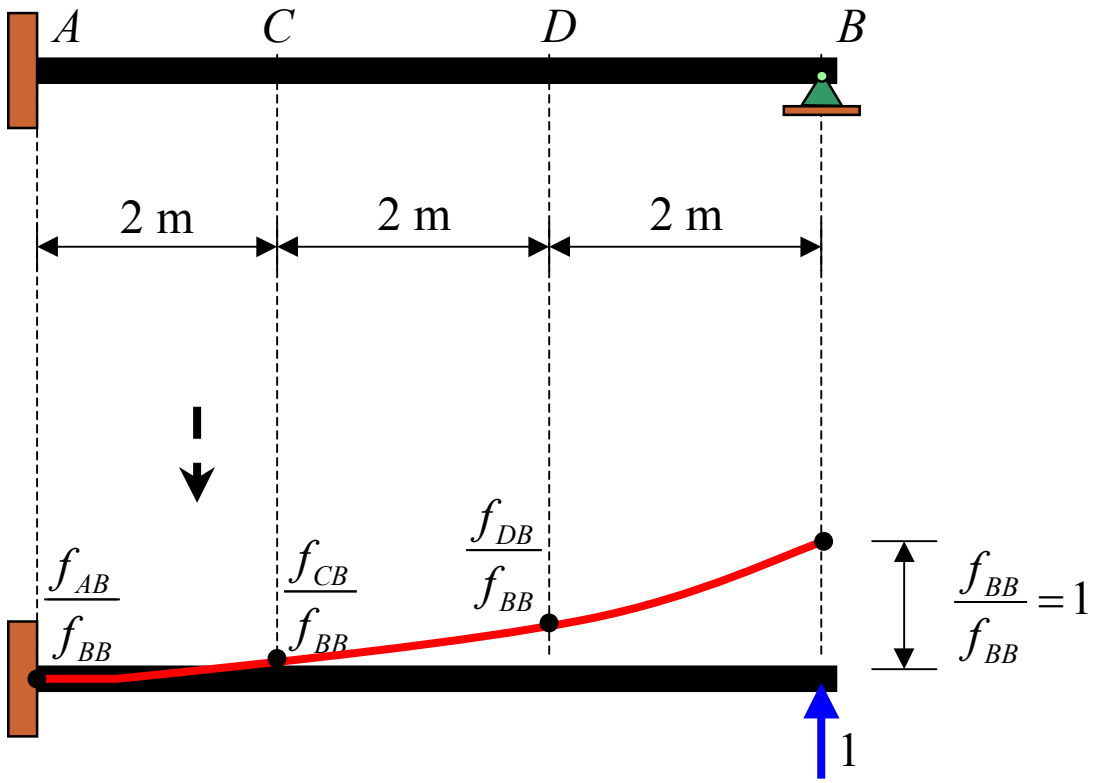
Draw the influence line for

- the vertical reaction at *A* and *B*
- shear at *C*
- bending moment at *A* and *C*

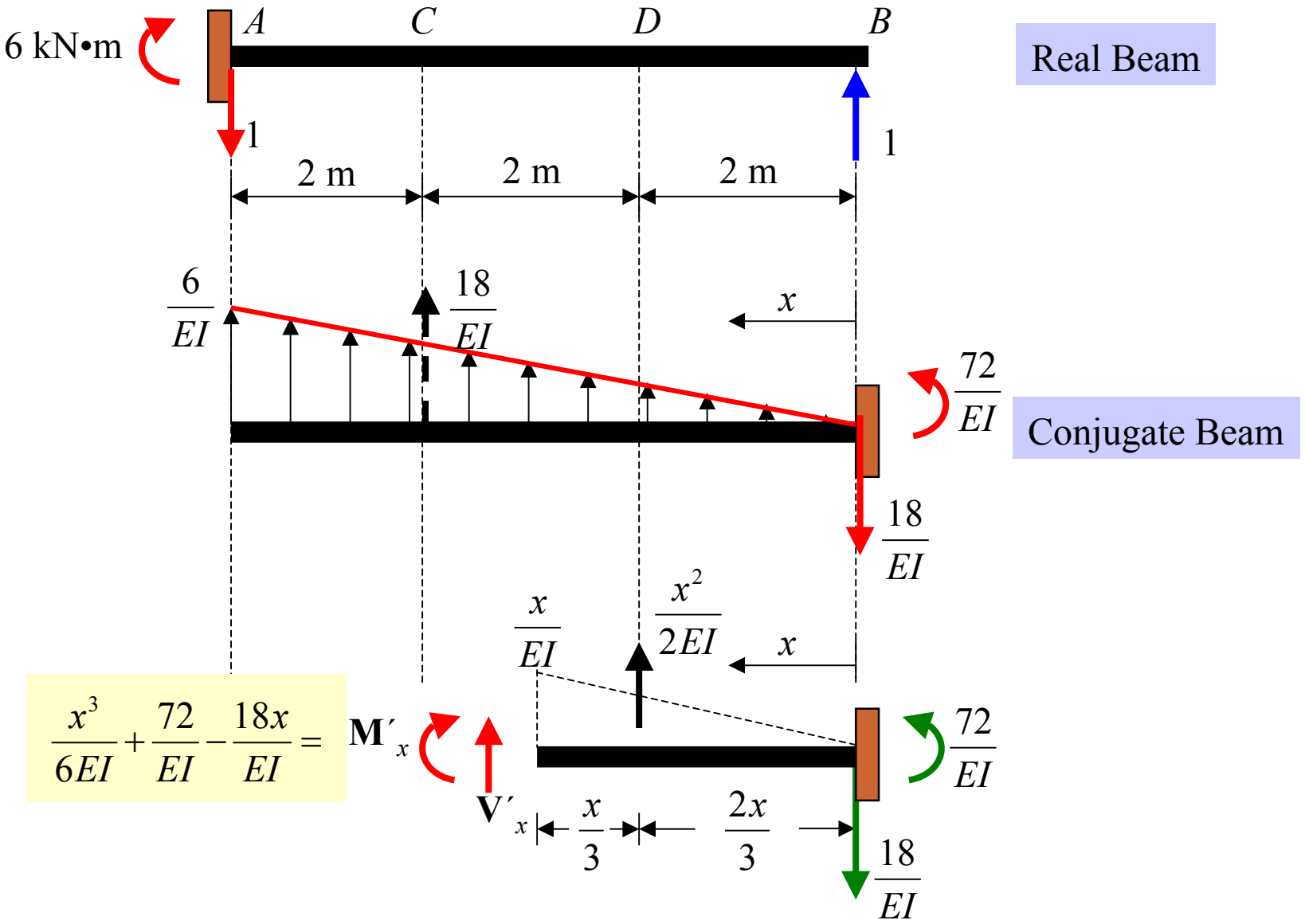
EI is constant . Plot numerical values every 2 m.

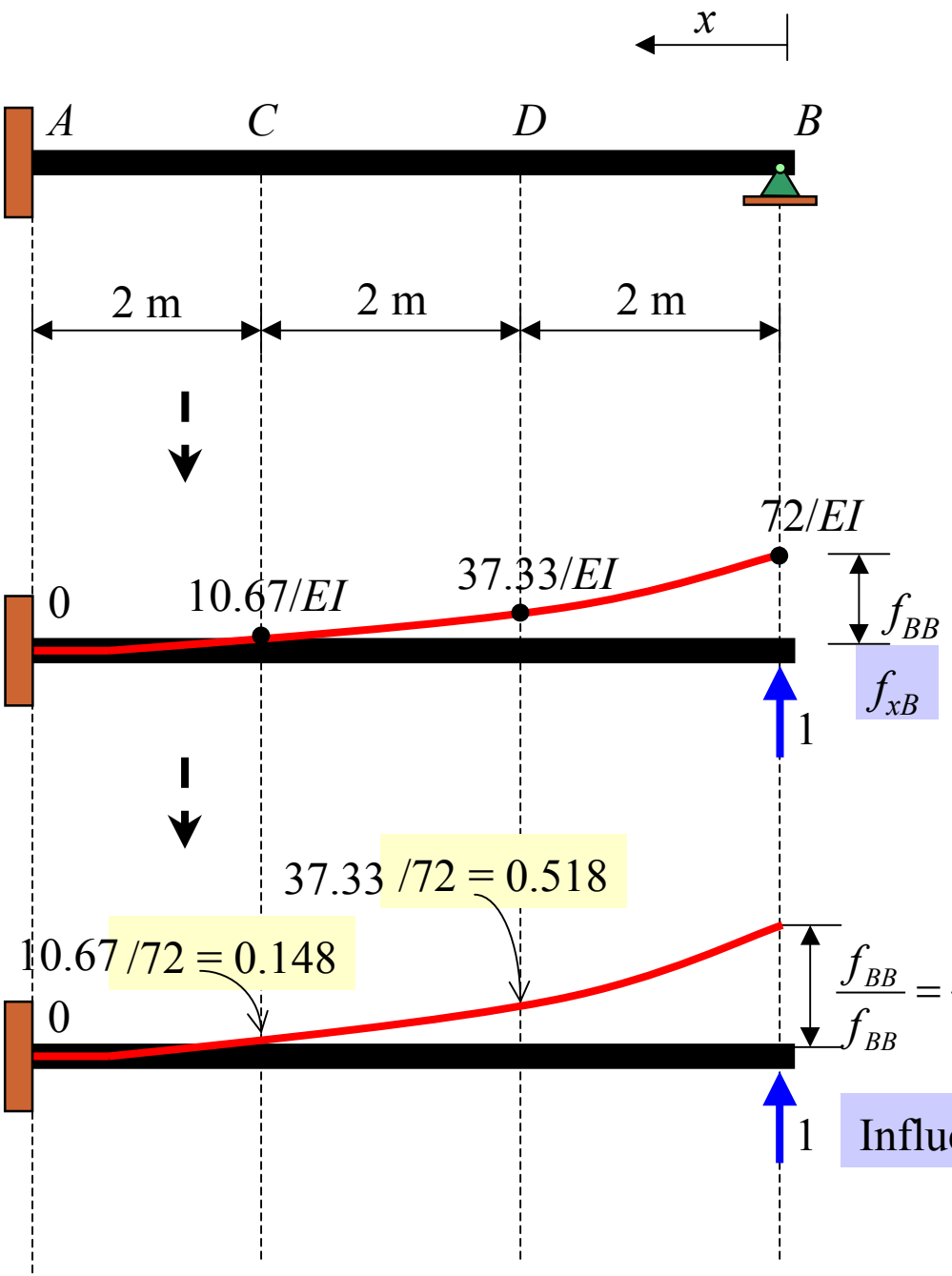


• **Influence line of R_B**



• **Find f_{xB} by conjugate beam**

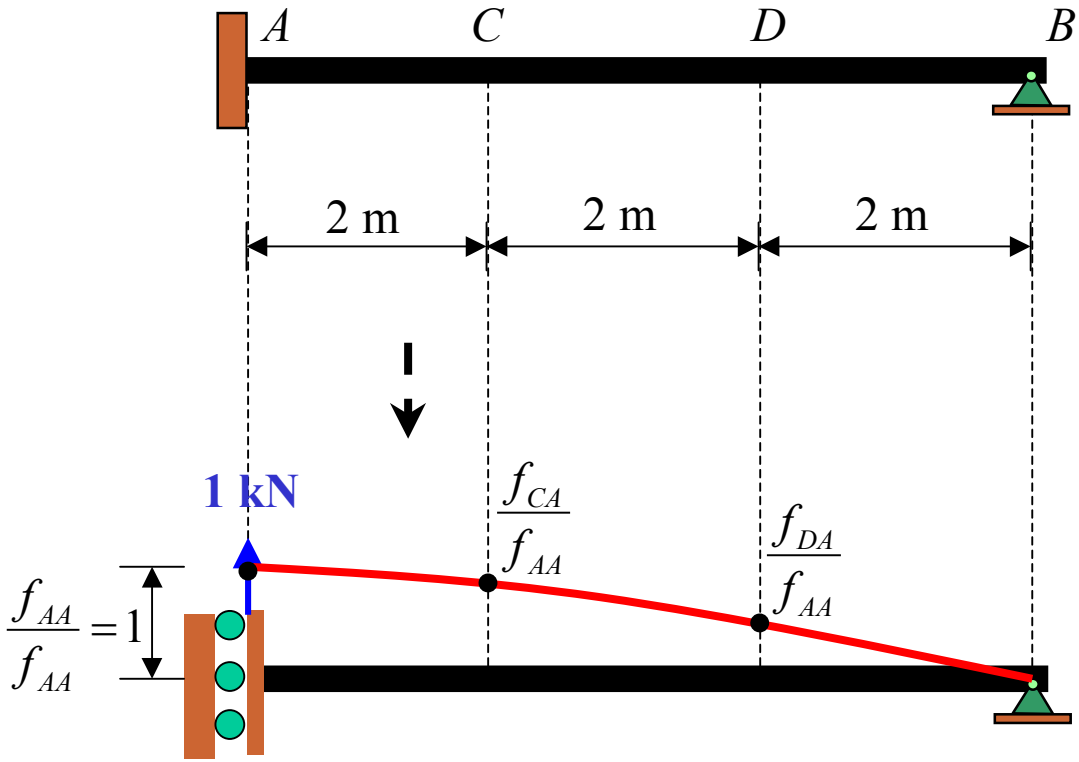




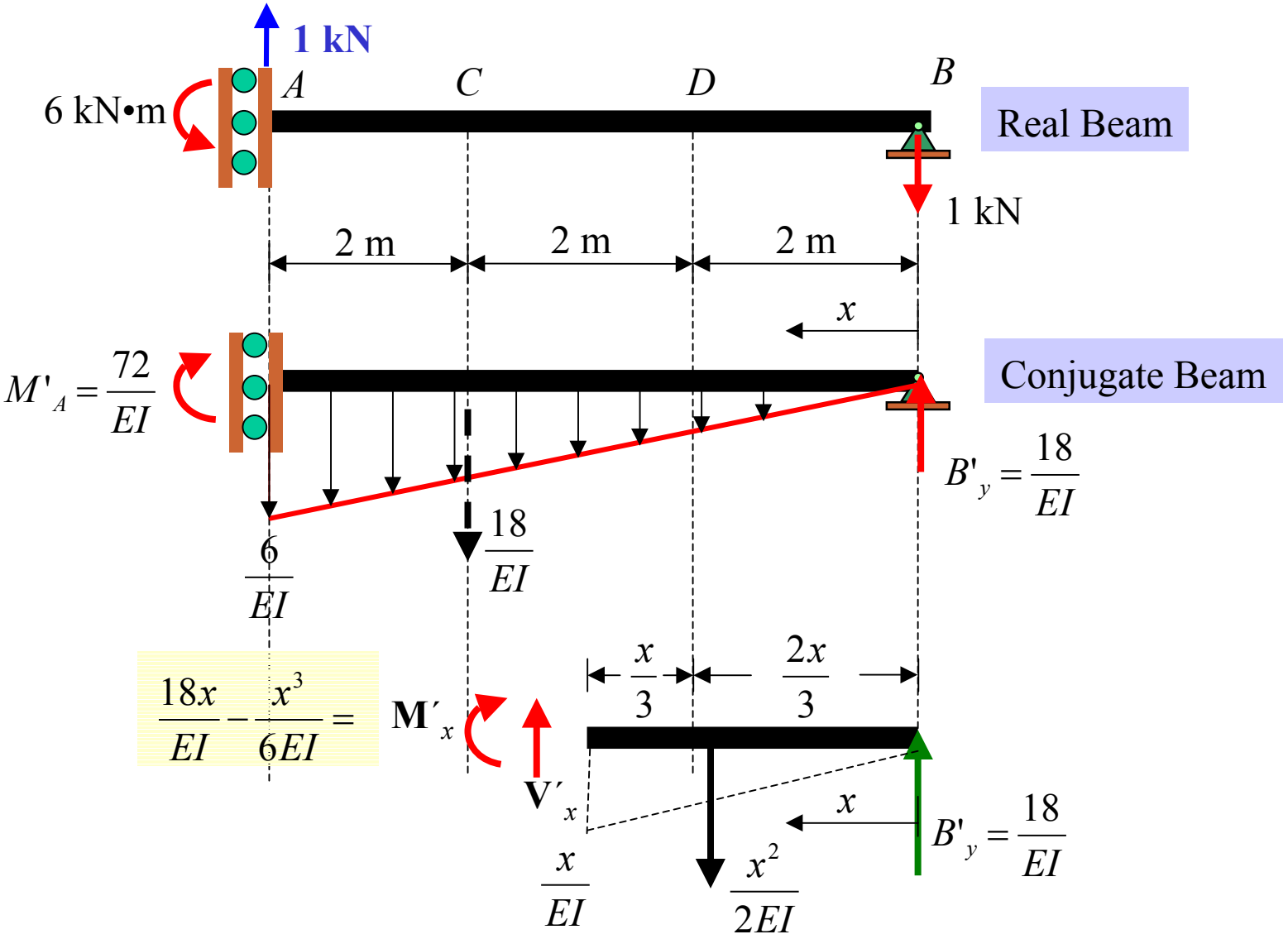
$$f_{xB} = M'_x = \frac{x^3}{6EI} + \frac{72}{EI} - \frac{18x}{EI}$$

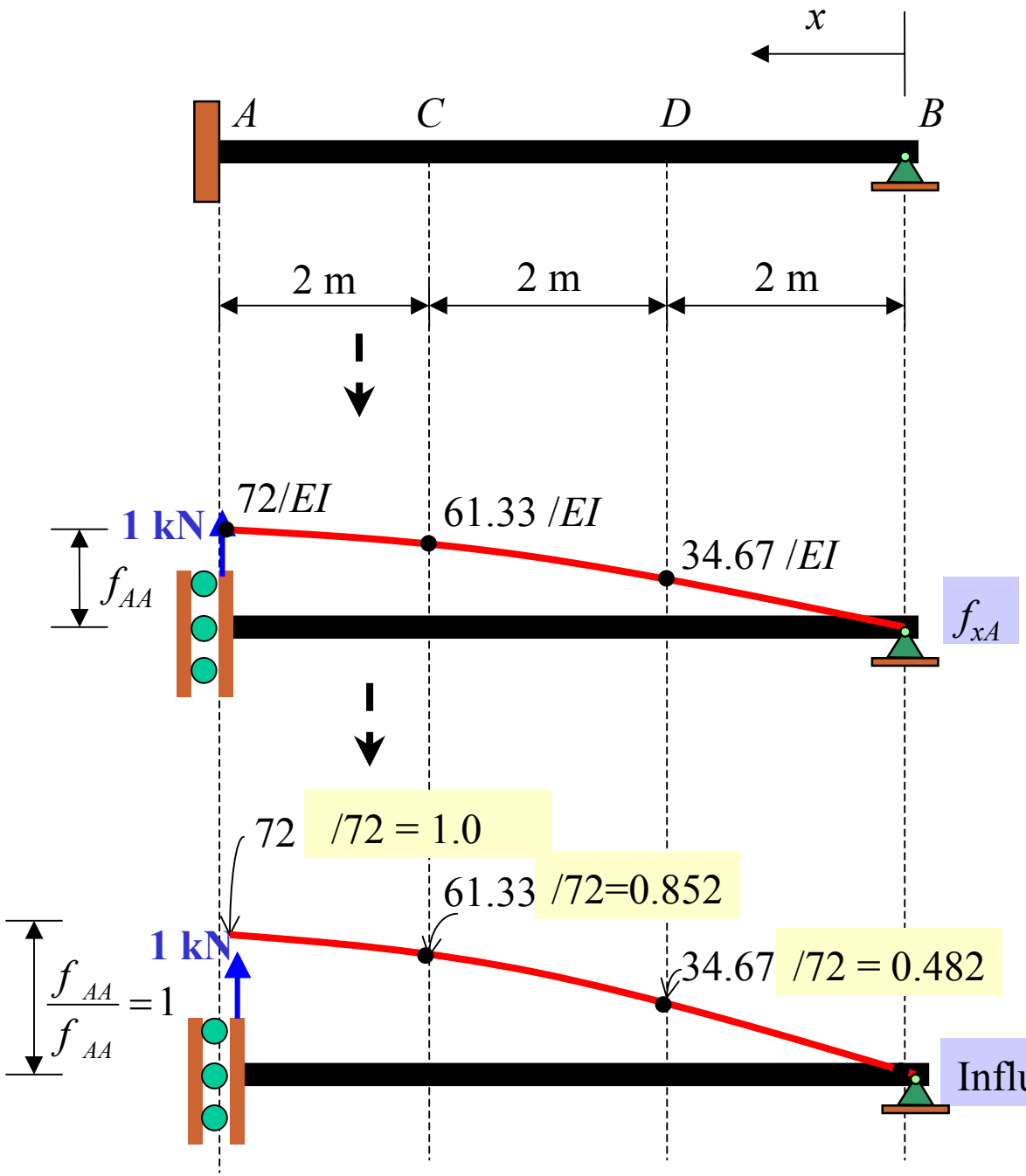
Point	x (m)	f_{xB}	f_{xB} / f_{BB}
B	0	$\frac{72}{EI}$	1
D	2	$\frac{37.33}{EI}$	0.518
C	4	$\frac{10.67}{EI}$	0.148
A	6	0	0

Influence line of R_A



• **Find f_{xA} by conjugate beam**

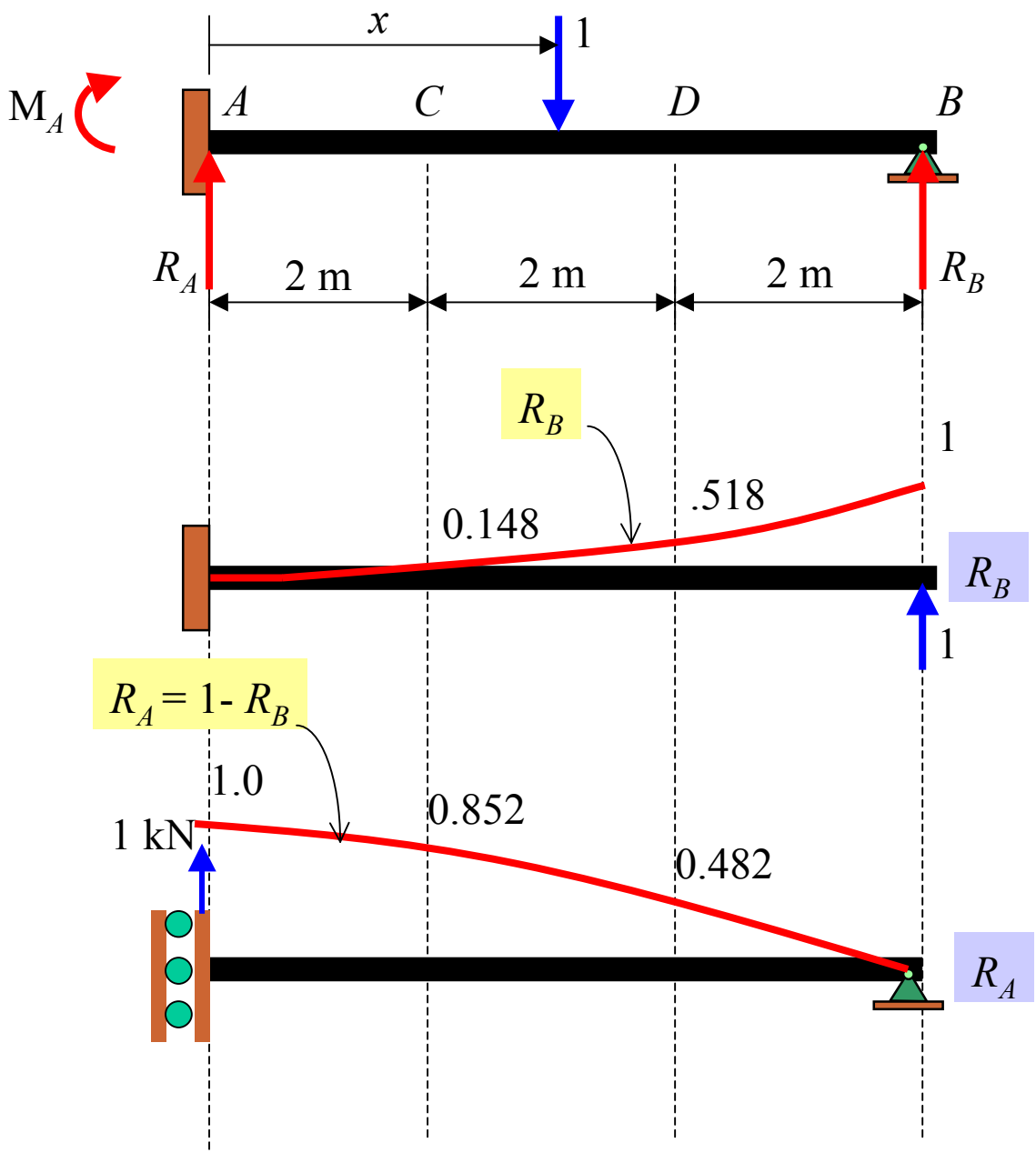




$$f_{xA} = M'_x = \frac{18x}{EI} - \frac{x^3}{6EI}$$

Point	x (m)	f_{xA}	f_{xA} / f_{AA}
B	0	0	0
D	2	$\frac{34.67}{EI}$	0.482
C	4	$\frac{61.33}{EI}$	0.852
A	6	$\frac{72}{EI}$	1.0

Alternate Method: Use equilibrium conditions for the influence line of R_A

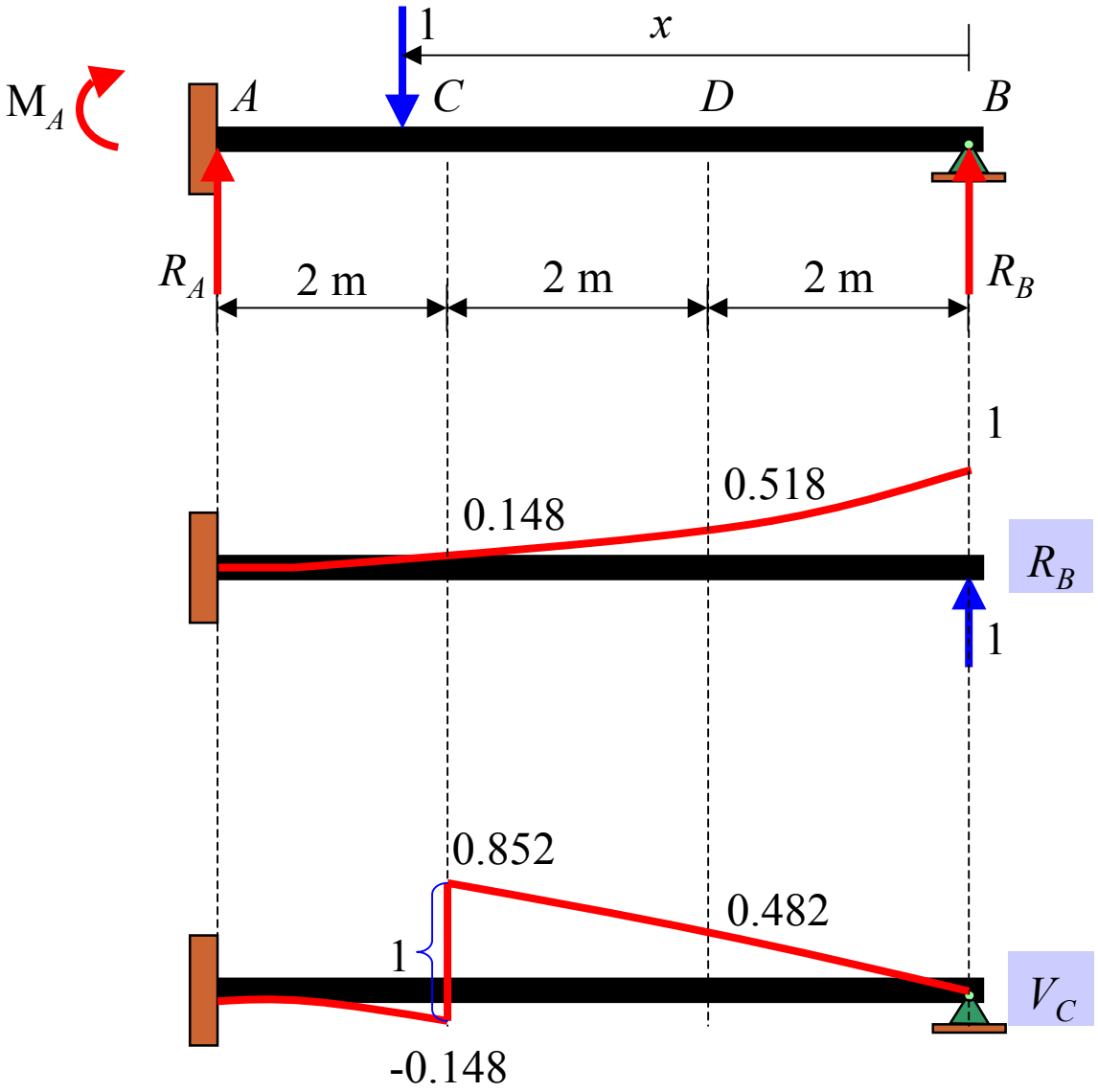


$$+\uparrow \Sigma F_y = 0;$$

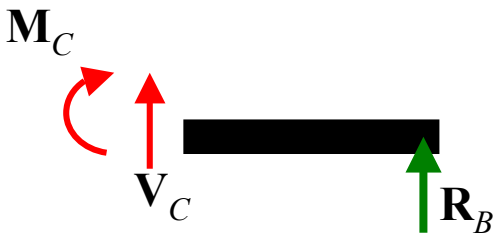
$$R_A + R_B - 1 = 0$$

$$R_B = 1 - R_A$$

Using equilibrium conditions for the influence line of V_C



• Unit load **to the left** of C

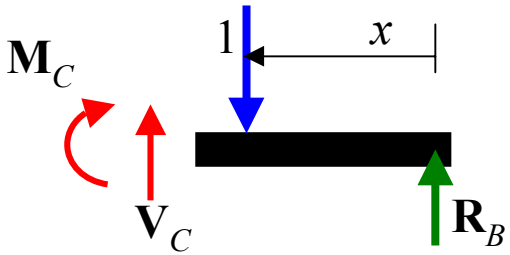


$$+\uparrow \Sigma F_y = 0;$$

$$+R_B + V_C = 0$$

$$V_C = -R_B$$

• Unit load **to the left** of C

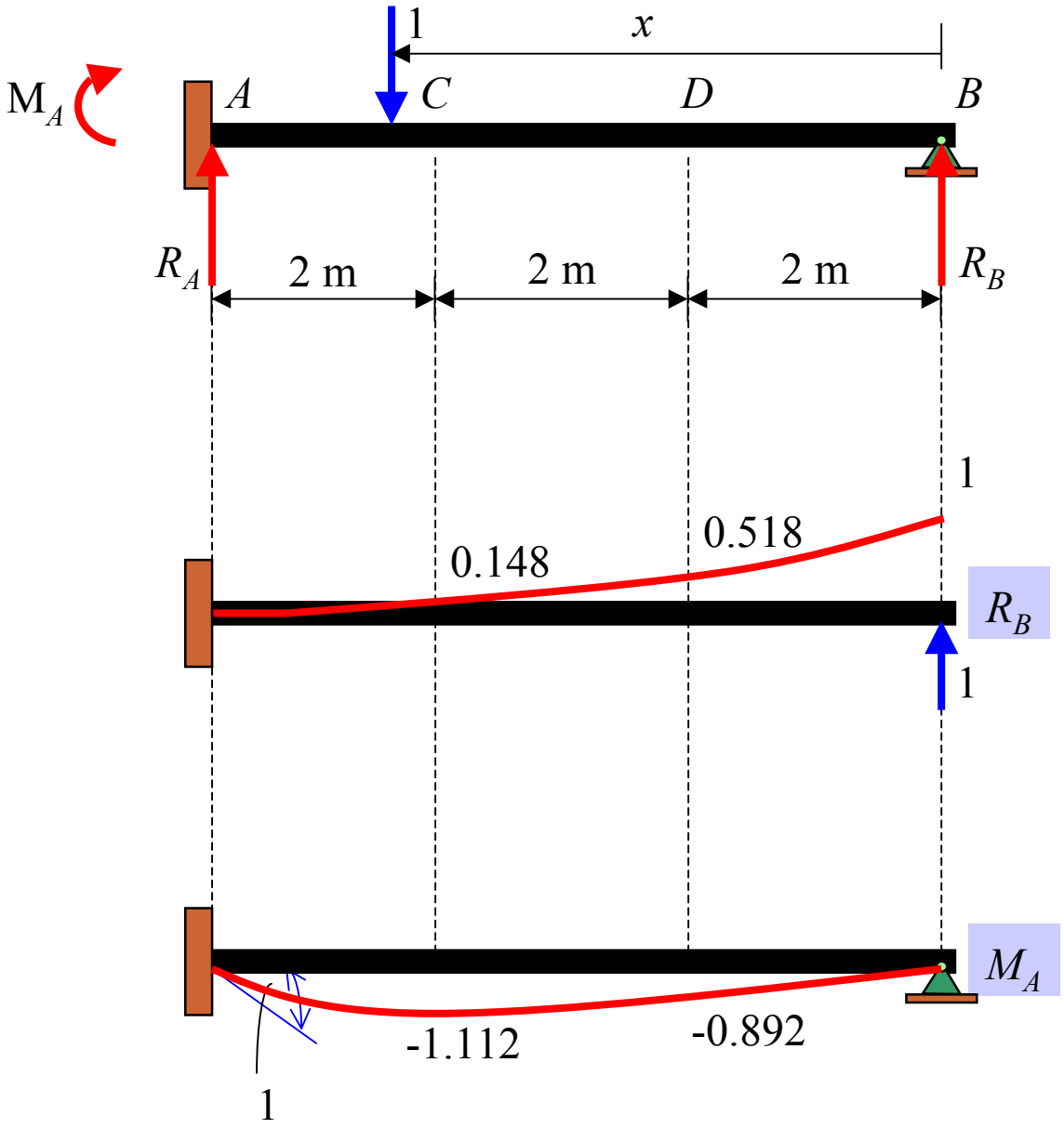


$$+\uparrow \Sigma F_y = 0;$$

$$+V_C - 1 + R_B = 0$$

$$V_C = 1 - R_B$$

Using equilibrium conditions for the influence line of M_A

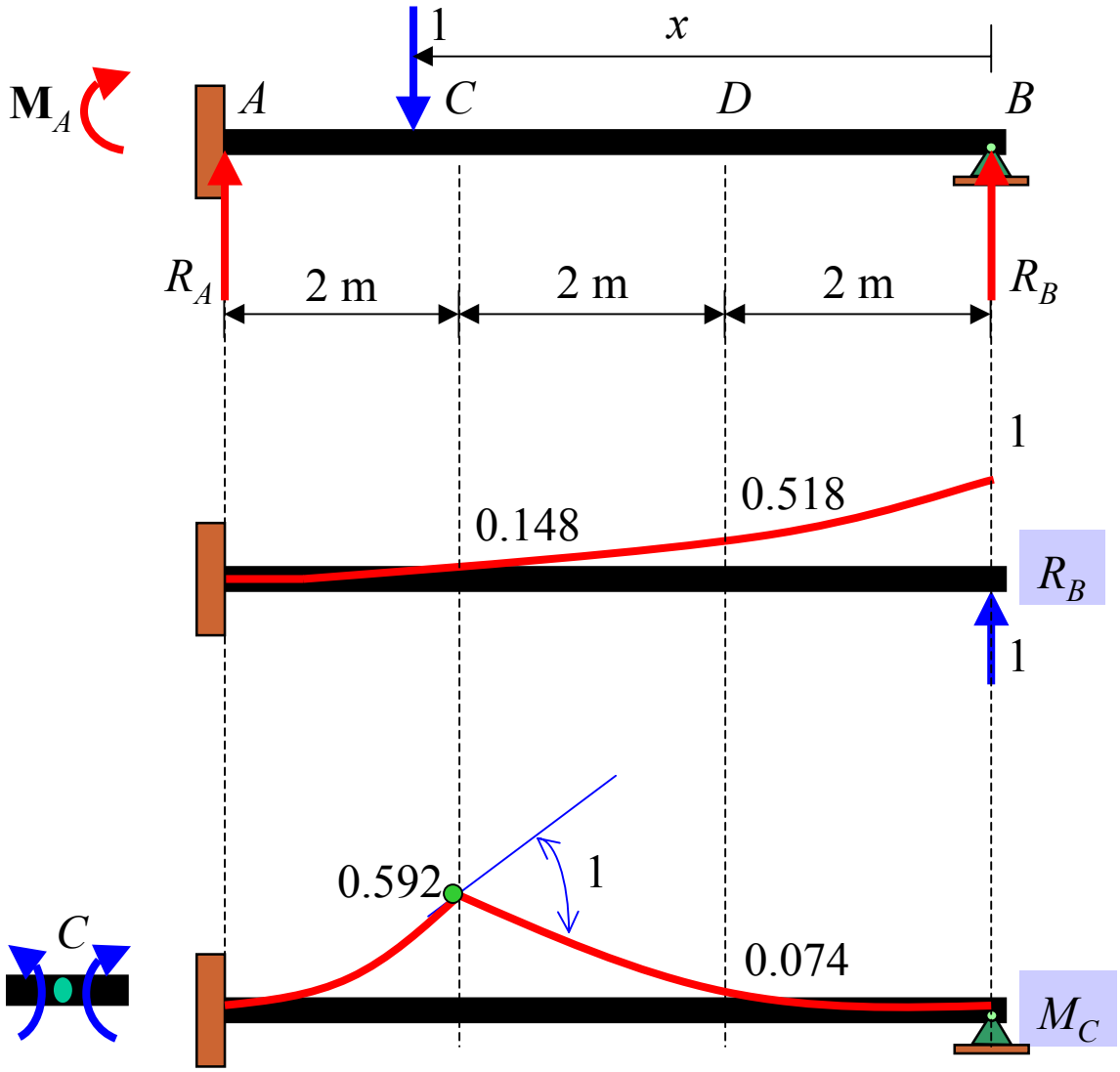


$$+\circlearrowleft \Sigma M_A = 0;$$

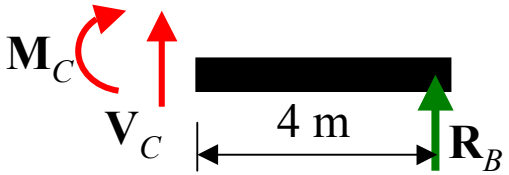
$$-M_A - 1(6-x) + 6R_B = 0$$

$$M_A = -6 + x + 6R_B$$

Using equilibrium conditions for the influence line of M_C



• Unit load **to the left** of C

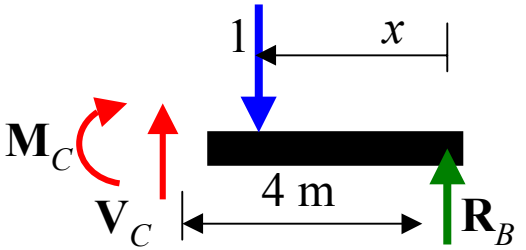


$$\circlearrowleft + \quad \Sigma M_C = 0;$$

$$-M_C + 4R_B = 0$$

$$M_C = 4R_B$$

• Unit load **to the left** of C



$$\circlearrowleft + \quad \Sigma M_C = 0;$$

$$-M_C - 1(4-x) + 4R_B = 0$$

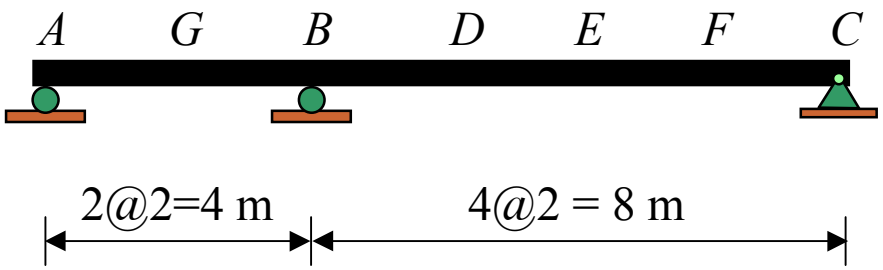
$$M_C = -4 + x + 4R_B$$

Example 2

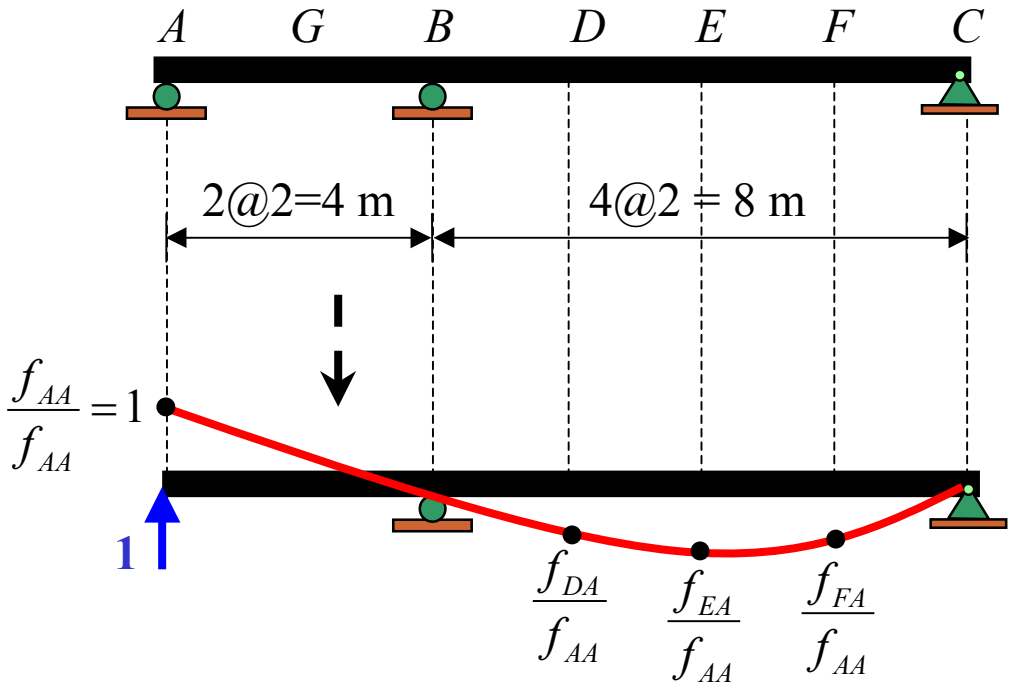
Draw the influence line and plot numerical values every 2 m for

- the vertical reaction at supports *A, B* and *C*
- Shear at *G* and *E*
- Bending moment at *G* and *E*

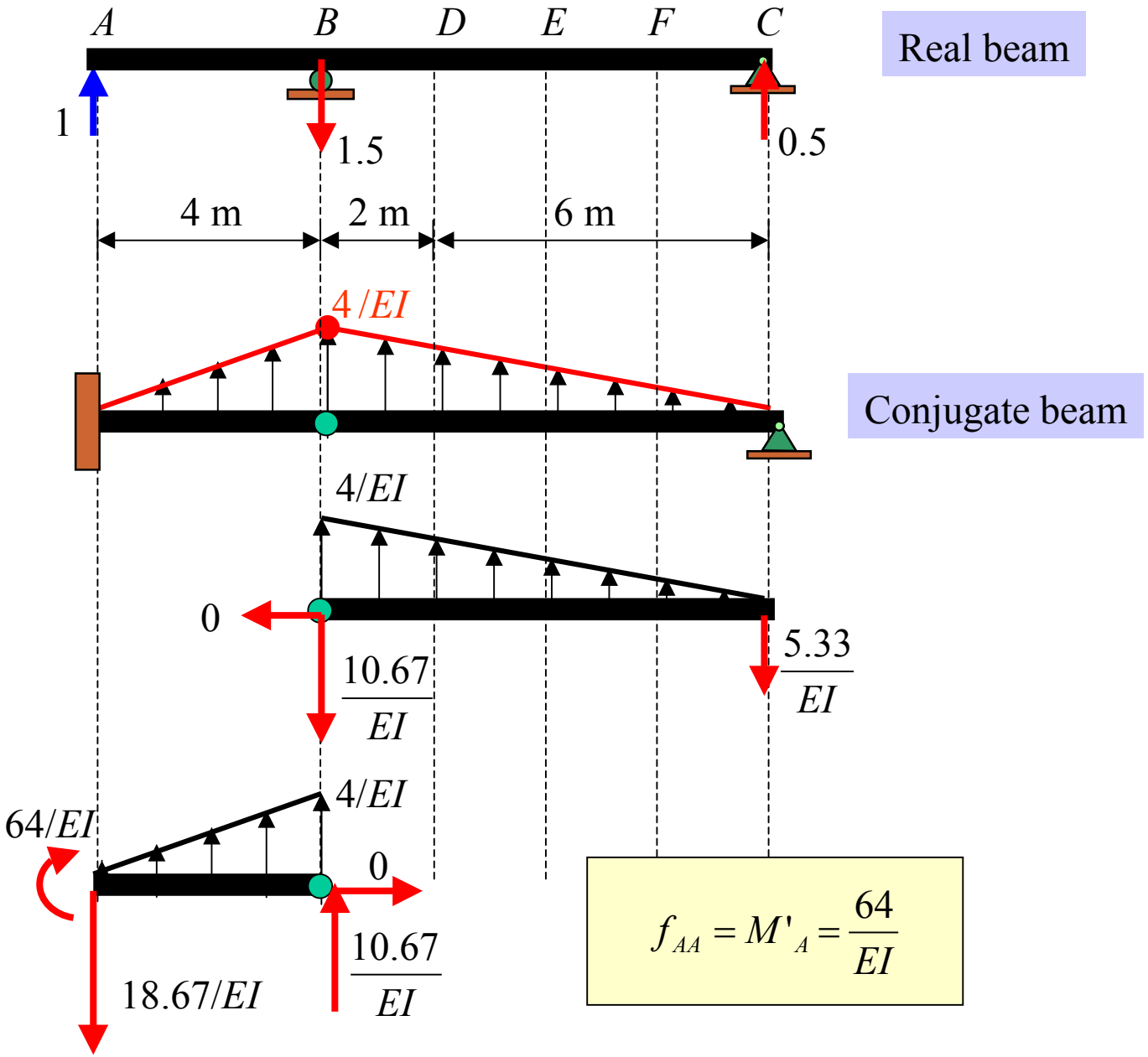
EI is constant.

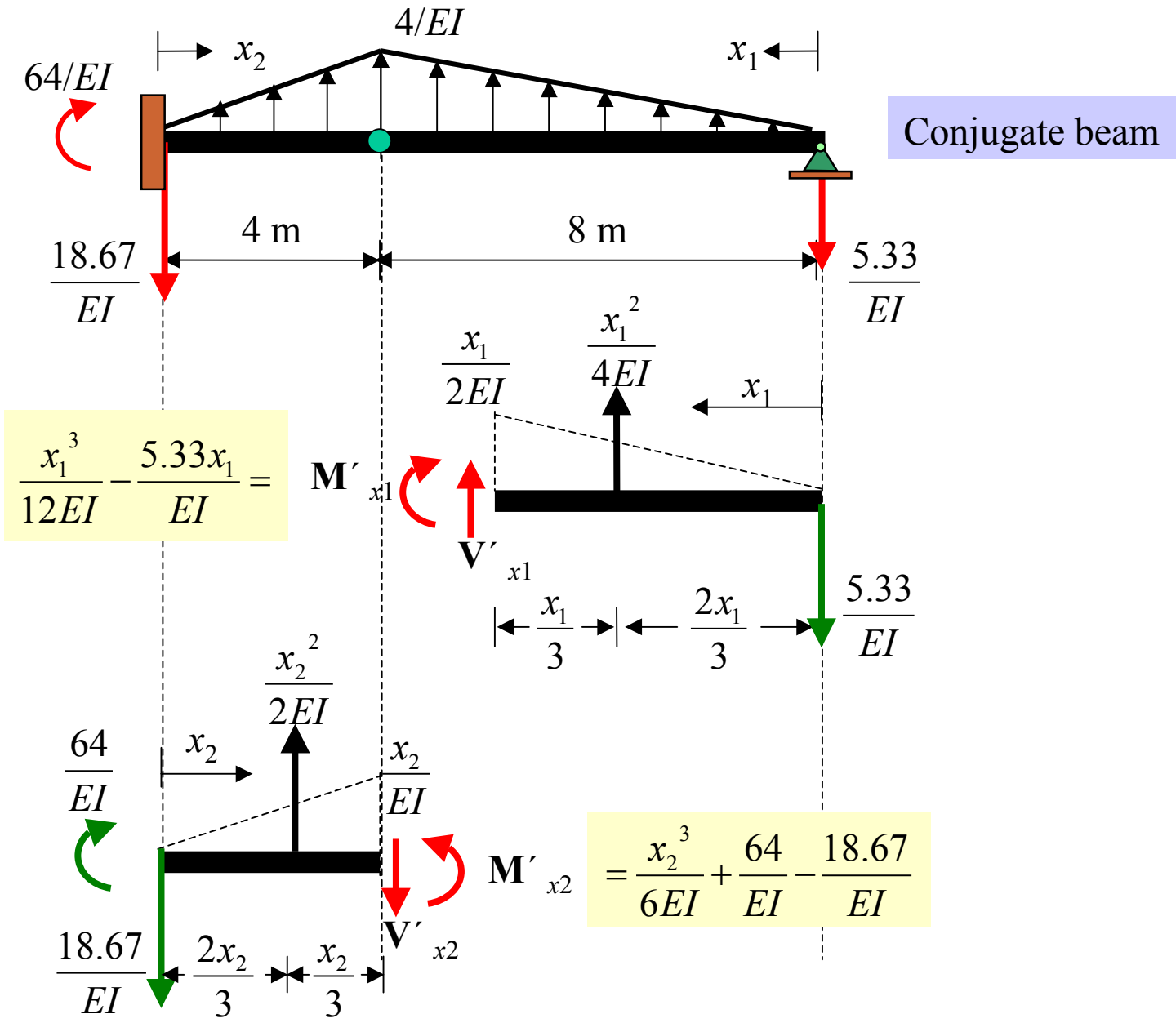


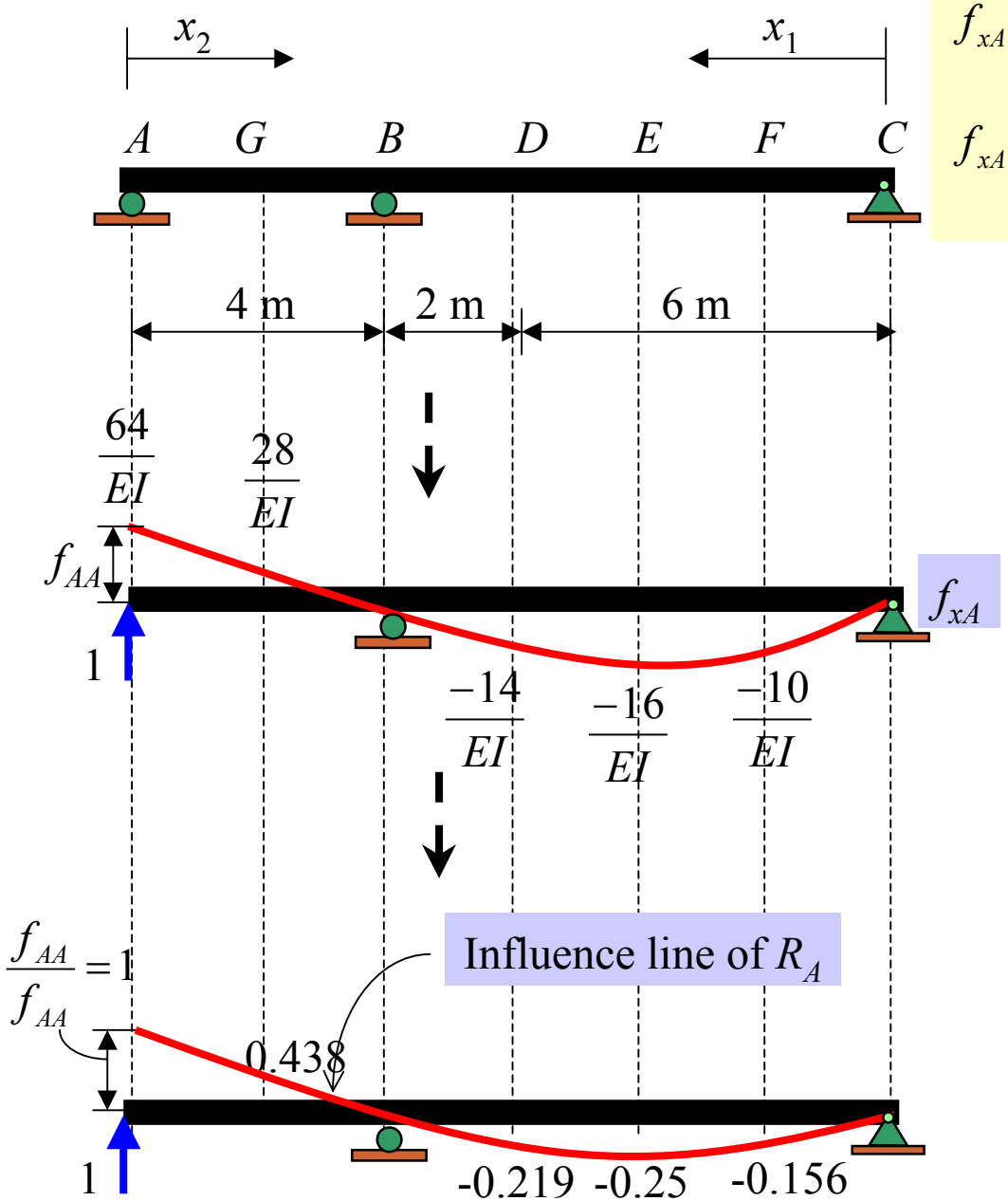
Influence line of R_A



• **Find f_{xA} by conjugate beam**




4/EI
x2
x1
64/EI
Conjugate beam
4 m
8 m
18.67/EI
5.33/EI
x1/2EI
x1²/4EI
x1
x1³/12EI − 5.33x1/EI =
M´x1
V´x1
x1/3
2x1/3
5.33/EI
x2²/2EI
64/EI
x2
x2/EI
M´x2
= x2³/6EI + 64/EI − 18.67/EI
V´x2
18.67/EI
2x2/3
x2/3

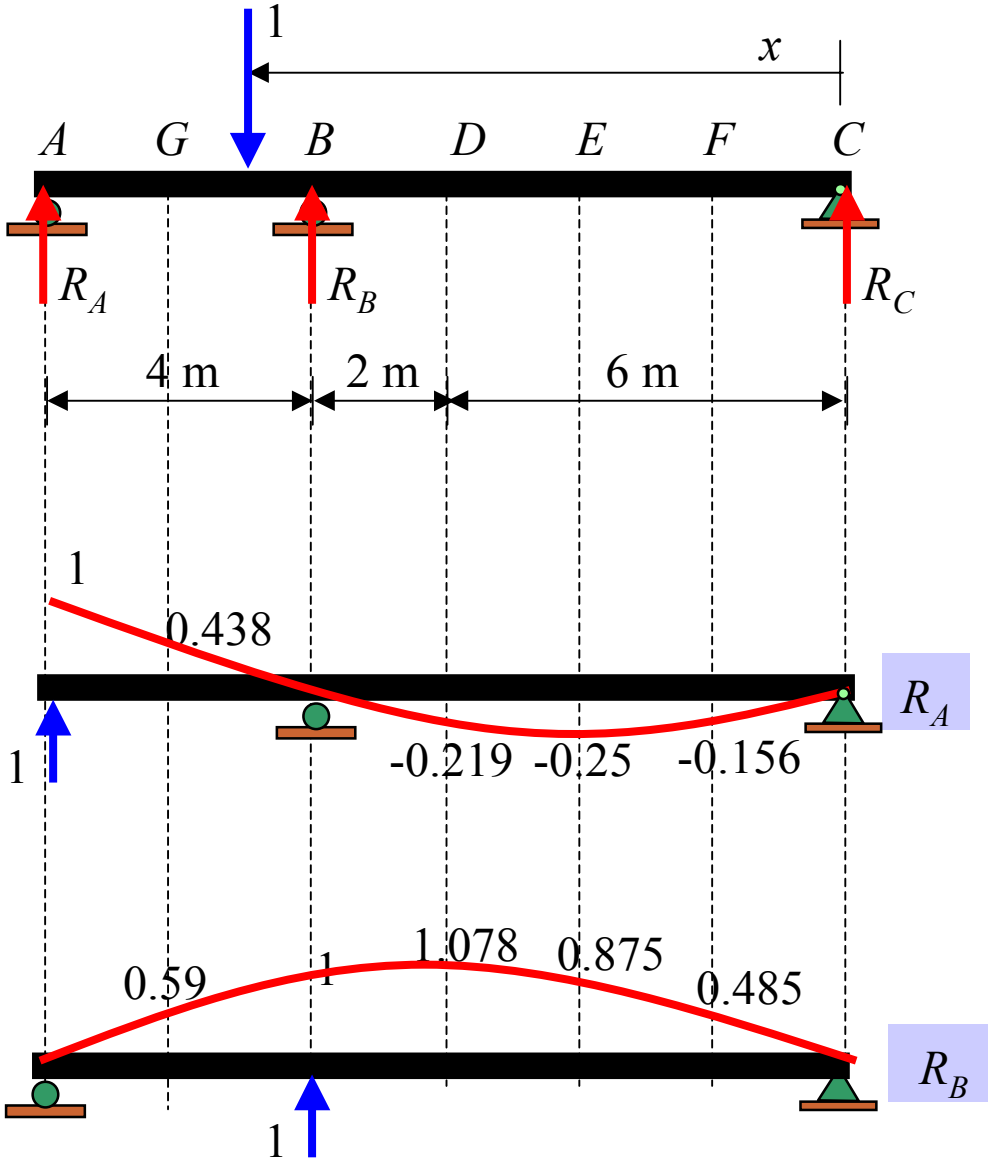


$$f_{xA} = M'_{x1} = \frac{x^3}{12EI} - \frac{5.33x}{EI}, \text{ for } B \text{ to } C$$

$$f_{xA} = M'_{x2} = \frac{x_2^{\,3}}{6EI} - \frac{18.67x_2}{EI} + \frac{64}{EI}, \text{ for } A \text{ to } B$$

Point	x (m)	f_{xA}	f_{xA} / f_{AA}
C	0	0	0
F	2	$-\frac{10}{EI}$	-0.1562
E	4	$-\frac{16}{EI}$	-0.25
D	6	$-\frac{14}{EI}$	-0.2188
B	8	0	0
G	10	$\frac{28}{EI}$	0.4375
A	12	$\frac{64}{EI}$	1

Using equilibrium conditions for the influence line of R_B



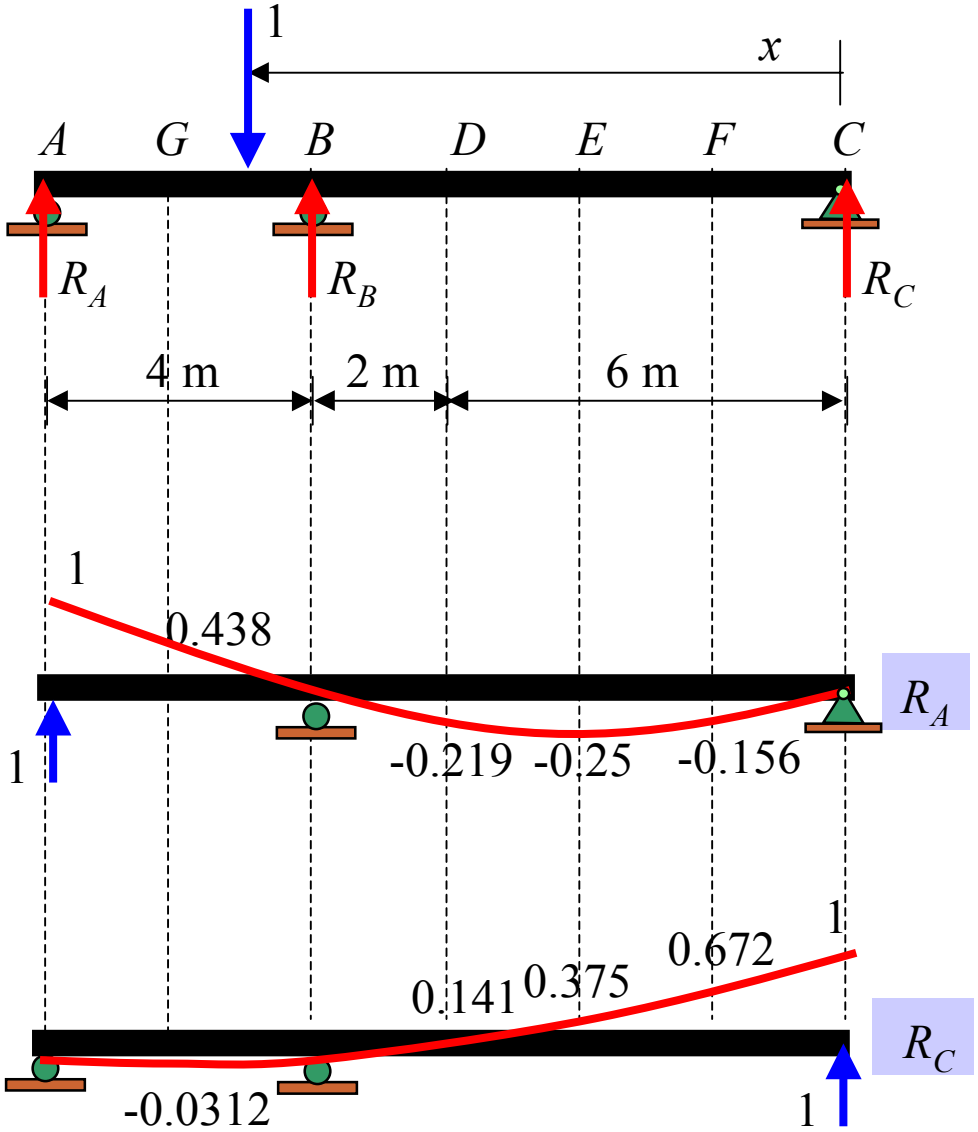
$$\circlearrowleft + \Sigma M_C = 0;$$

$$-8R_B + x - 12R_A = 0$$

$$R_B = \frac{x}{8} - \frac{12}{8} R_A$$

Point	x (m)	R_A	R_B
C	0	0	0
F	2	-0.1562	0.485
E	4	-0.25	0.875
D	6	-0.2188	1.078
B	8	0	1
G	10	0.4375	0.5939
A	12	1	0

Using equilibrium conditions for the influence line of R_C



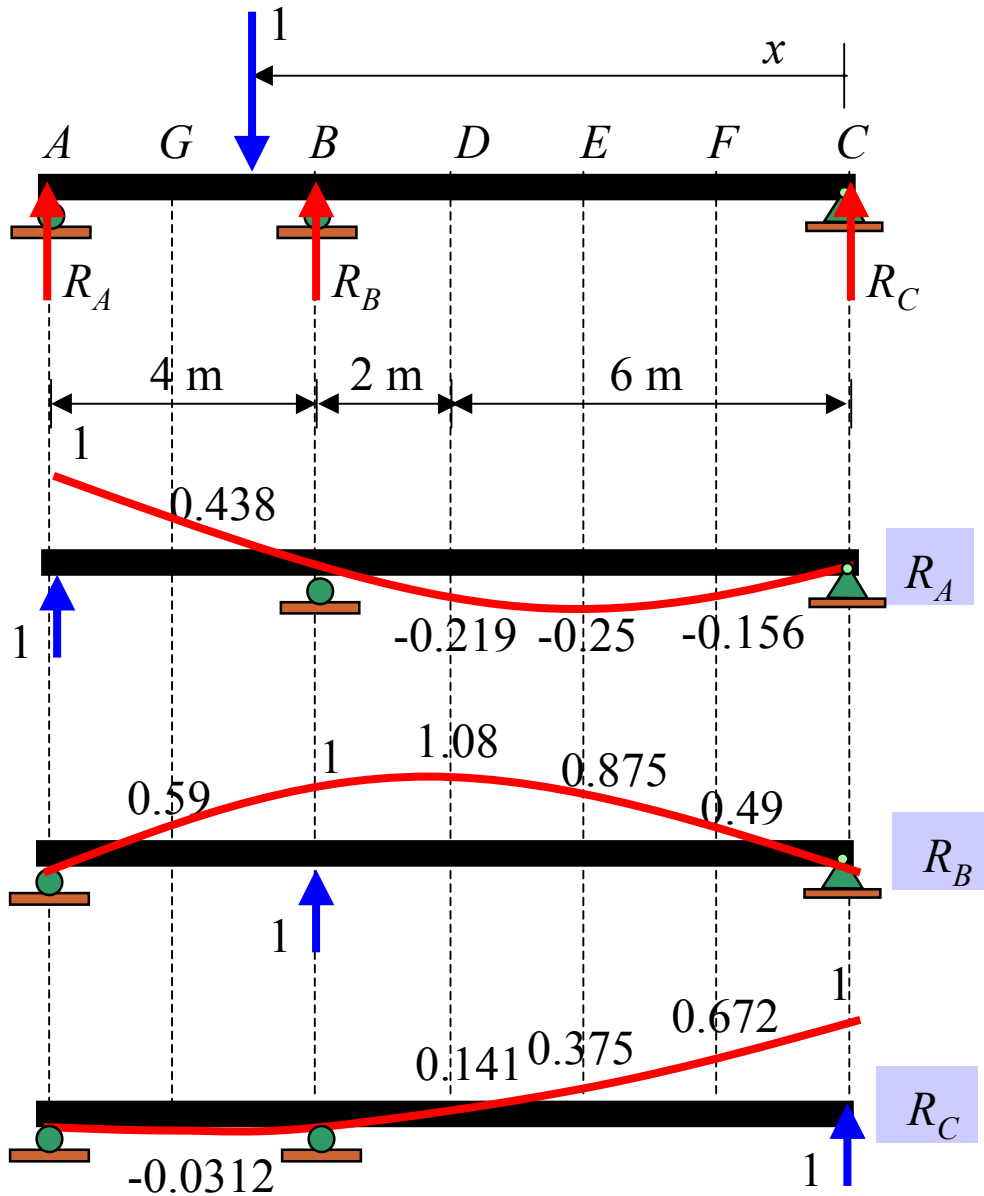
$$(+\Sigma M_B = 0;$$

$$4R_A - 1(x-8) - 8R_C = 0$$

$$R_C = 0.5R_A - \frac{x}{8} + 1$$

Point	x (m)	R_A	R_C
C	0	0	1
F	2	-0.1562	0.6719
E	4	-0.25	0.375
D	6	-0.2188	0.1406
B	8	0	0
G	10	0.4375	-0.0312
A	12	1	0

• Check $\Sigma F_y = 0$

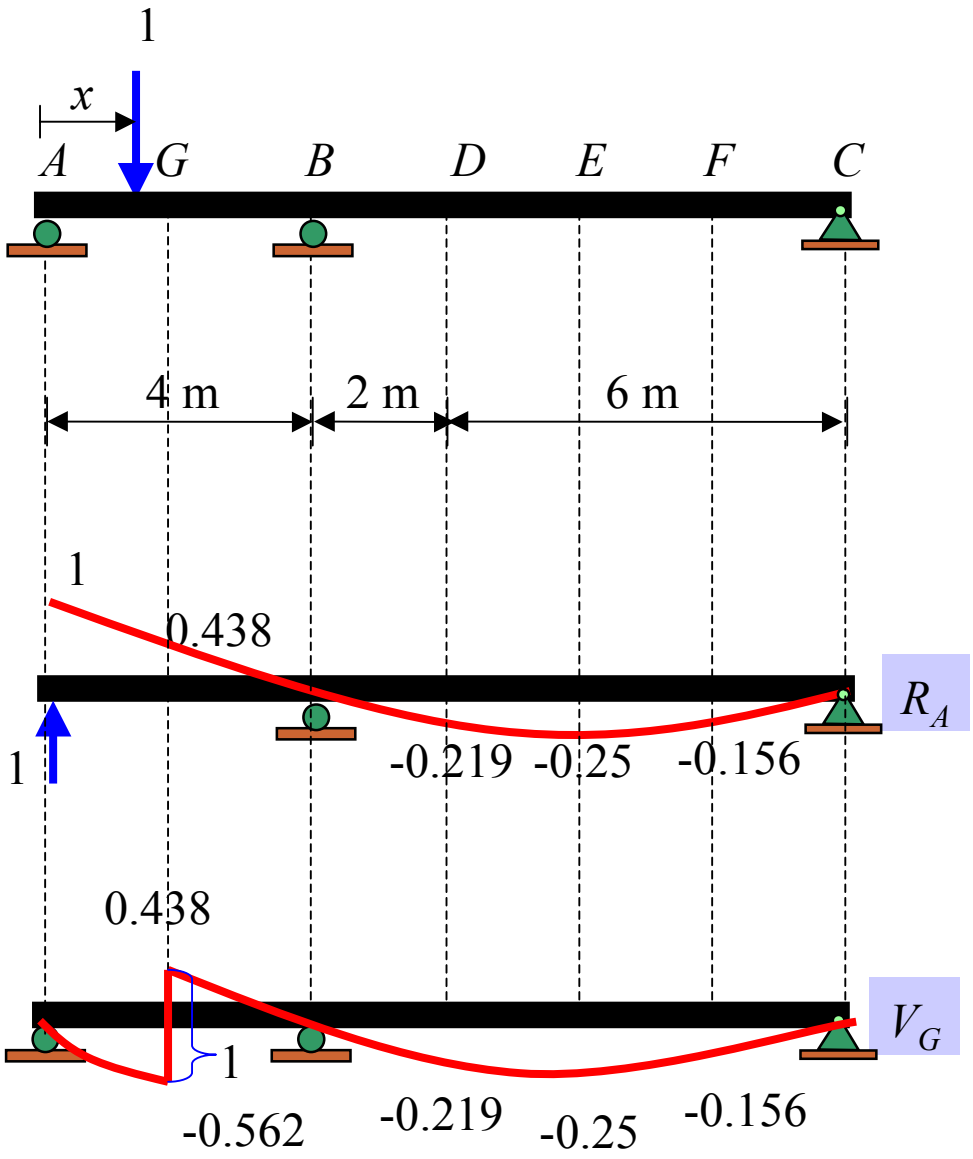


$$+\uparrow \Sigma F_y = 0;$$

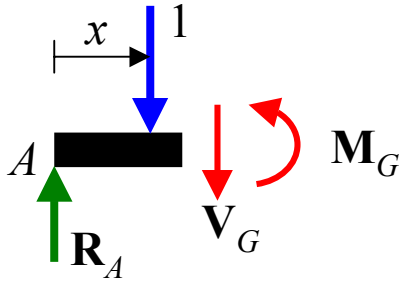
$$R_A + R_B + R_C = 1$$

Point	R_A	R_B	R_C	ΣR
C	0	0	1	1
F	-0.1562	0.485	0.6719	1
E	-0.25	0.875	0.375	1
D	-0.2188	1.078	0.1406	1
B	0	1	0	1
G	0.4375	0.5939	-0.0312	1
A	1	0	0	1

Using equilibrium conditions for the influence line of V_G



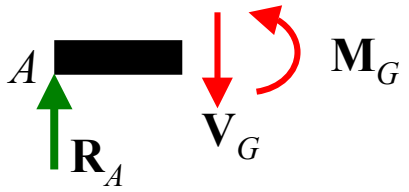
• Unit load **to the left** of G



$$+\uparrow \quad \Sigma F_y = 0; \quad R_A - 1 - V_G = 0$$

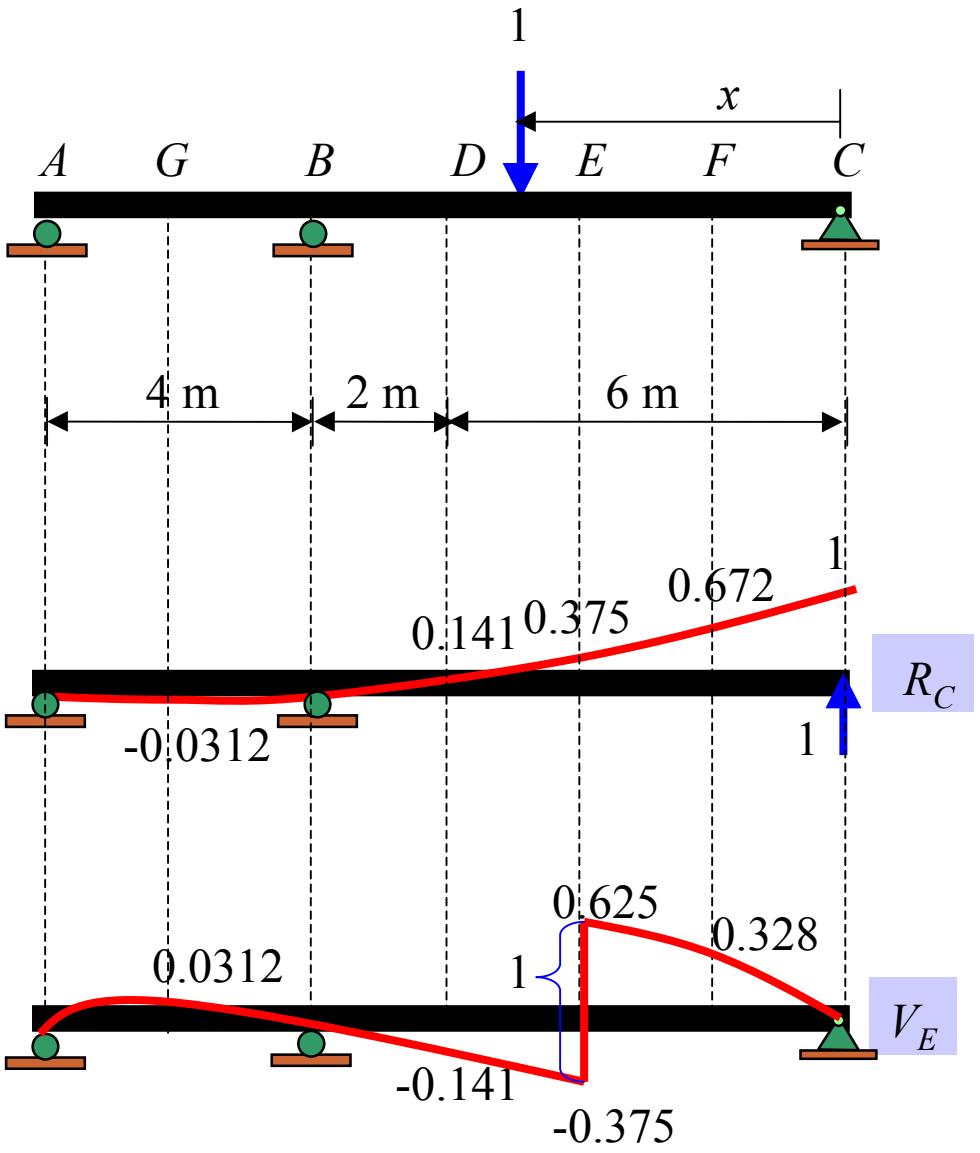
$$V_G = R_A - 1$$

• Unit load **to the right** of G



$$V_G = R_A$$

Using equilibrium conditions for the influence line of V_E



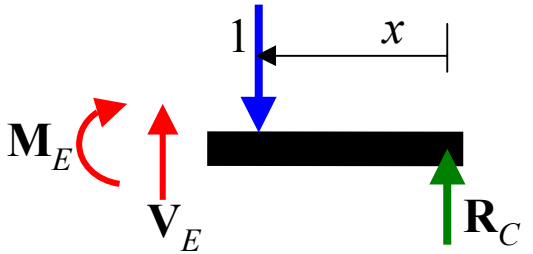
• Unit load **to the left** of E



$$+\uparrow \quad \Sigma F_y = 0; \quad R_C + V_E = 0$$

$$\mathbf{V}_E = -R_C$$

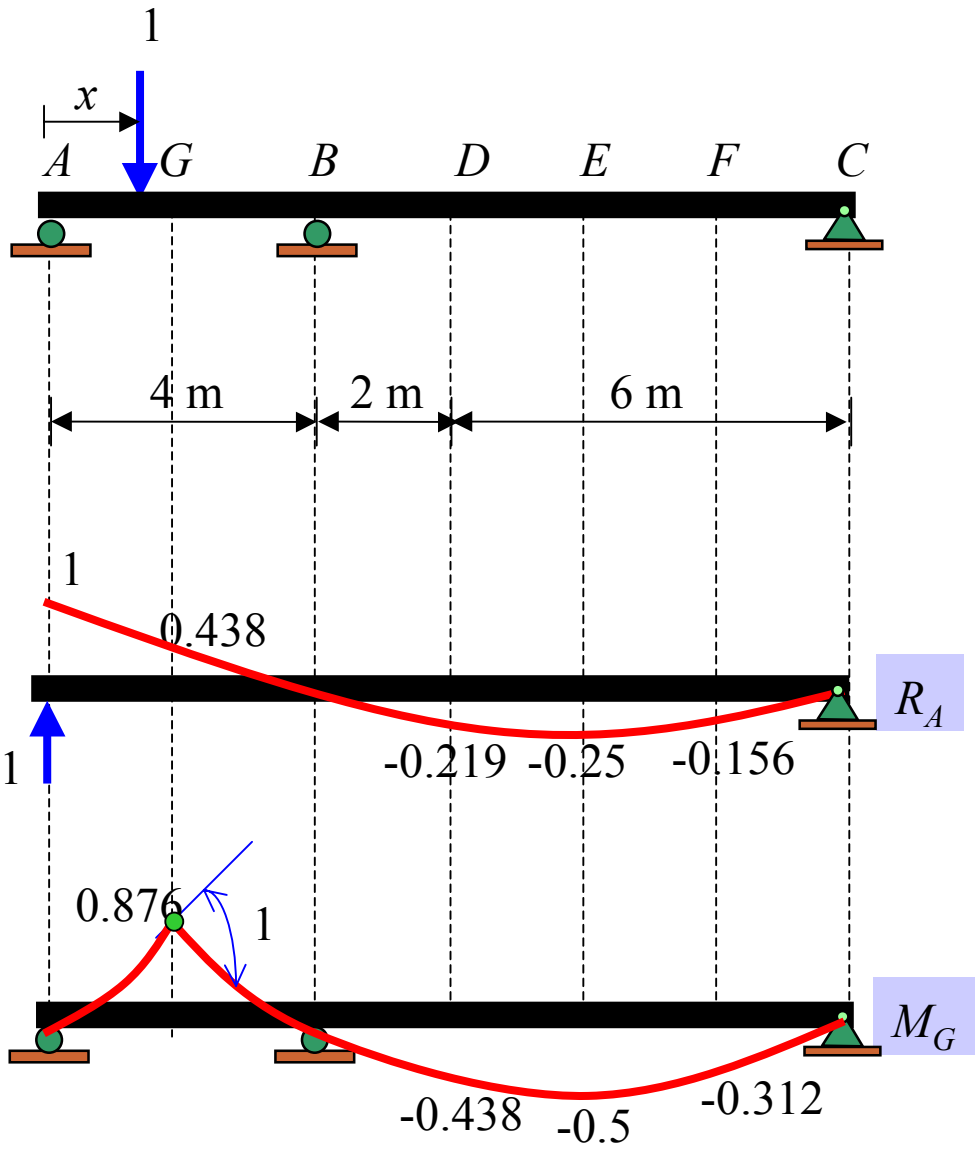
• Unit load **to the right** of E



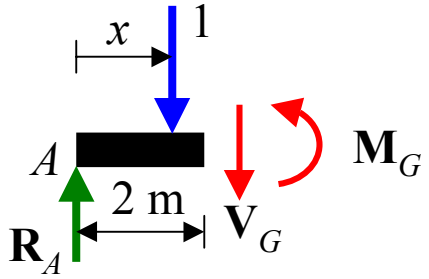
$$+\uparrow \quad \Sigma F_y = 0; \quad V_E - 1 + R_C = 0$$

$$\mathbf{V}_E = 1 - R_C$$

Using equilibrium conditions for the influence line of M_G



- Unit load **to the left** of G

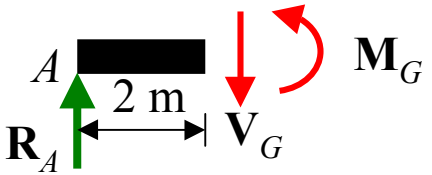


$$+\circlearrowleft \quad \Sigma M_G = 0;$$

$$M_G + 1(2 - x) - 2R_A = 0$$

$$M_G = -2 + x + 2R_A$$

- Unit load **to the right** of G

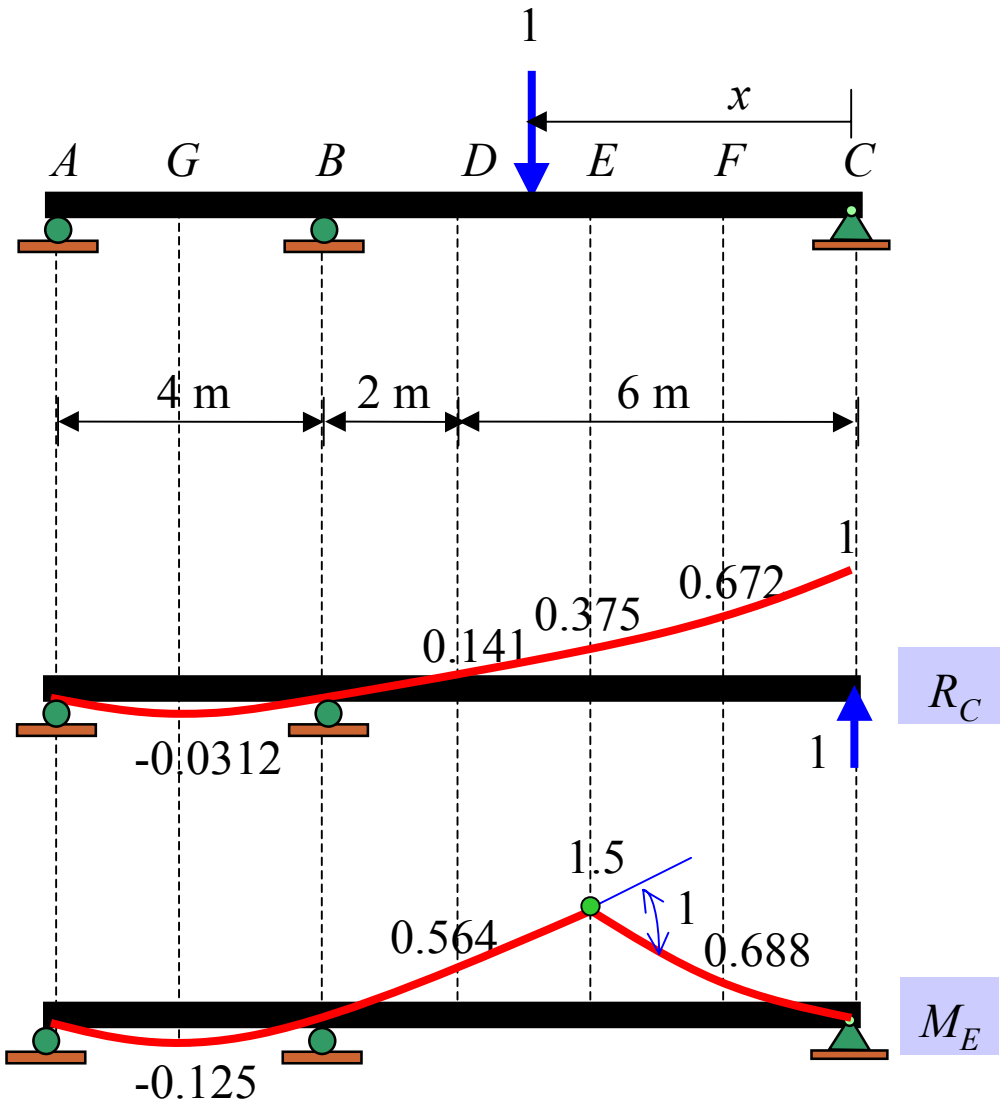


$$+\circlearrowleft \quad \Sigma M_G = 0;$$

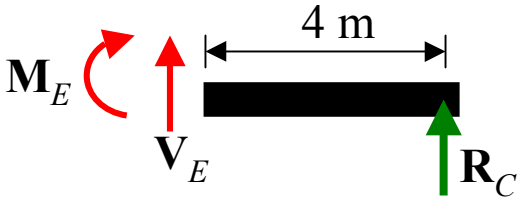
$$M_G - 2R_A = 0$$

$$M_G = 2R_A$$

Using equilibrium conditions for the influence line of M_E



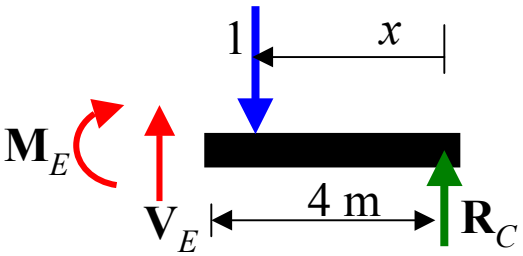
• Unit load **to the left** of E



$$\circlearrowleft + \quad \Sigma M_E = 0;$$

$$M_E = 4R_C$$

• Unit load **to the right** of E



$$\circlearrowleft + \Sigma M_E = 0;$$

$$-M_E - 1(4-x) + 4R_C = 0$$

$$M_E = -4 + x + 4R_C$$

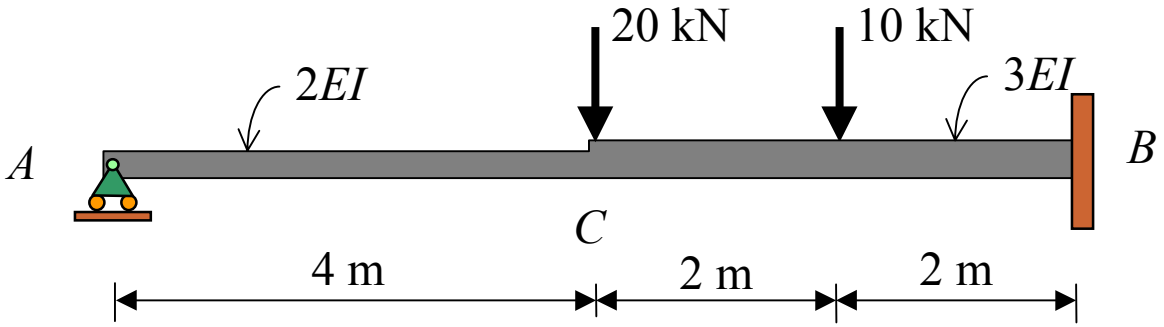
Example 3

For the beam shown

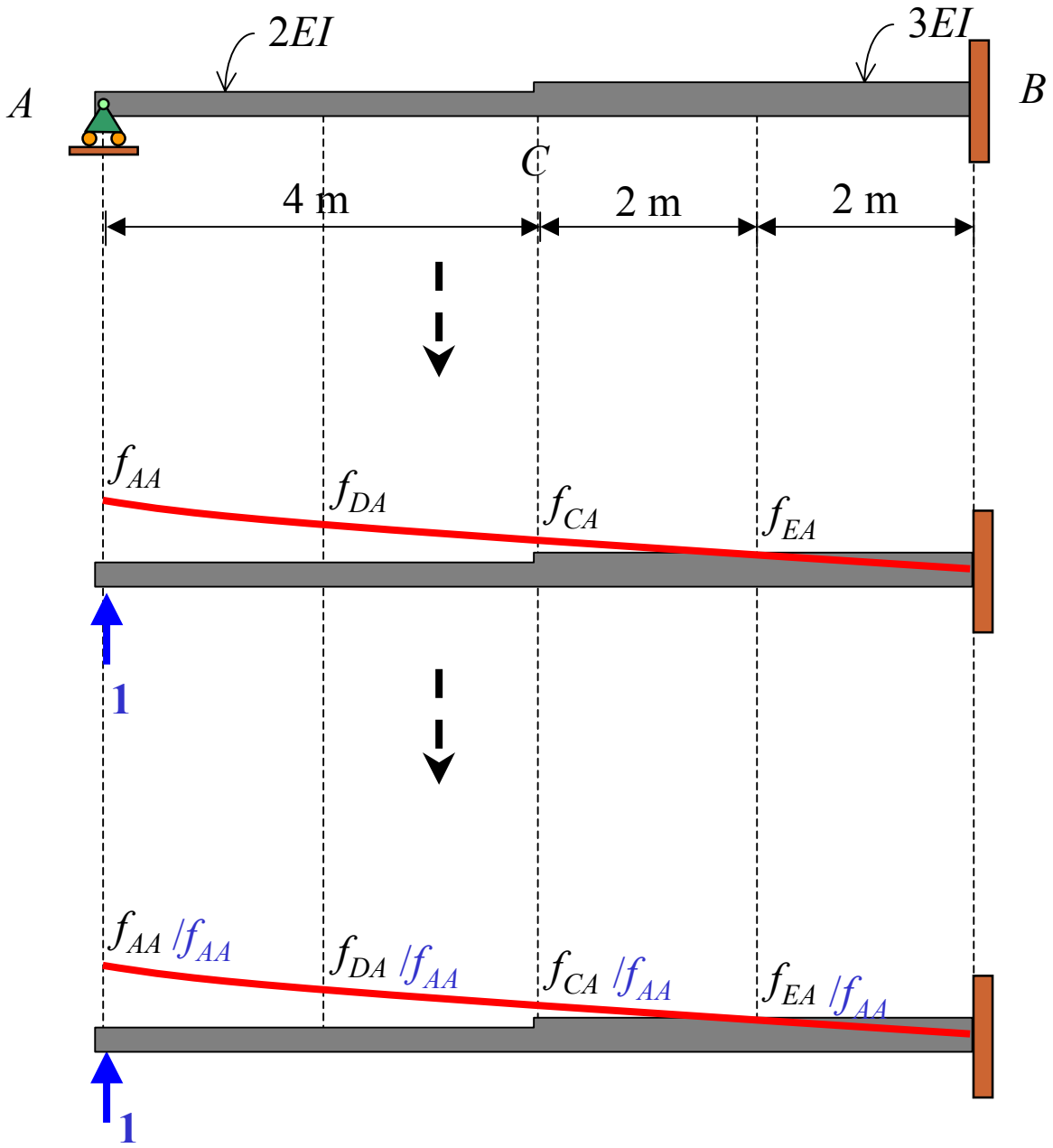
(a) Draw **quantitative influence lines** for the **reaction** at supports *A* and *B*, and **bending moment** at *B*.

(b) Determine all the reactions at supports, and also draw its quantitative shear, bending moment diagrams, and qualitative deflected curve for

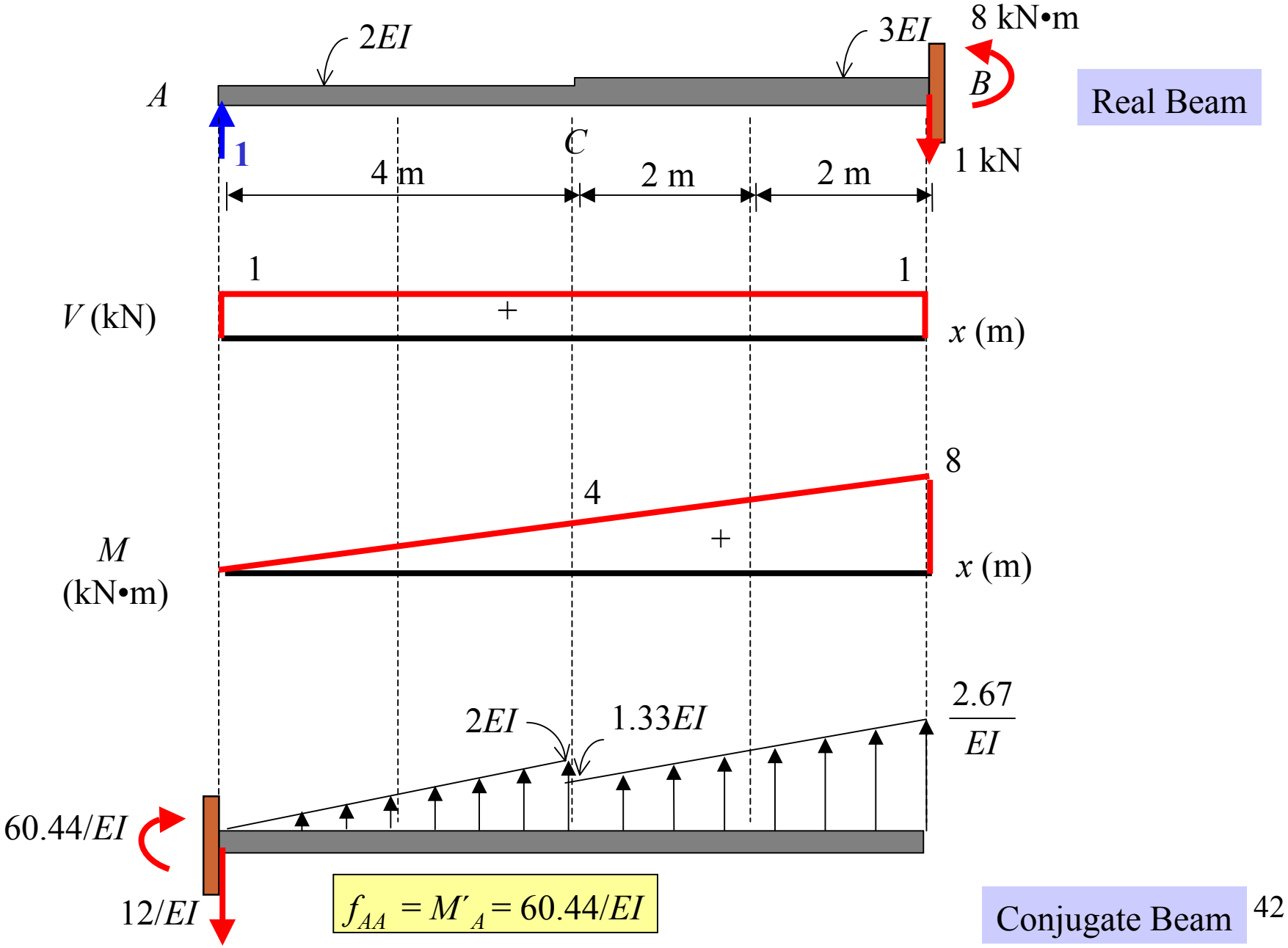
- Only 10 kN downward at 6 m from *A*
- Both 10 kN downward at 6 m from *A* and 20 kN downward at 4 m from *A*



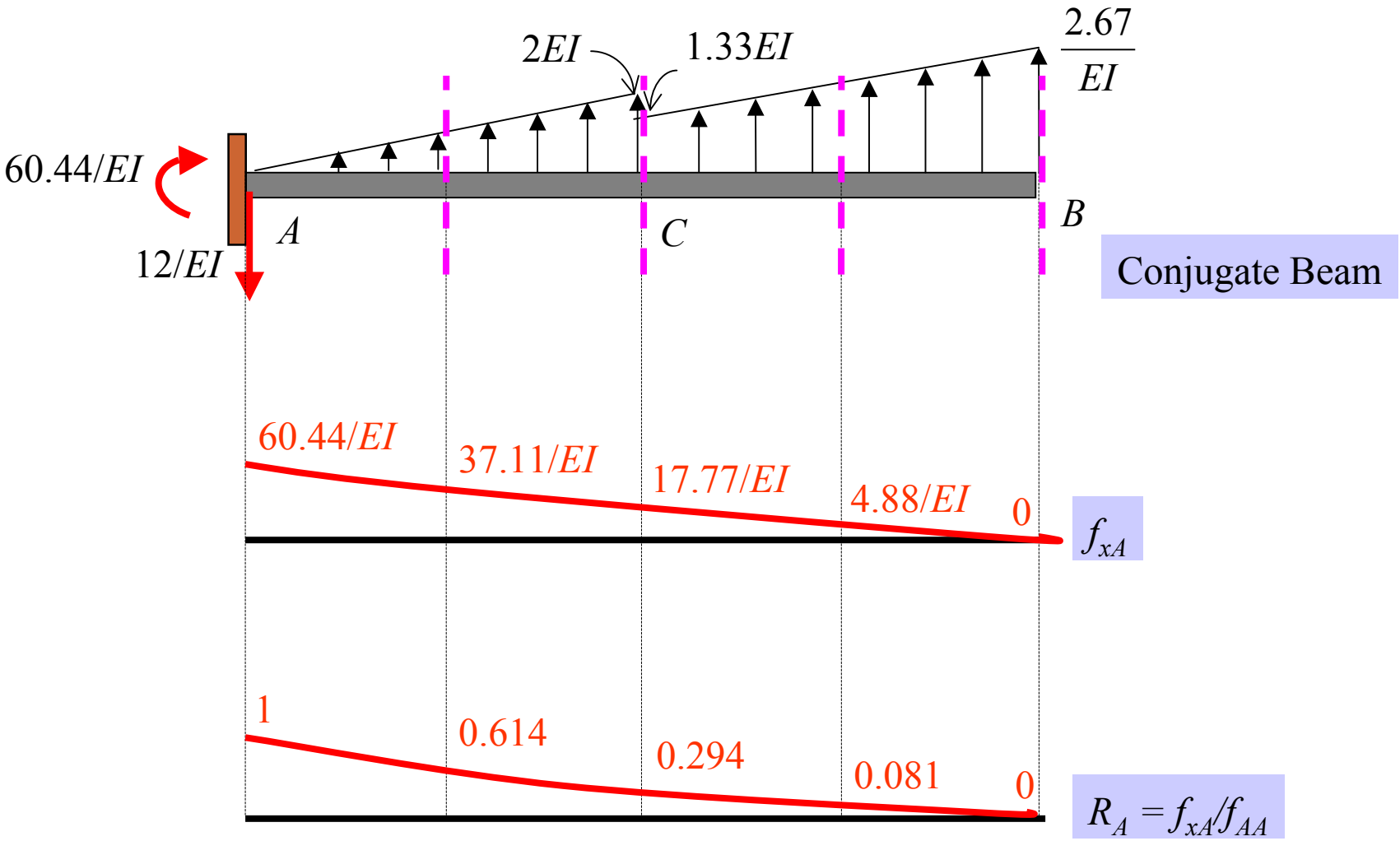
Influence line of R_A



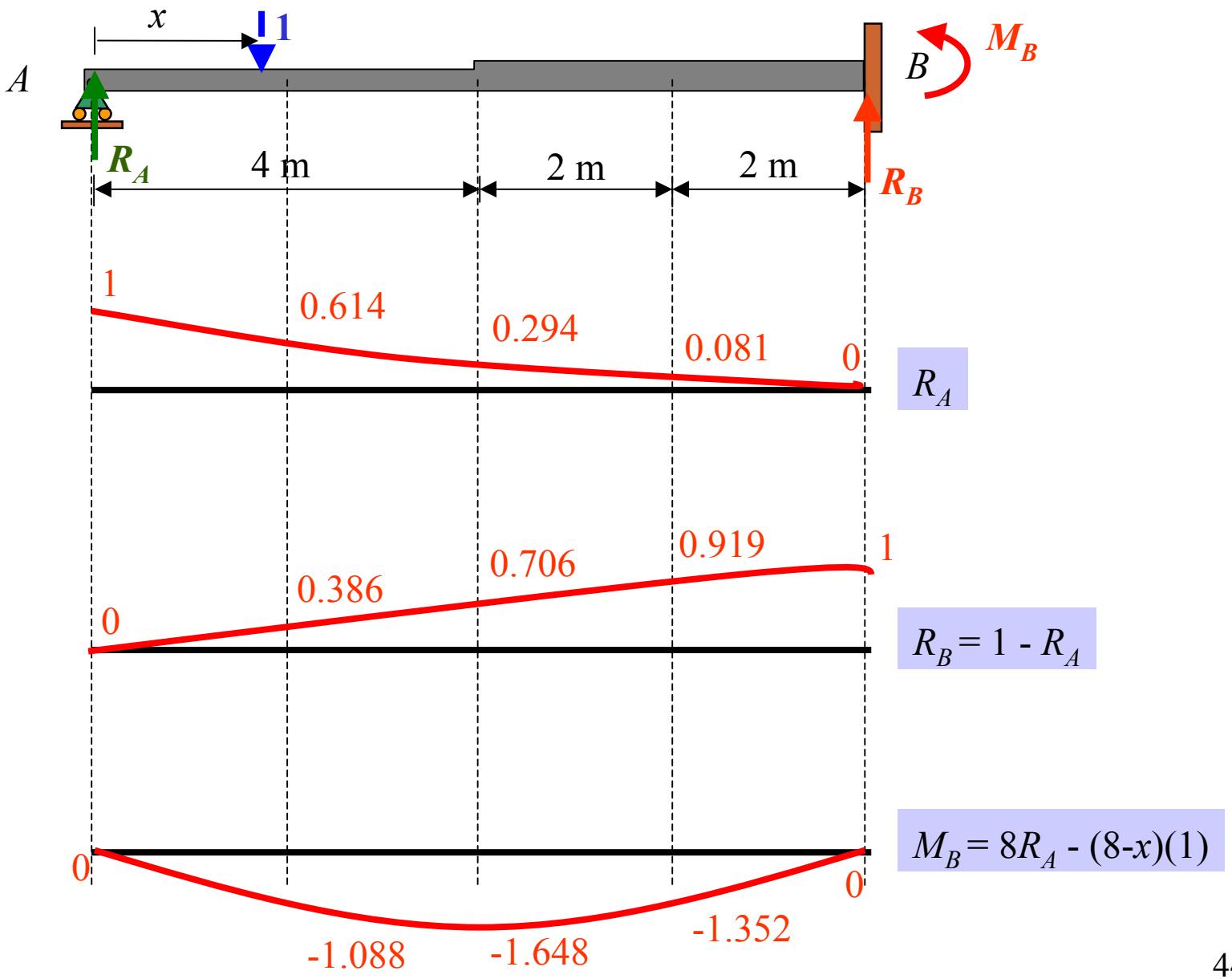
• **Find f_{xA} by conjugate beam**



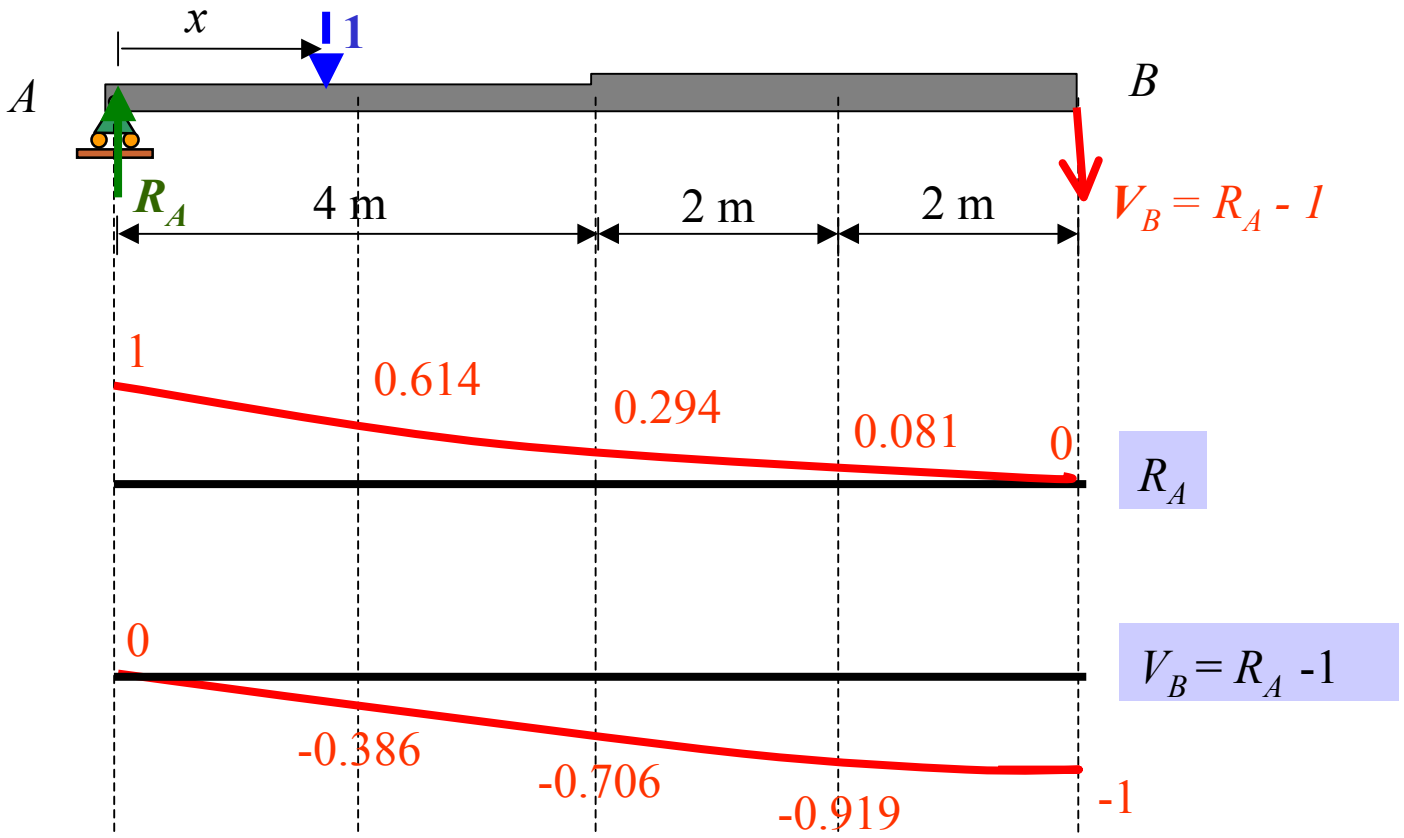
• **Quantitative influence line of R_A**



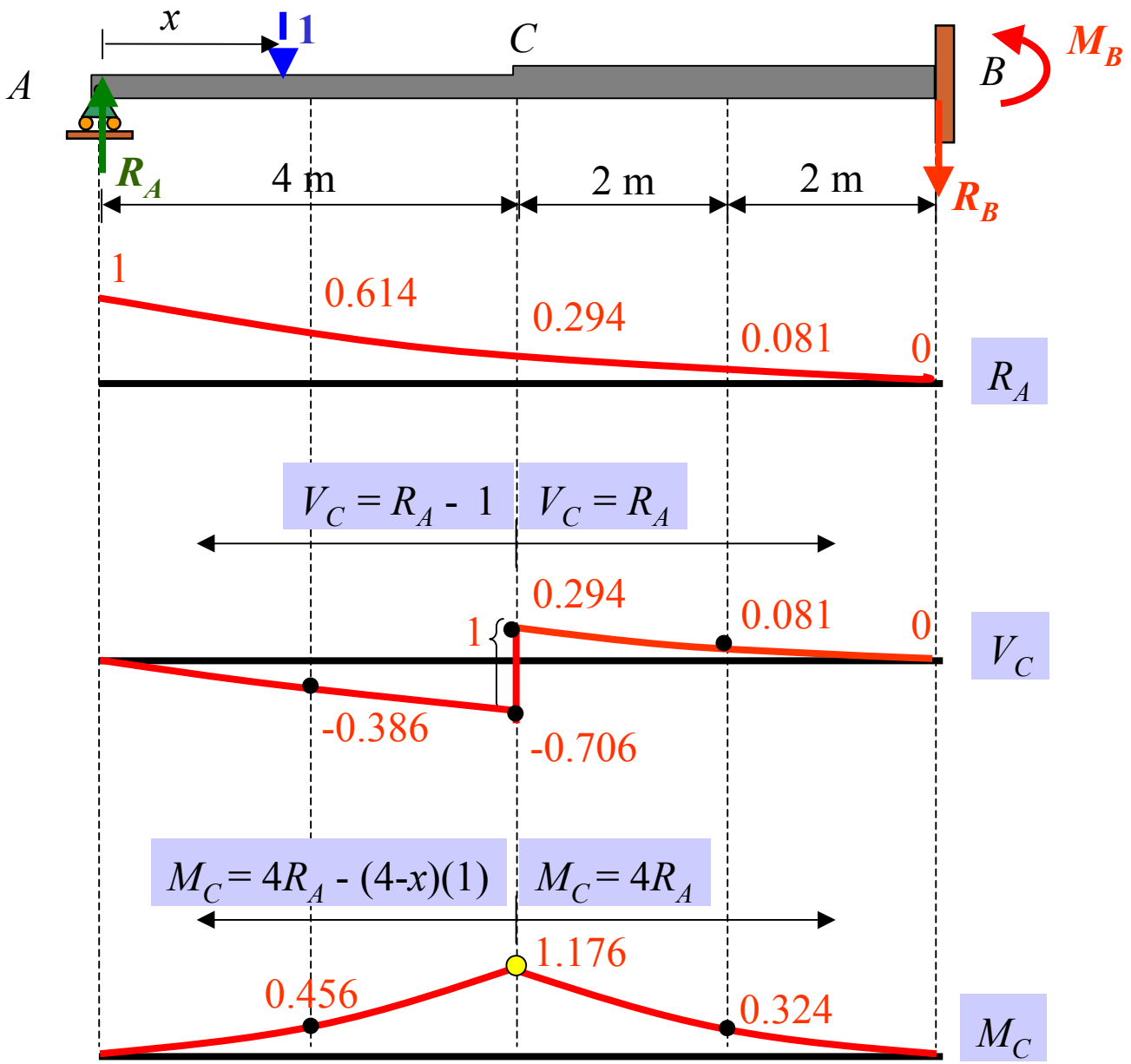
Using equilibrium conditions for the influence line of R_B and M_B



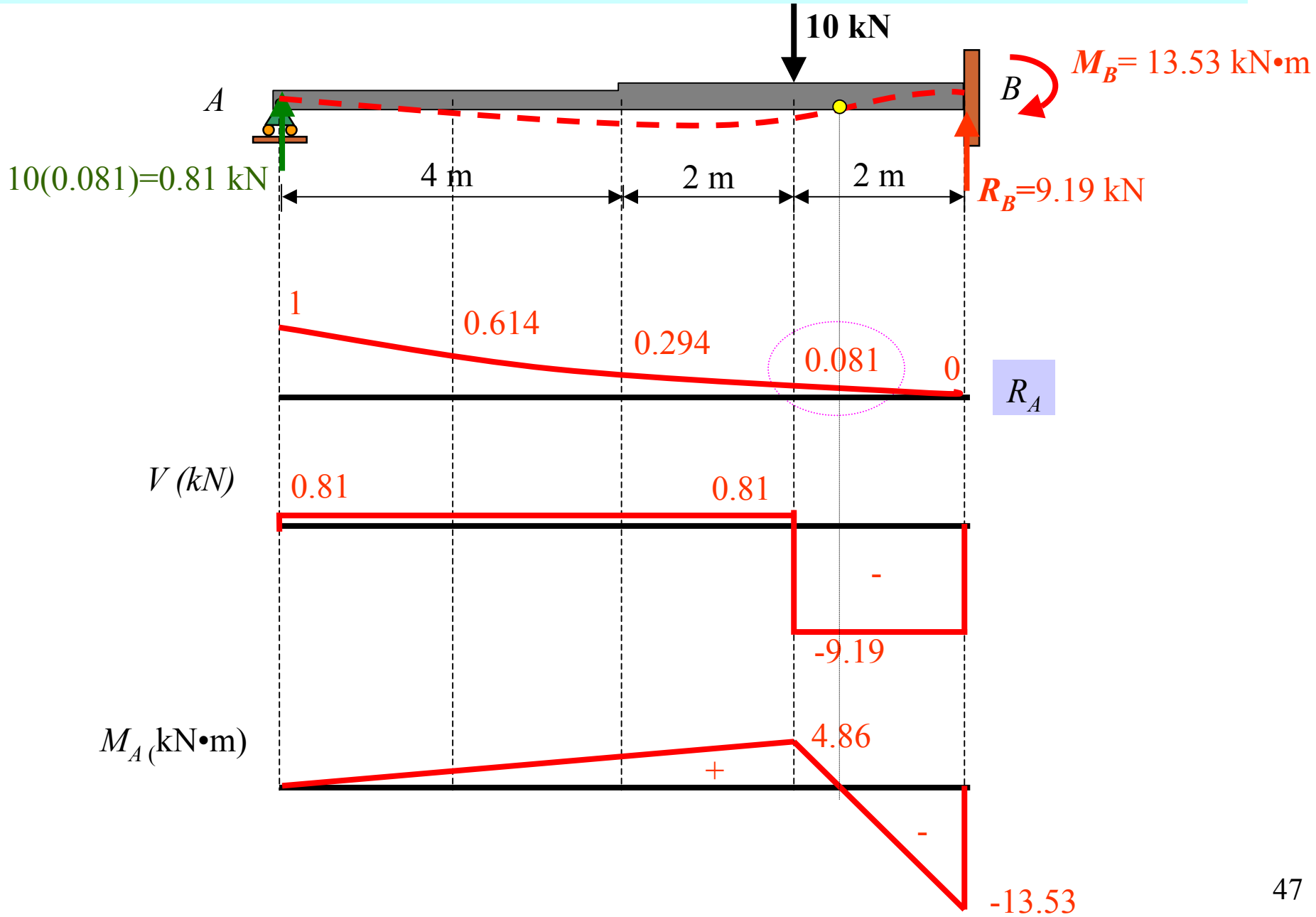
Using equilibrium conditions for the influence line of V_B



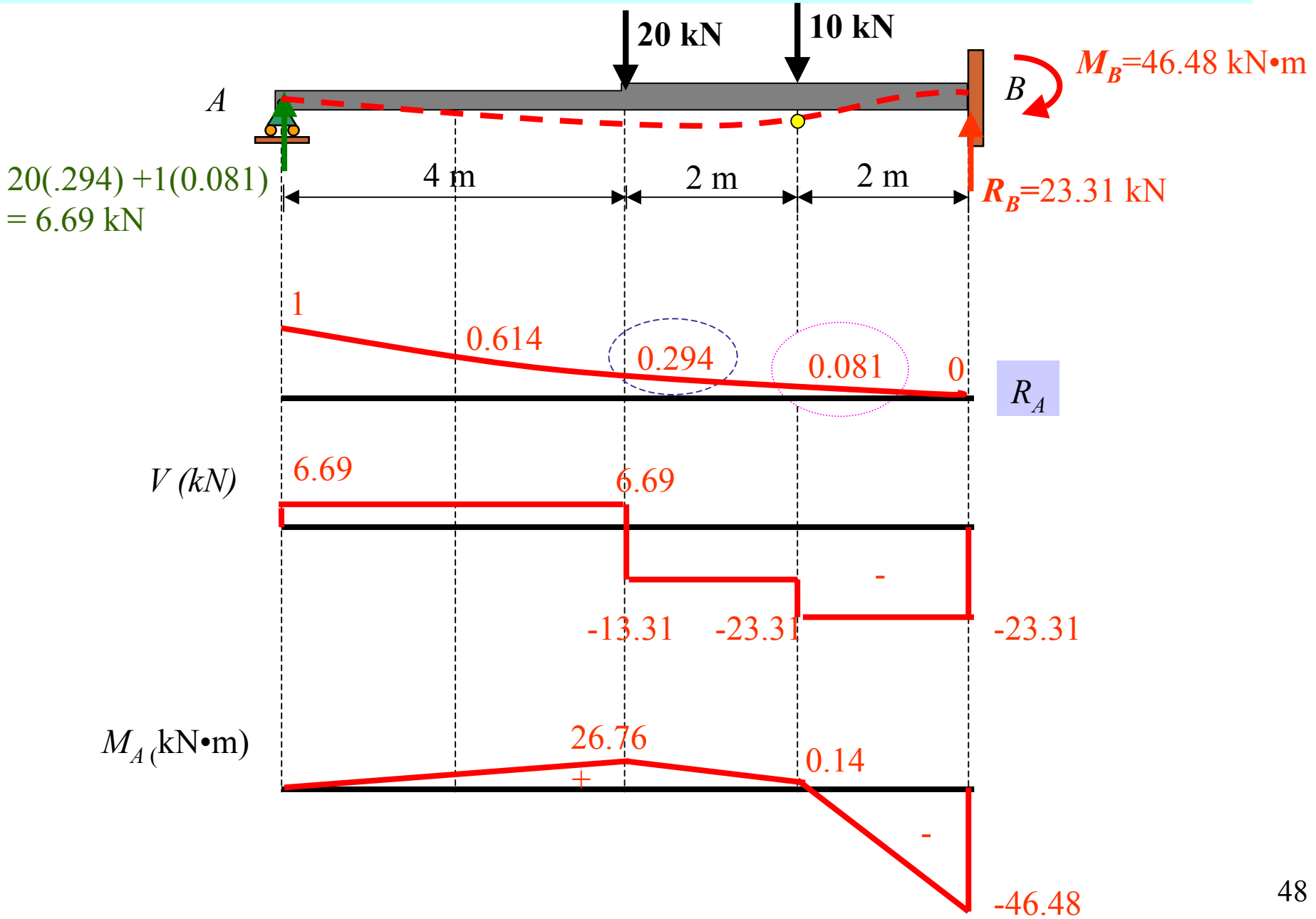
Using equilibrium conditions for the influence line of V_C and M_C



The quantitative shear and bending moment diagram and qualitative deflected curve



The quantitative shear and bending moment diagram and qualitative deflected curve

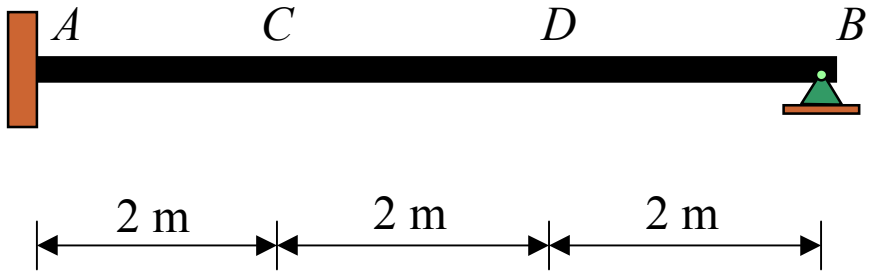


APPENDIX

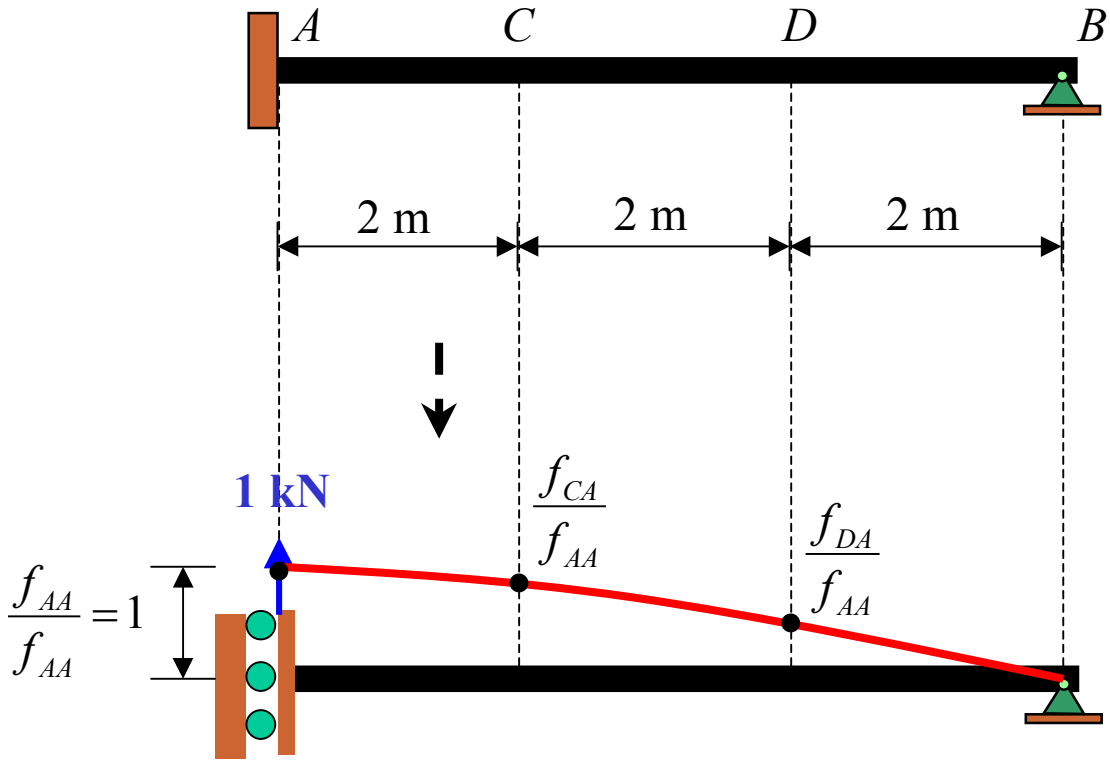
- ***Muller-Breslau for the influence line of reaction, shear and moment***
- ***Influence lines for MDOF beams***

Example 1

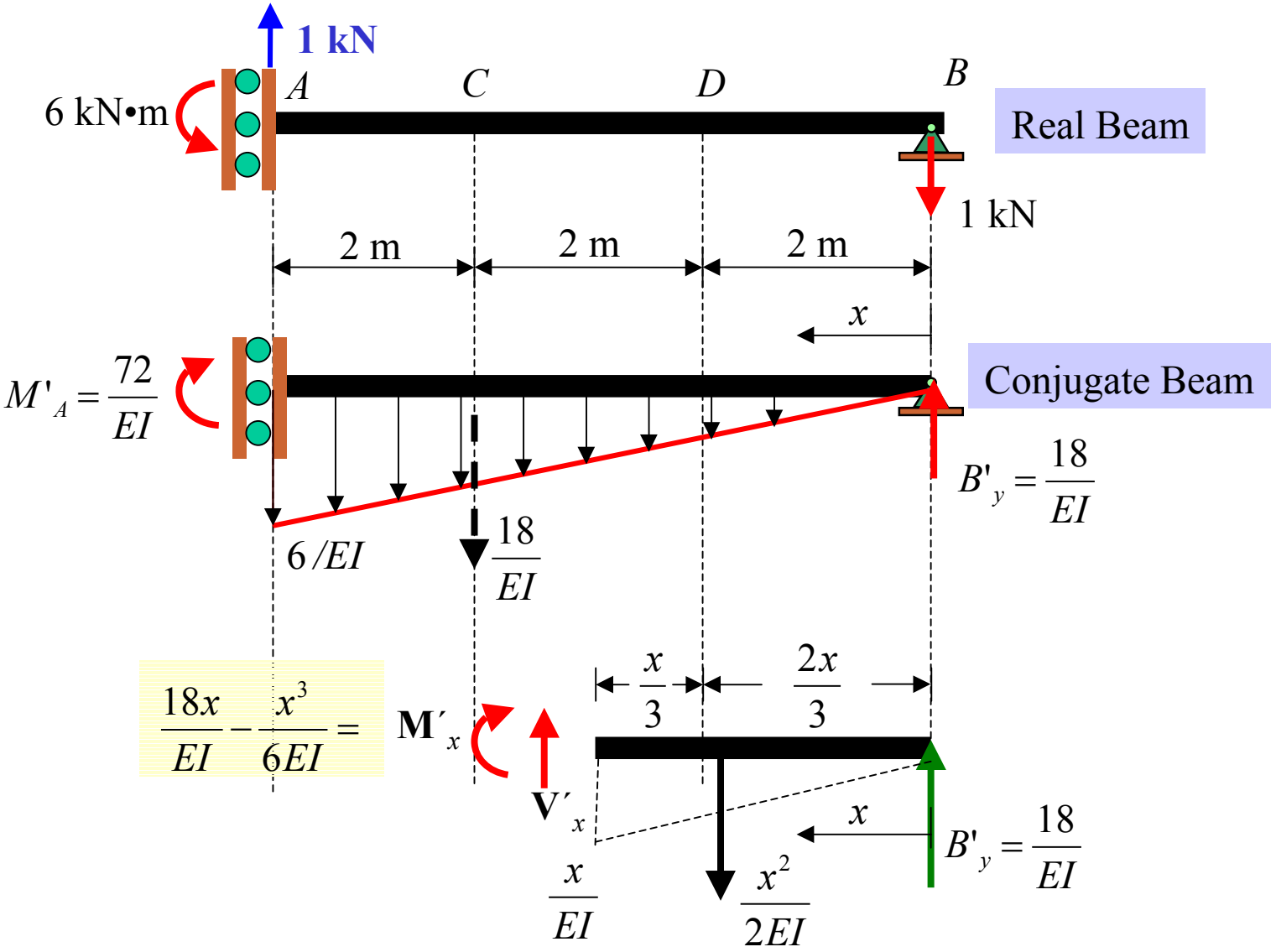
Draw the influence line for
- the vertical **reaction at *B***

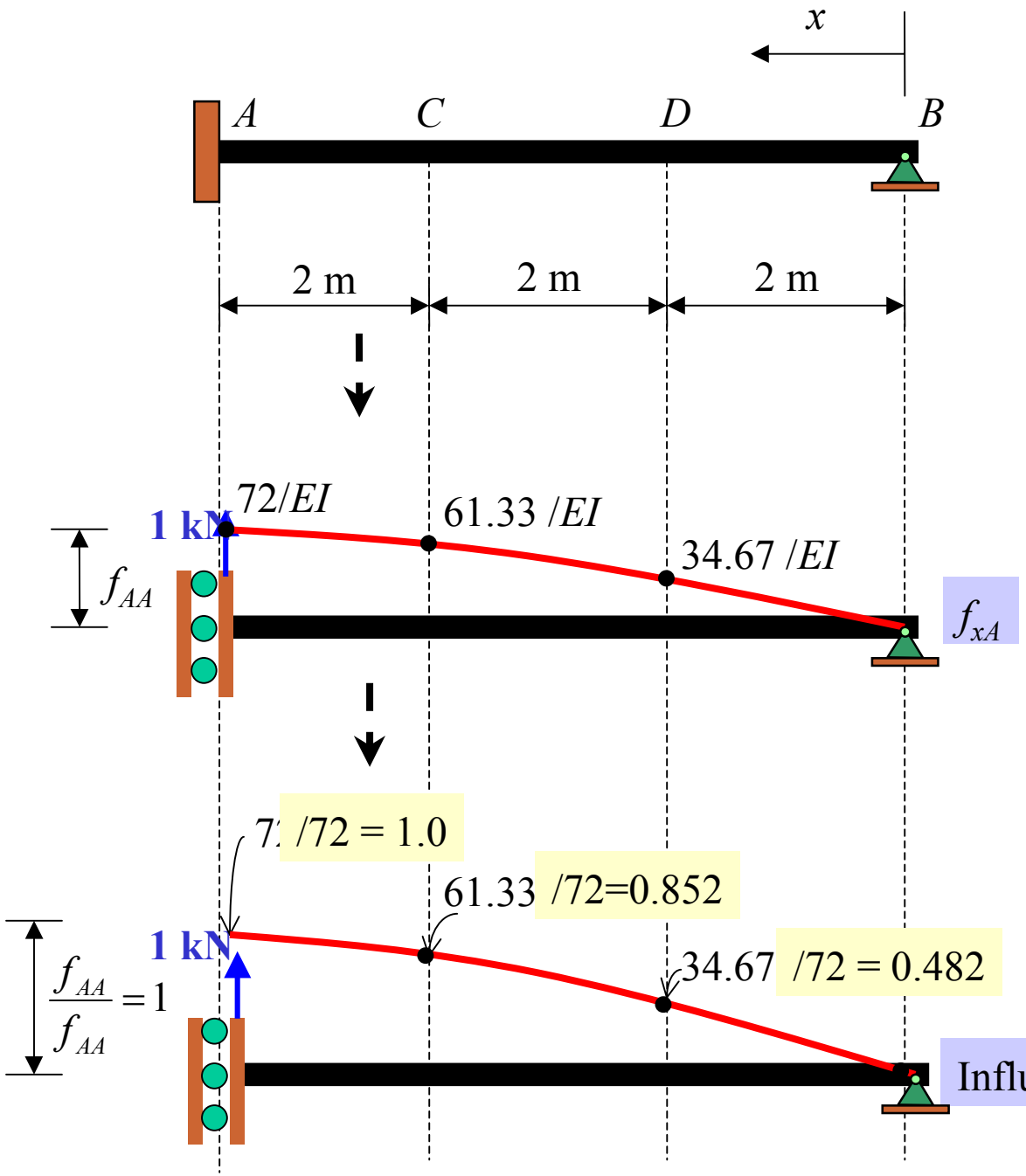


Influence line of R_A



• **Find f_{xA} by conjugate beam**

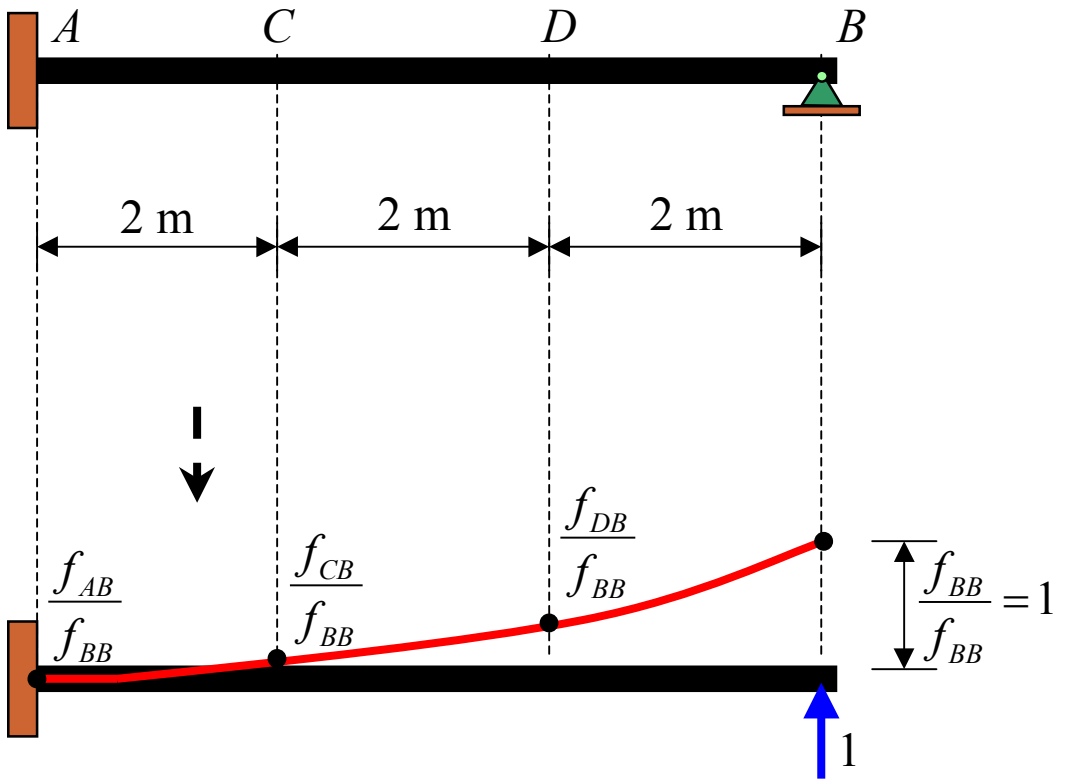




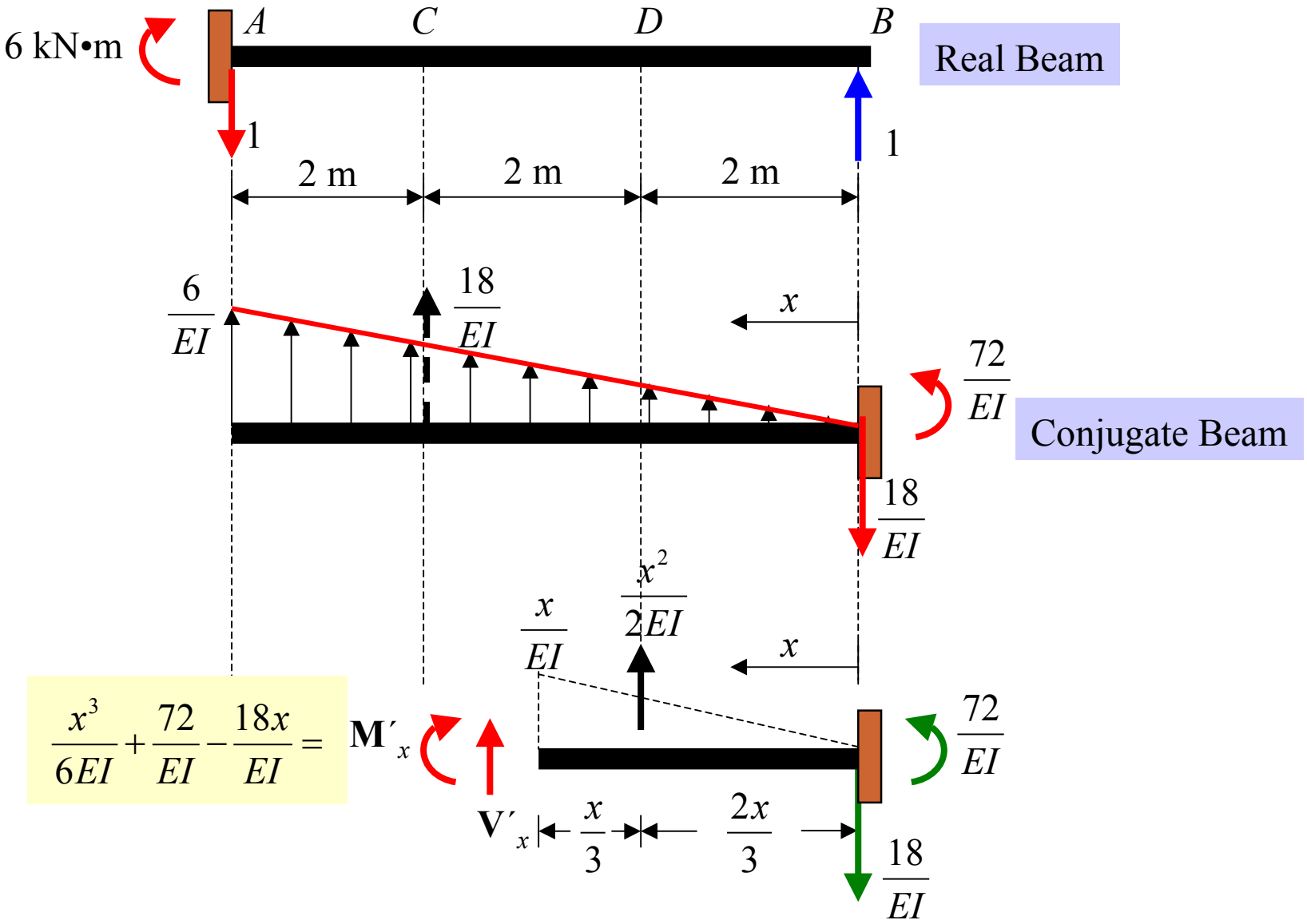
$$f_{xA} = M'_x = \frac{18x}{EI} - \frac{x^3}{6EI}$$

Point	x (m)	f_{xA}	f_{xA} / f_{AA}
B	0	0	0
D	2	$\frac{34.67}{EI}$	0.482
C	4	$\frac{61.33}{EI}$	0.852
A	6	$\frac{72}{EI}$	1.0

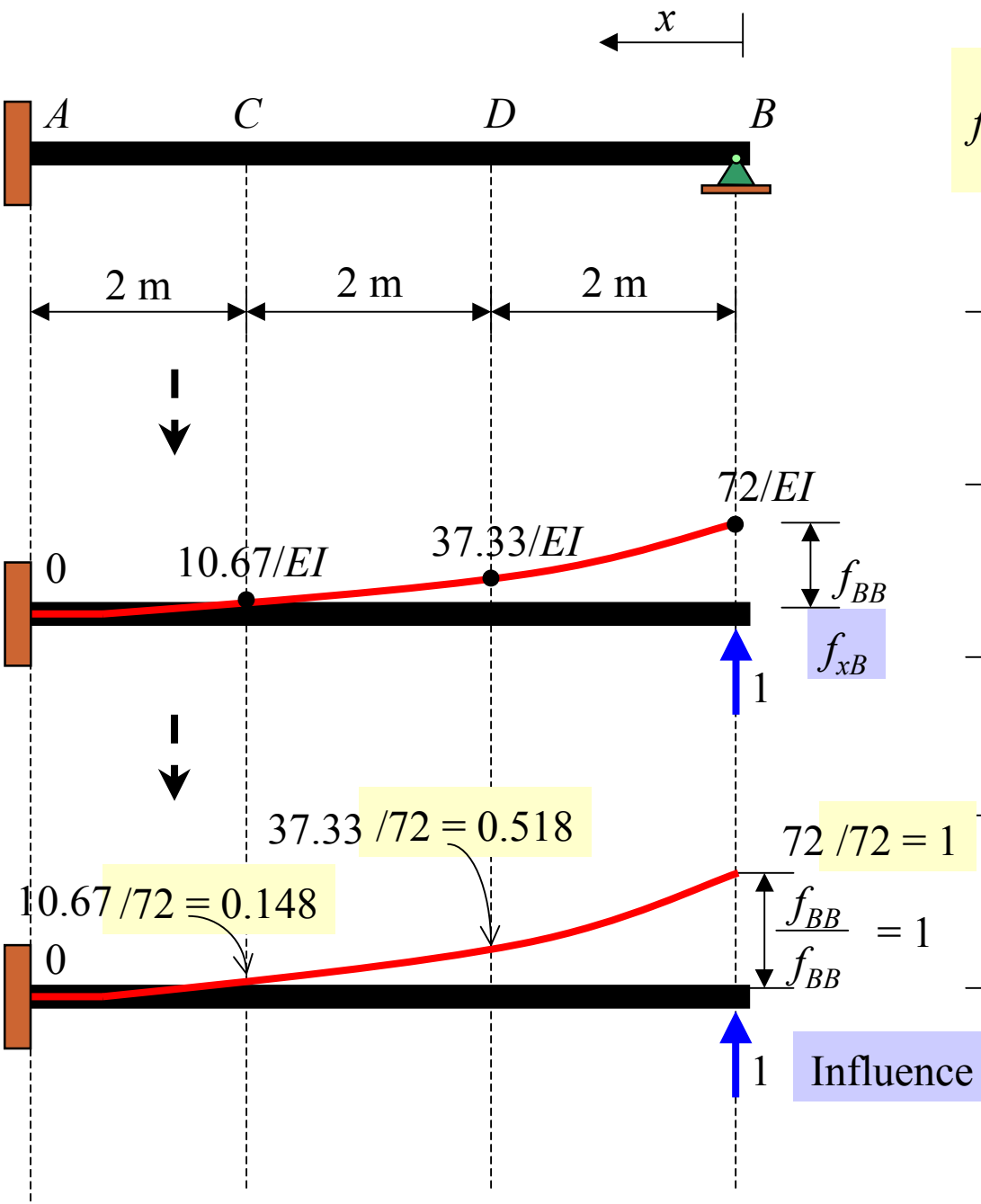
• **Influence line of R_B**



• **Find f_{xB} by conjugate beam**



$$\frac{x^3}{6EI}+\frac{72}{EI}-\frac{18x}{EI}=$$

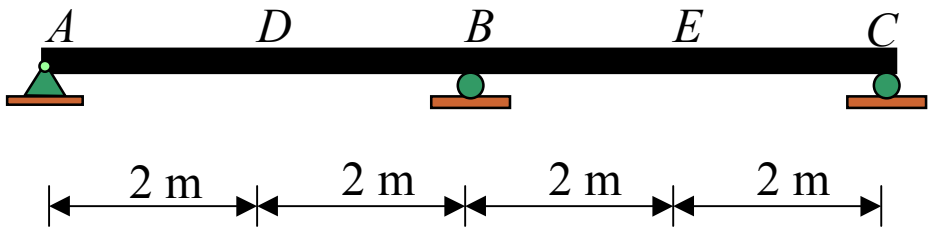


$$f_{xB} = M'_x = \frac{x^3}{6EI} + \frac{72}{EI} - \frac{18x}{EI}$$

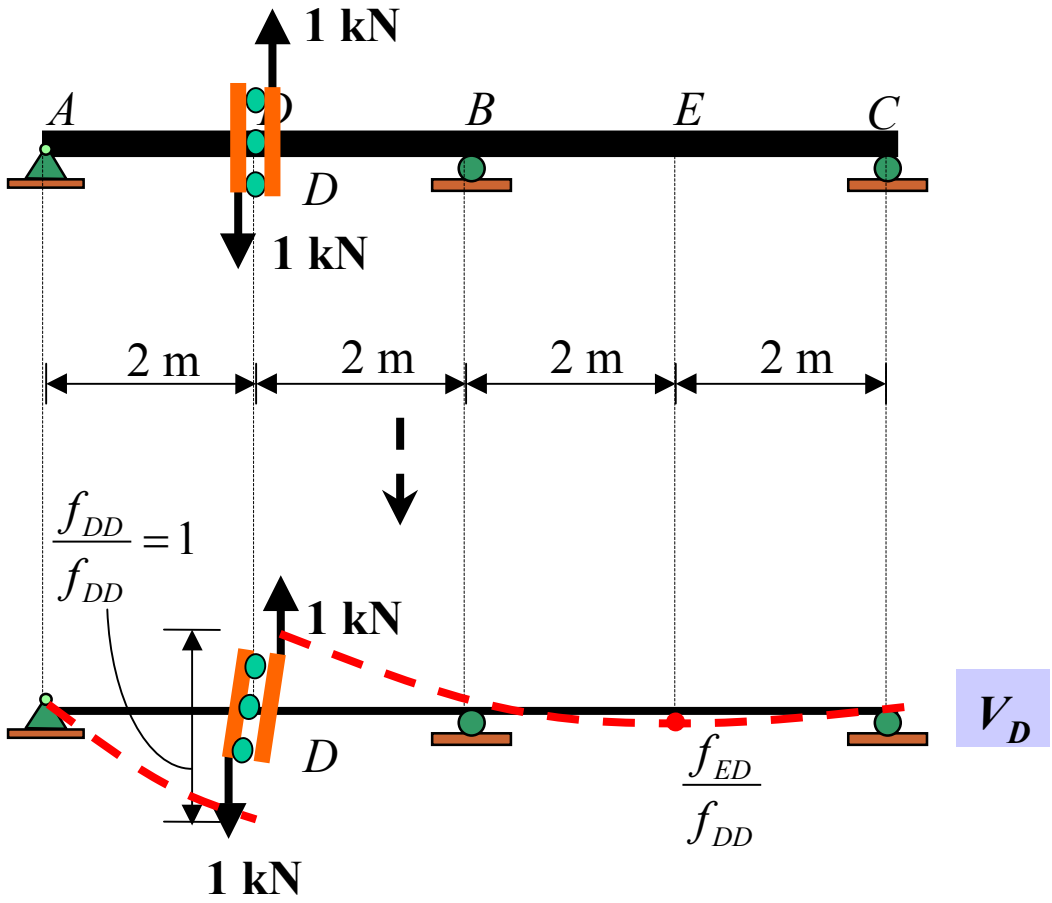
Point	x (m)	f_{xB}	f_{xB} / f_{BB}
B	0	$\frac{72}{EI}$	1
D	2	$\frac{37.33}{EI}$	0.518
C	4	$\frac{10.67}{EI}$	0.148
A	6	0	0

Example 2

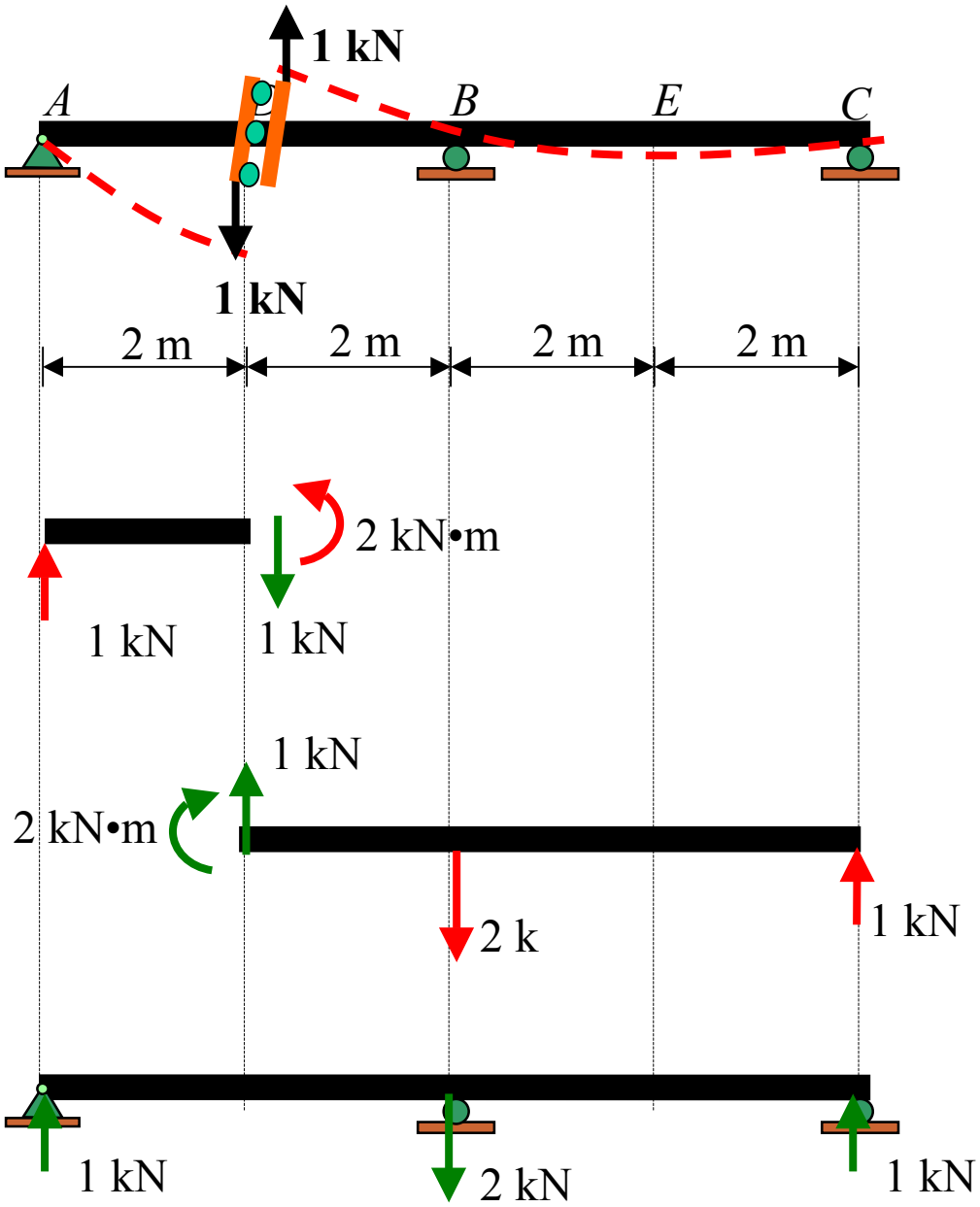
For the beam shown
(a) Draw the influence line for the **shear at *D*** for the beam
(b) Draw the influence line for the **bending moment at *D*** for the beam
EI is constant.Plot numerical values every 2 m.

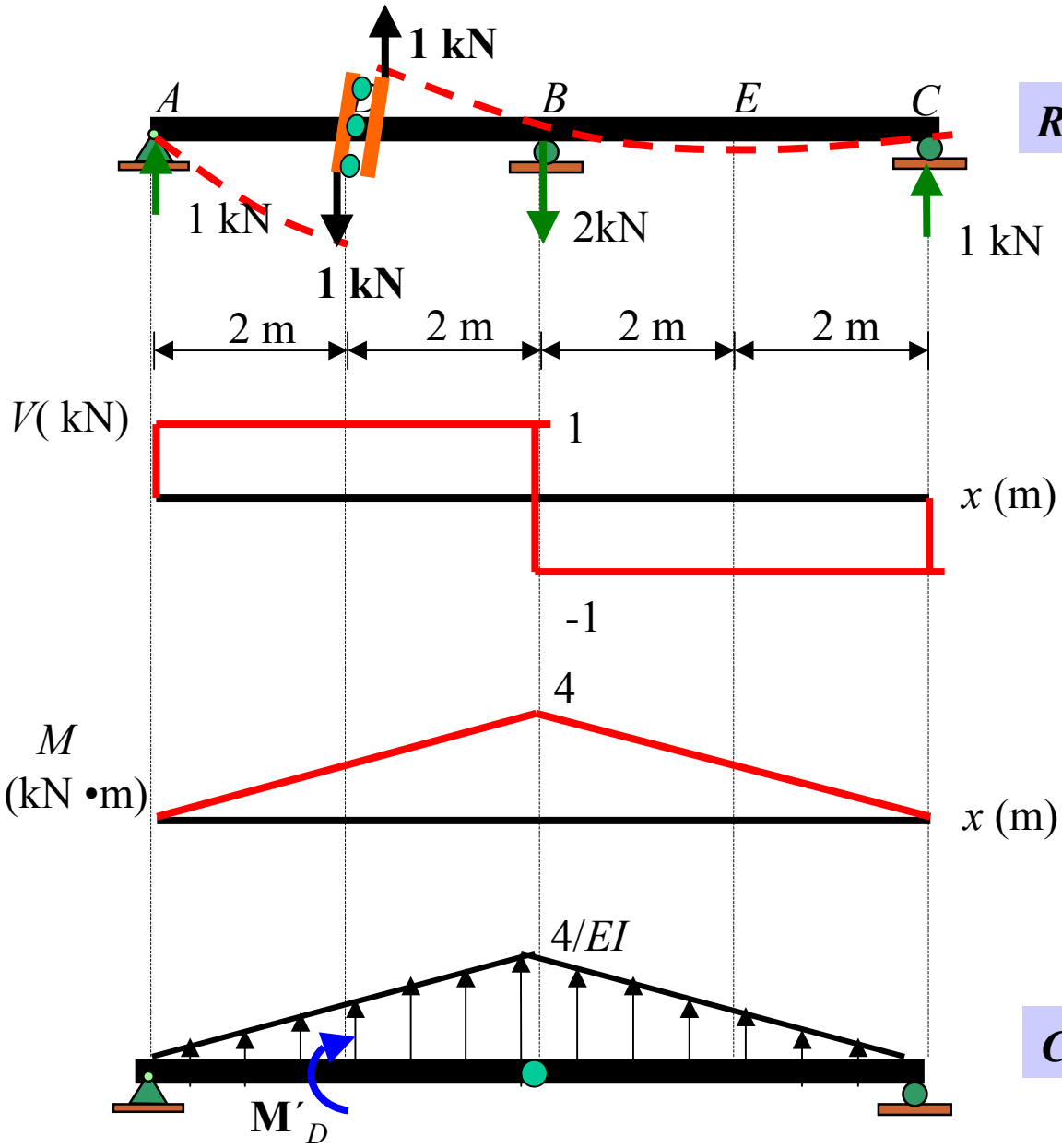


The influence line for the shear at *D*

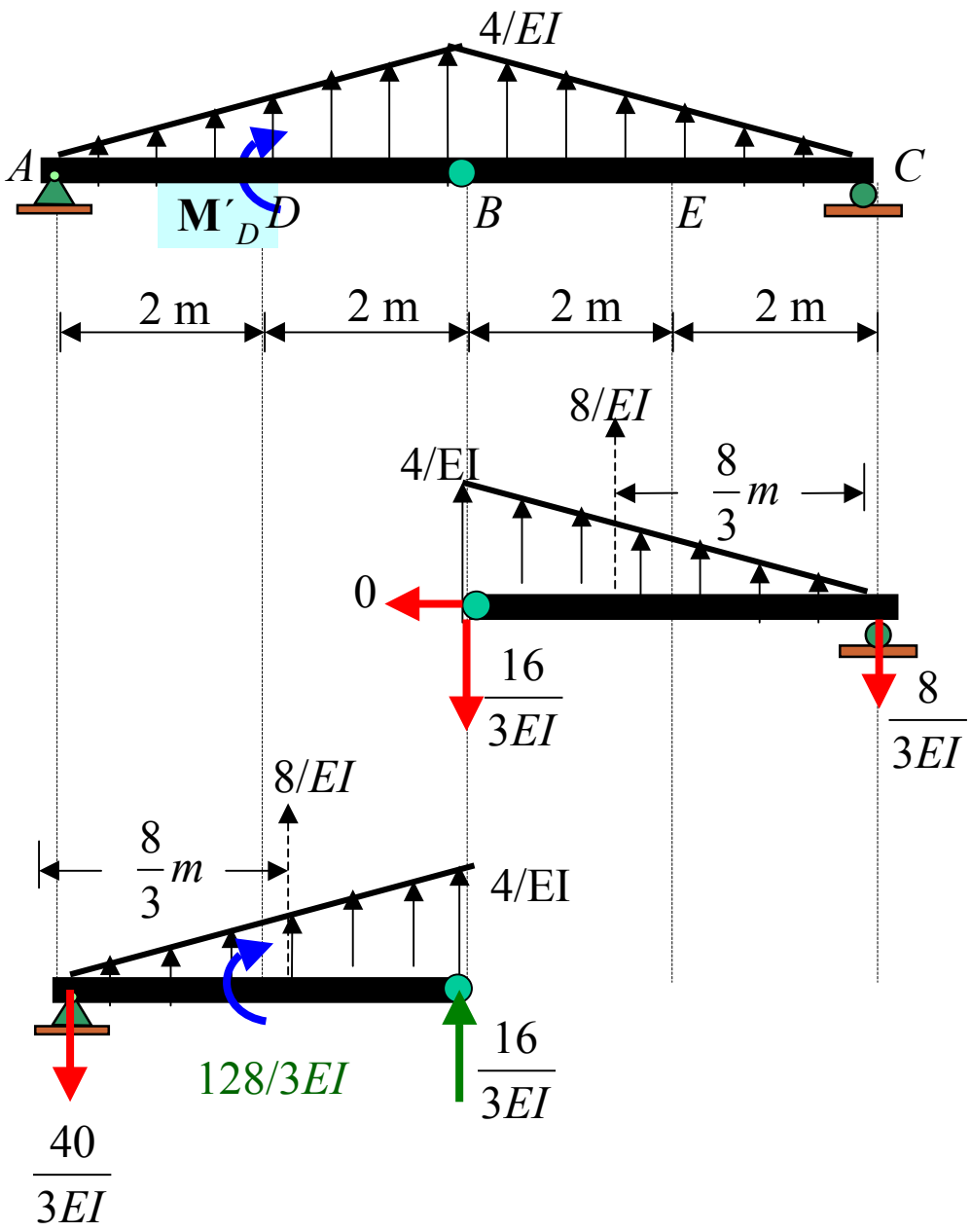


• **Using conjugate beam for find f_{xD}**

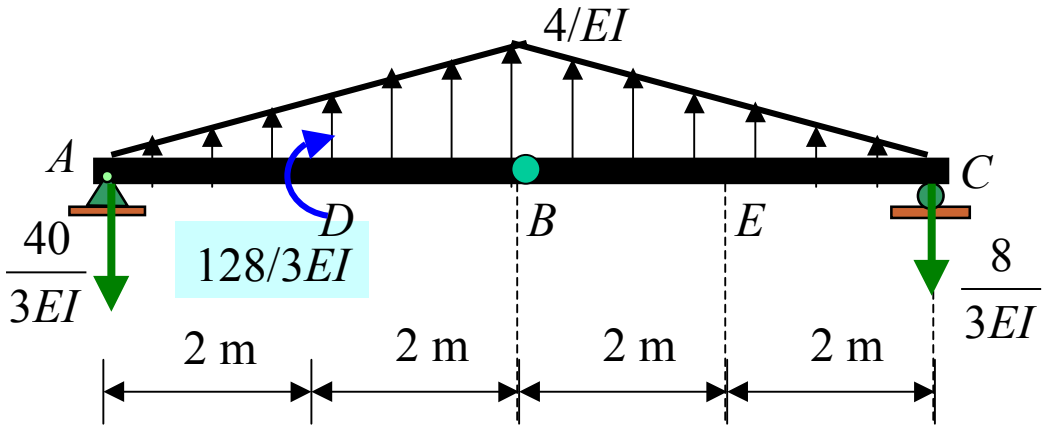



1 kN
A
D
B
E
C
Real beam
1 kN
2kN
1 kN
1 kN
2 m
2 m
2 m
2 m
V(kN)
1
x (m)
-1
4
M
(kN •m)
x (m)
4/EI
M´D
Conjugate beam

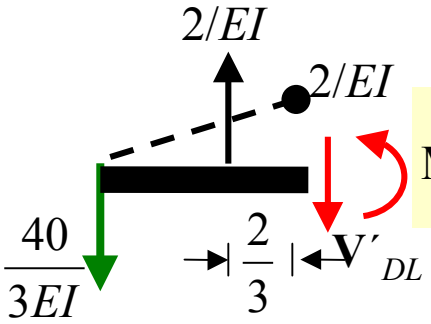
• **Determine M'_D at D**



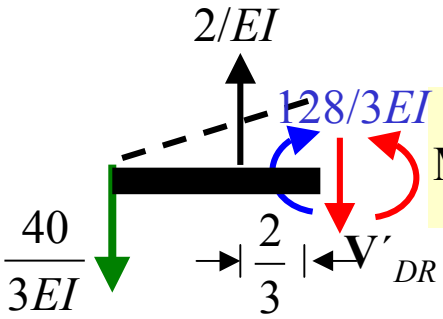
Conjugate beam



Conjugate beam

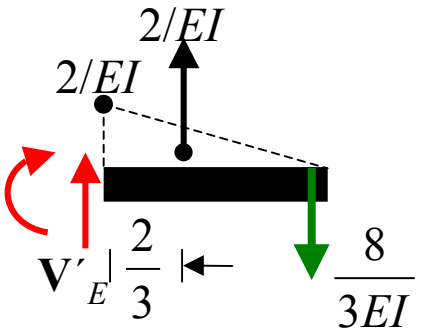


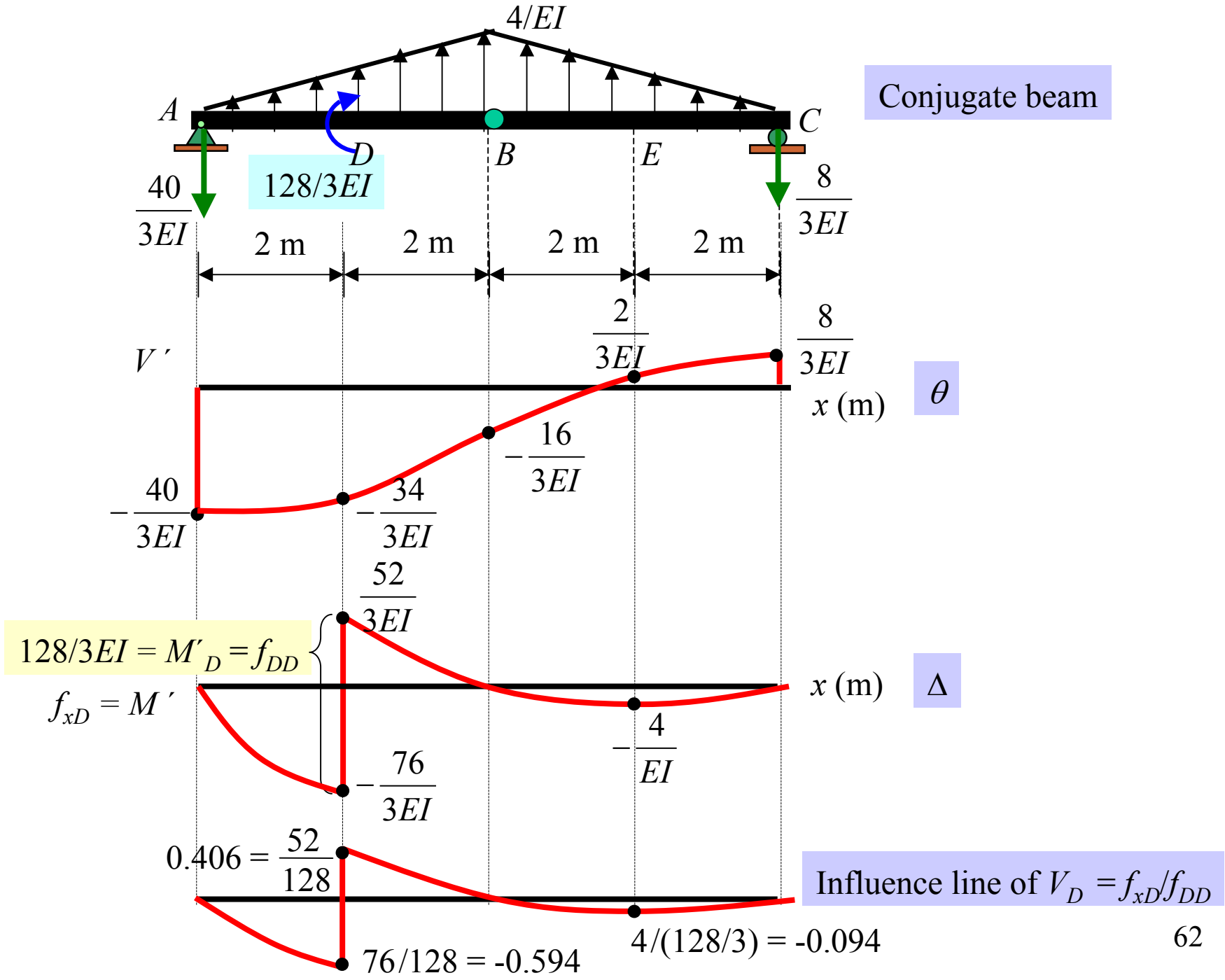
$$\mathbf{M}'_{DL} = (\frac{2}{EI})(\frac{2}{3}) - (\frac{40}{3EI})(2) = -\frac{76}{3EI}$$



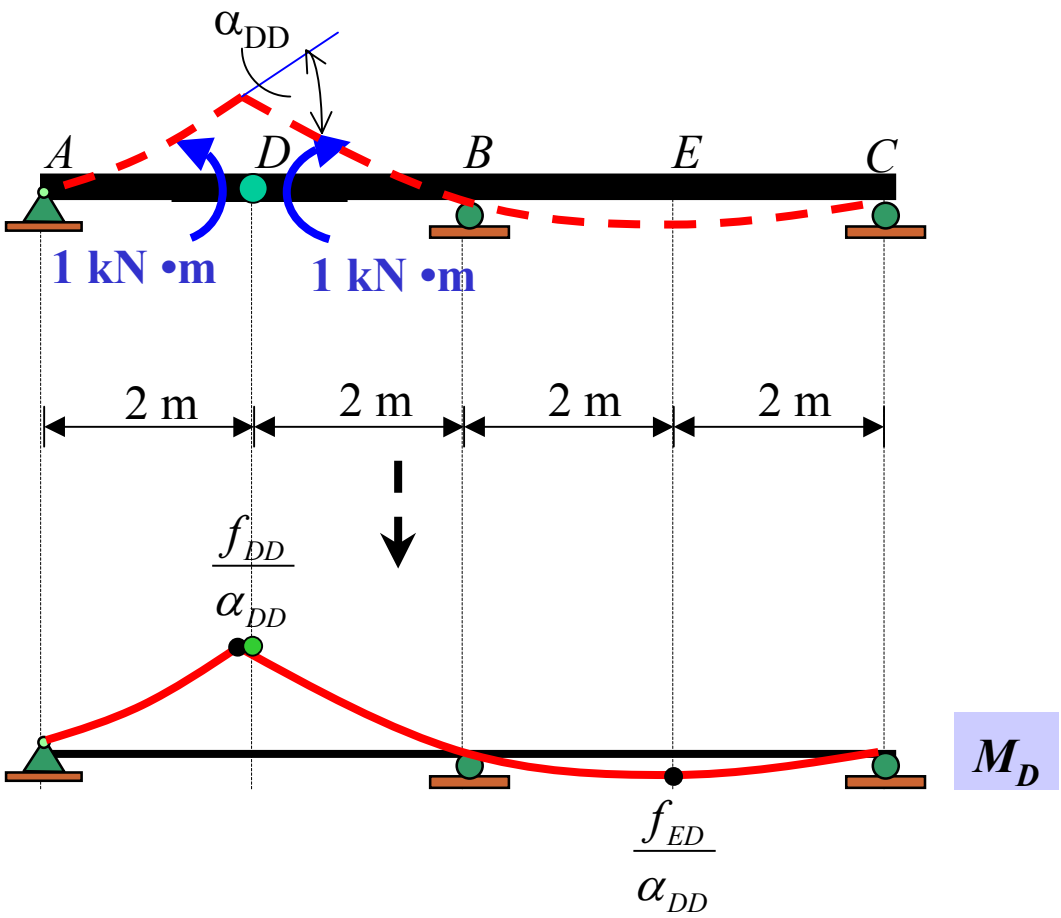
$$\mathbf{M}'_{DR} = (\frac{2}{EI})(\frac{2}{3}) + \frac{128}{3EI} - (\frac{40}{3EI})(2) = \frac{52}{3EI}$$

$$-\frac{4}{EI} = (\frac{2}{EI})(\frac{2}{3}) - (\frac{8}{3EI})(2) = \mathbf{M}'_{E}$$

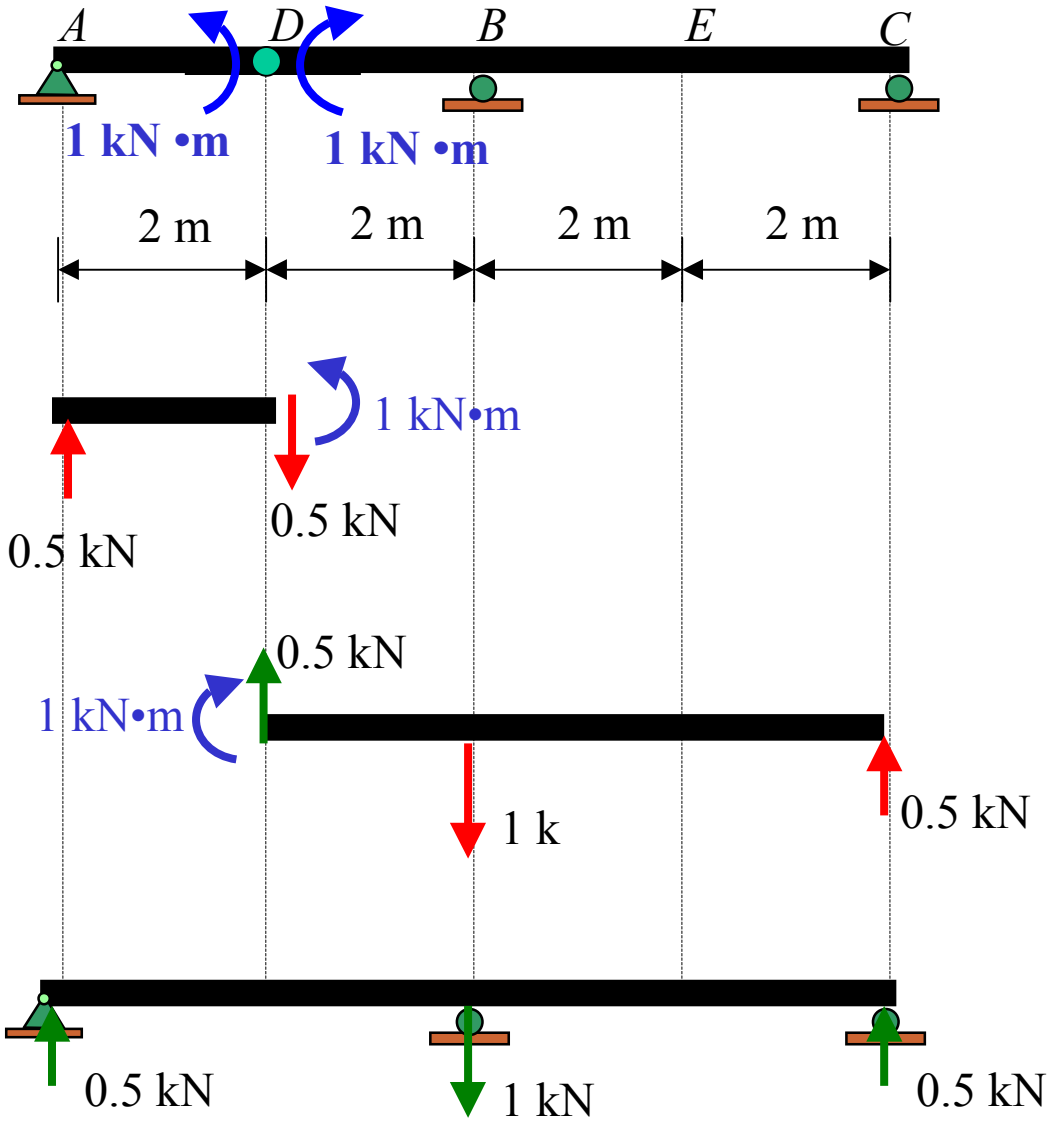


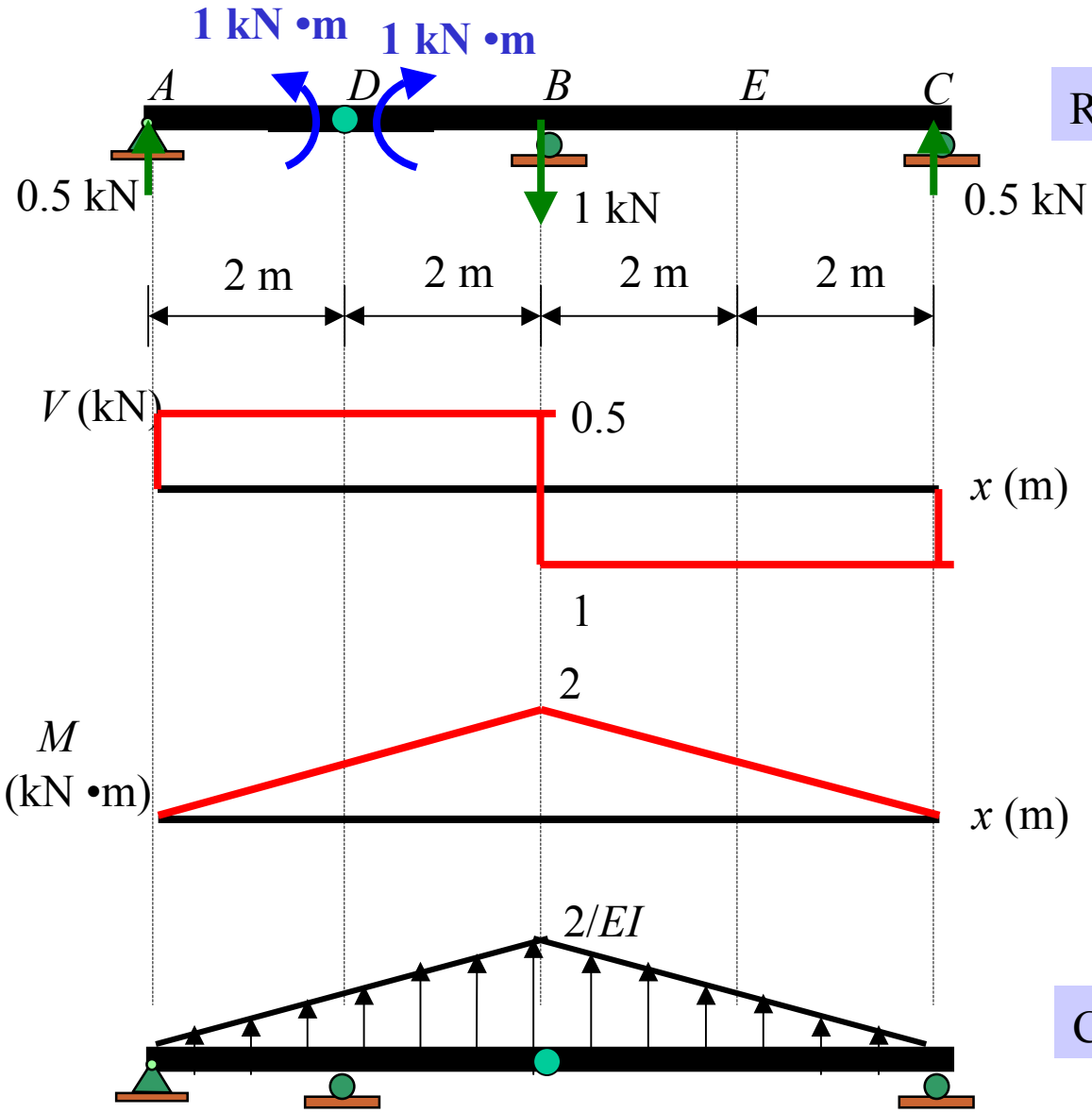

4/EI
A
D
B
E
C
Conjugate beam
128/3EI
40/3EI
8/3EI
2 m
2 m
2 m
2 m
V′
2/3EI
8/3EI
x (m)
θ
−16/3EI
−40/3EI
−34/3EI
52/3EI
128/3EI = M′D = fDD
fxD = M′
x (m)
Δ
−4/EI
−76/3EI
0.406 = 52/128
Influence line of VD = fxD/fDD
4/(128/3) = -0.094
76/128 = -0.594

The influence line for the bending moment at *D*



• **Using conjugate beam for find f_{xD}**

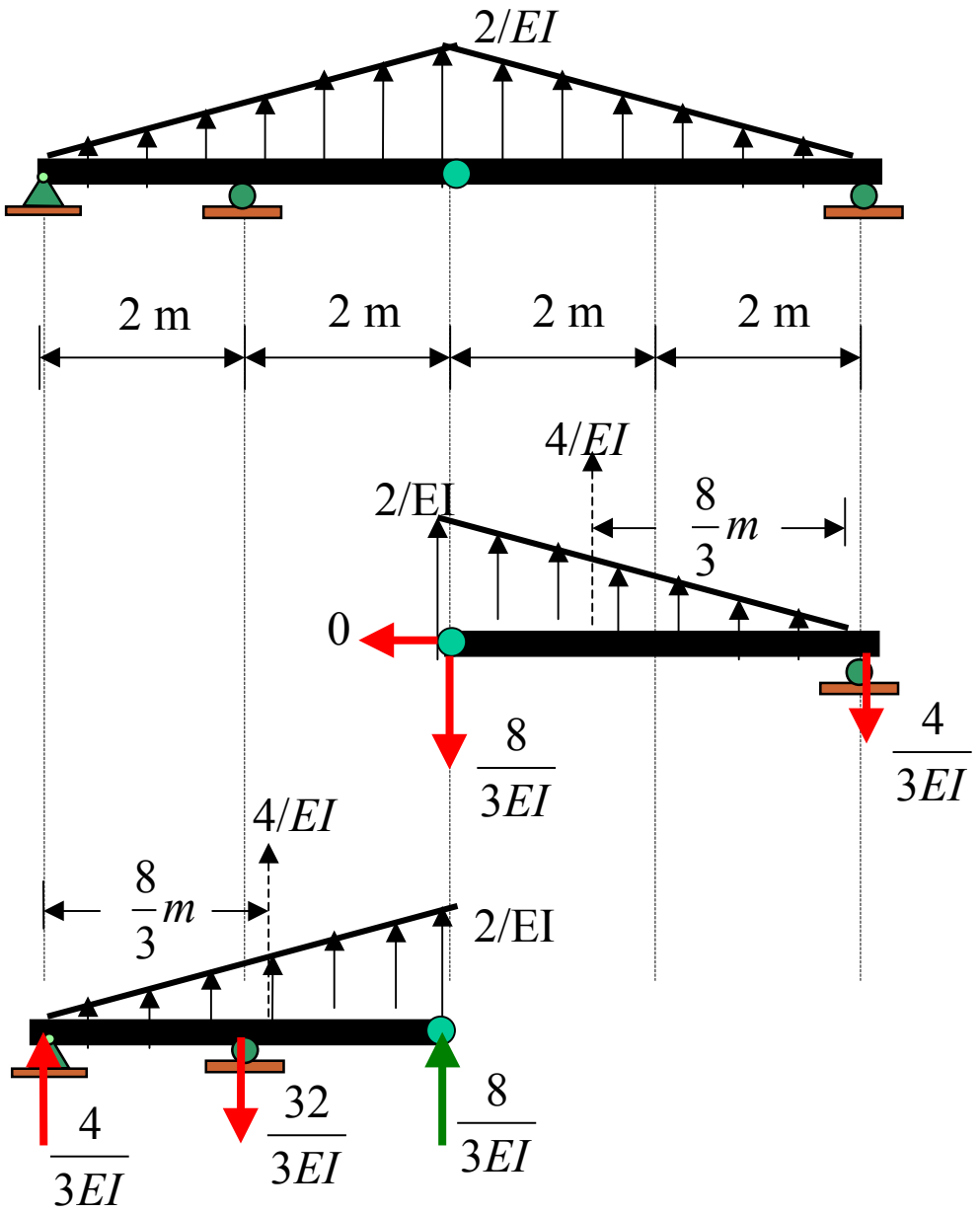




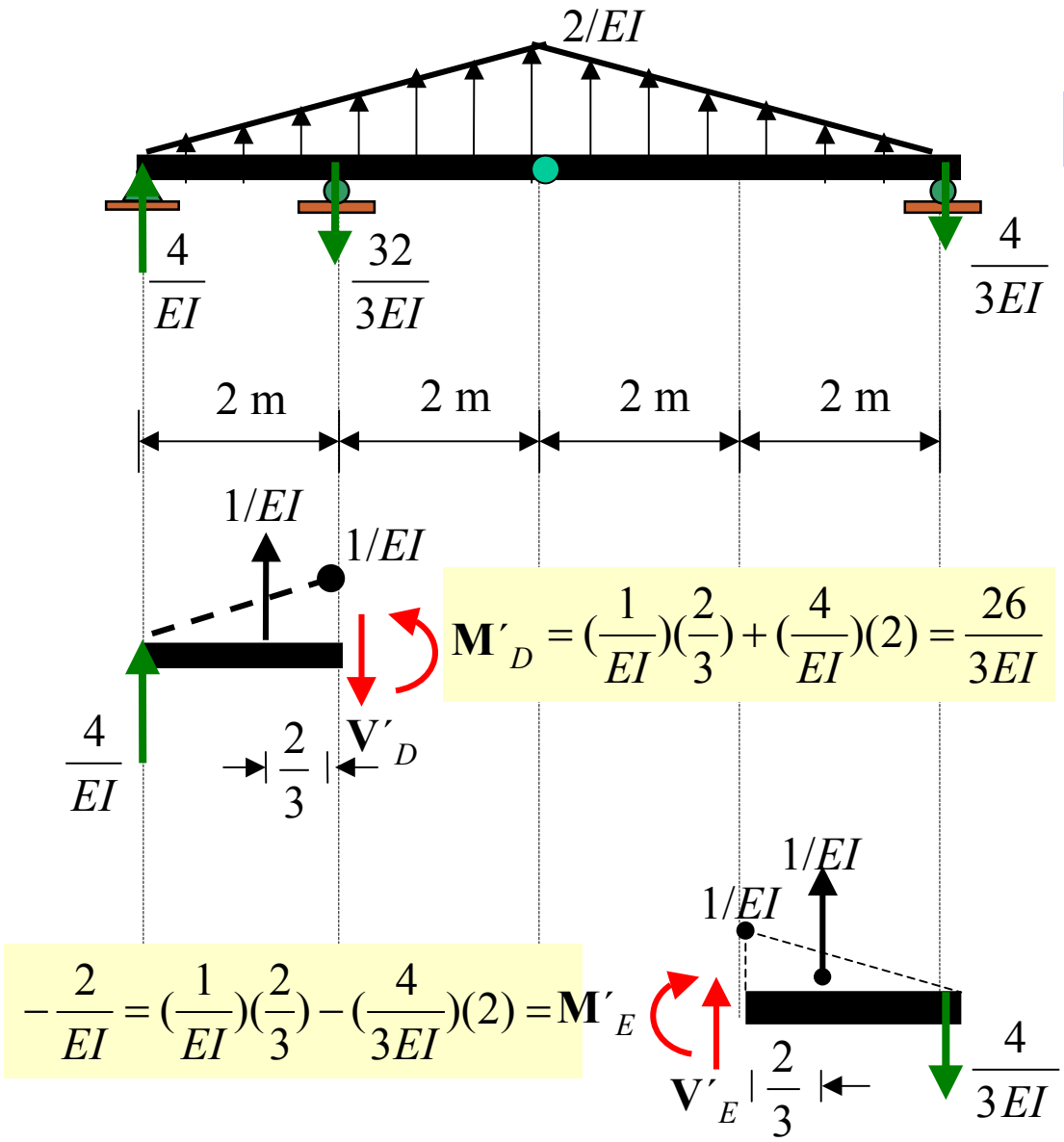
Real beam

Conjugate beam

Conjugate beam

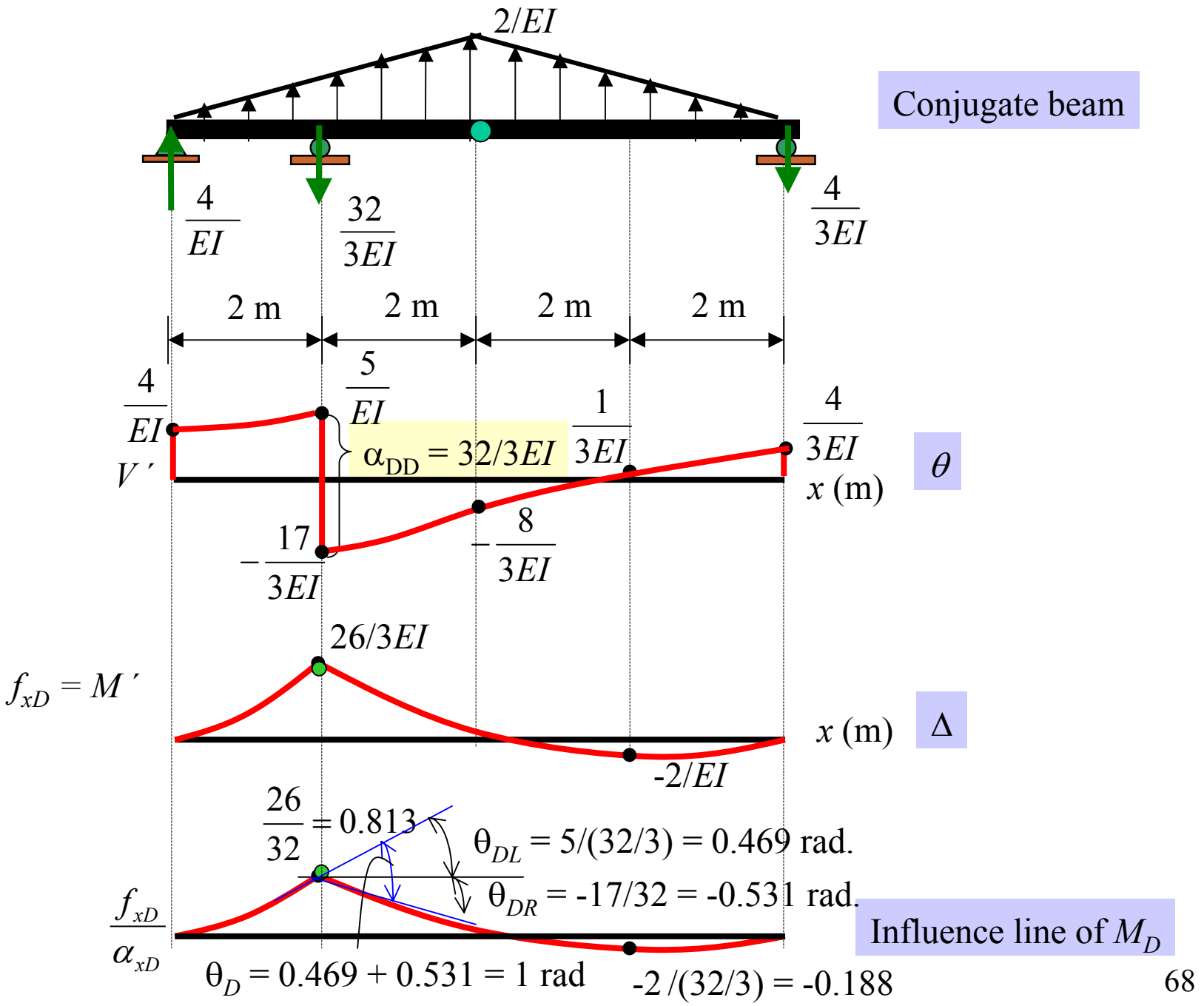


Conjugate beam



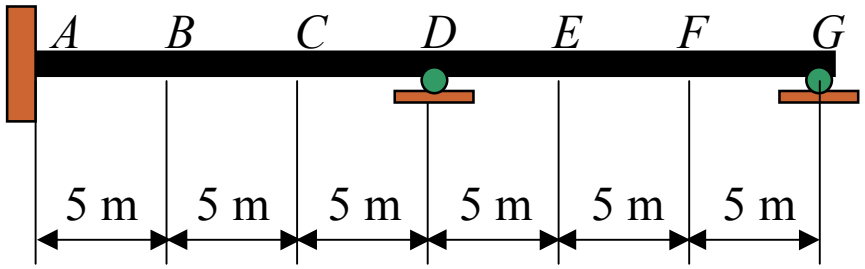
$$\mathbf{M}'_D = (\frac{1}{EI})(\frac{2}{3}) + (\frac{4}{EI})(2) = \frac{26}{3EI}$$

$$-\frac{2}{EI} = (\frac{1}{EI})(\frac{2}{3}) - (\frac{4}{3EI})(2) = \mathbf{M}'_E$$

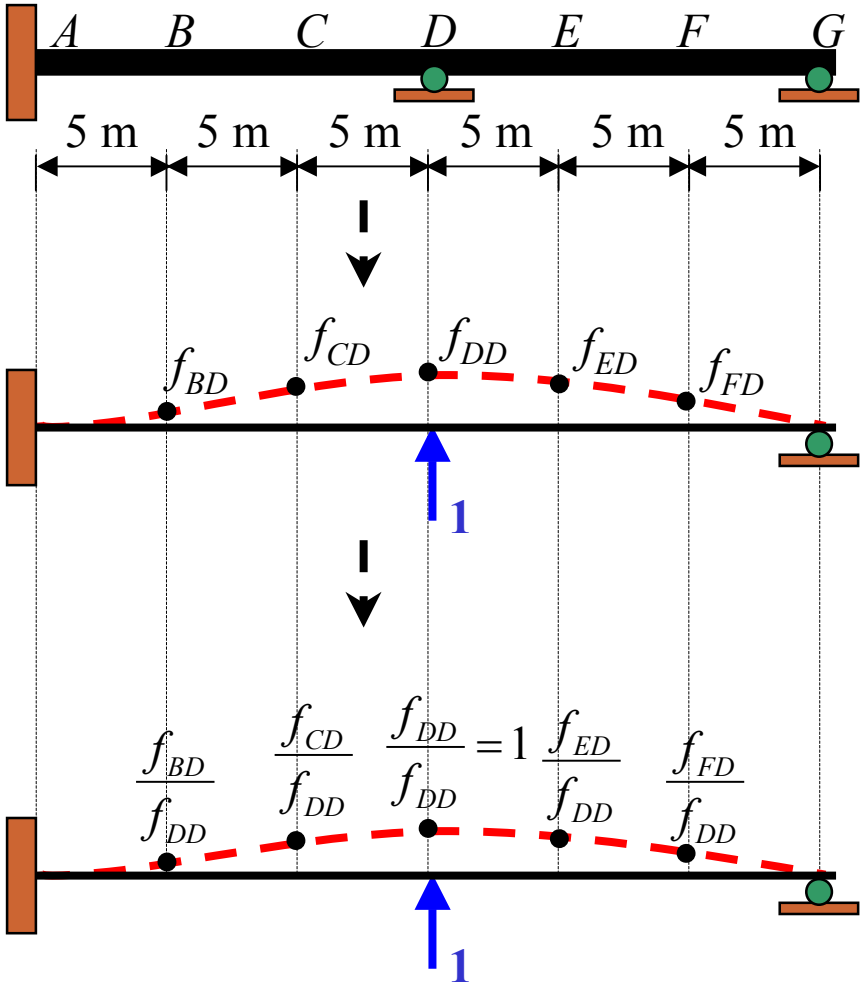

2/EI
4/EI
32/3EI
4/3EI
2 m
2 m
2 m
2 m
4/EI
5/EI
1/3EI
4/3EI
α_DD = 32/3EI
V′
x (m)
θ
-17/3EI
-8/3EI
26/3EI
f_xD = M′
x (m)
Δ
-2/EI
26/32 = 0.813
θ_DL = 5/(32/3) = 0.469 rad.
θ_DR = -17/32 = -0.531 rad.
f_xD / α_xD
θ_D = 0.469 + 0.531 = 1 rad
-2 /(32/3) = -0.188
Conjugate beam
Influence line of M_D

Example 3

Draw the influence line for the reactions at supports for the beam shown in the figure below. *EI* is constant.

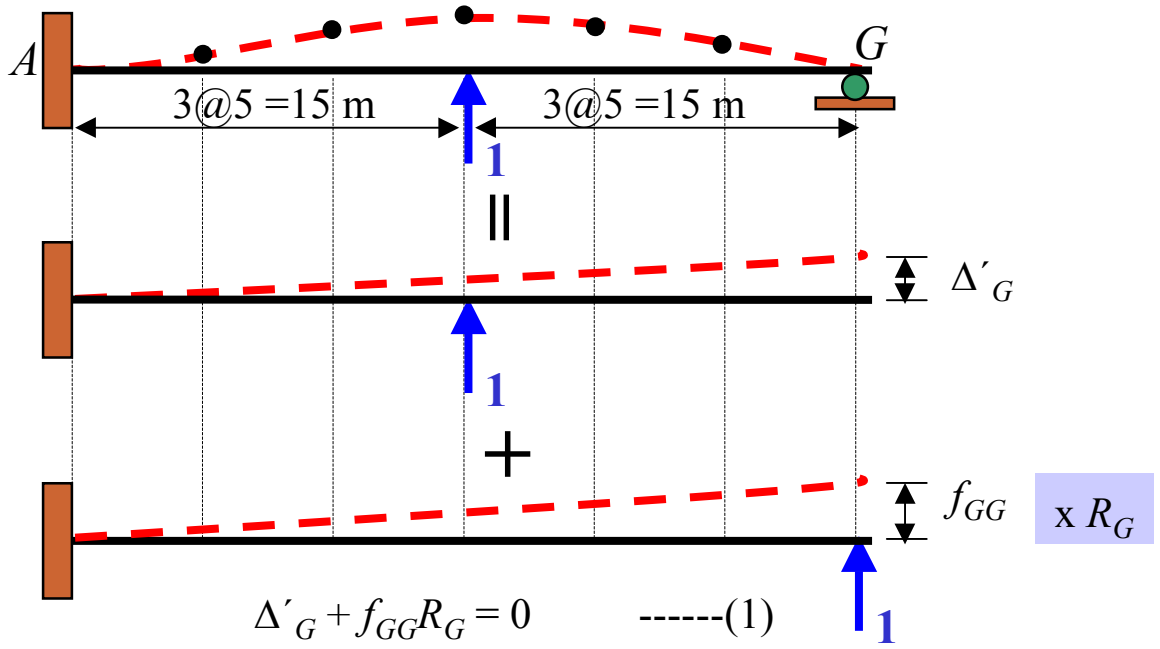


Influence line for R_D

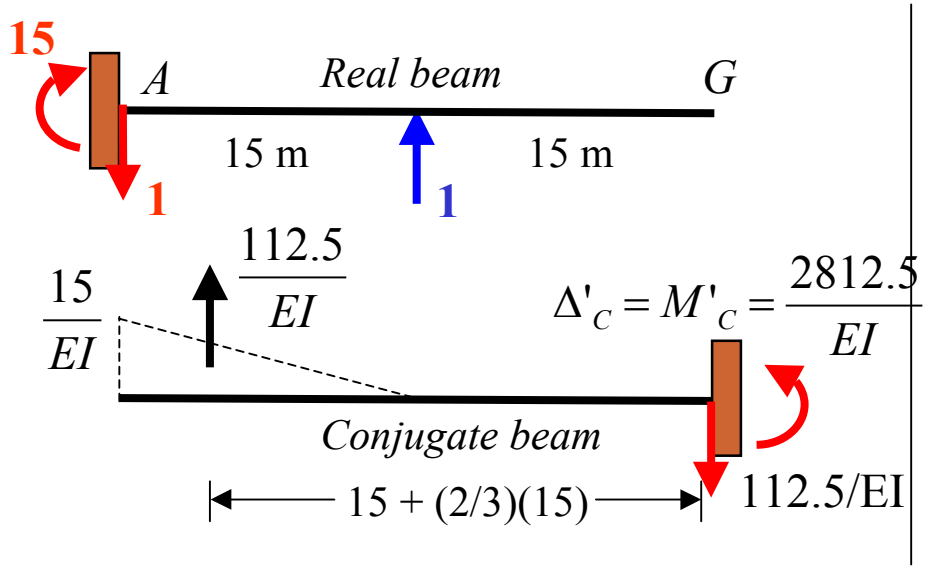


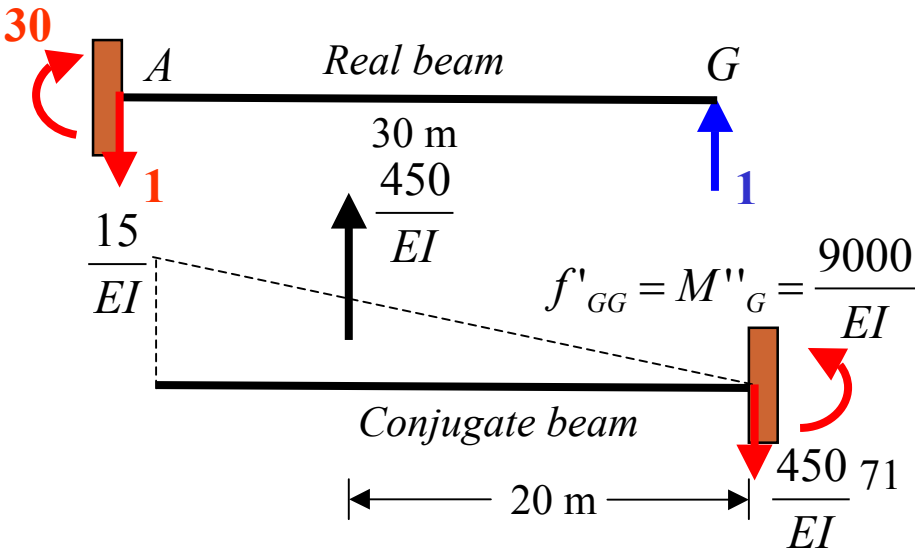
f_{XD}/f_{DD} = *Influence line for* R_D

• **Use the consistency deformation method**



- Use conjugate beam for find Δ'_G and f_{GG}

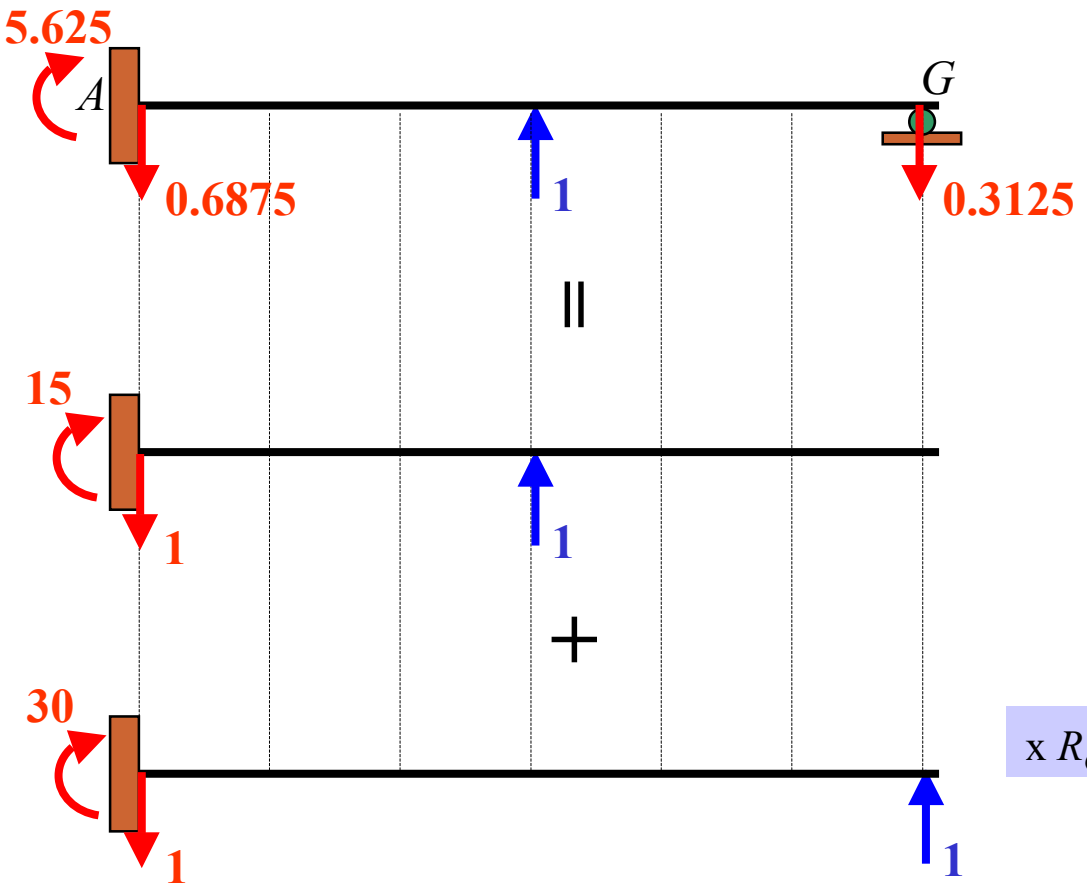




Substitute Δ'_G and f_{GG} in (1) :

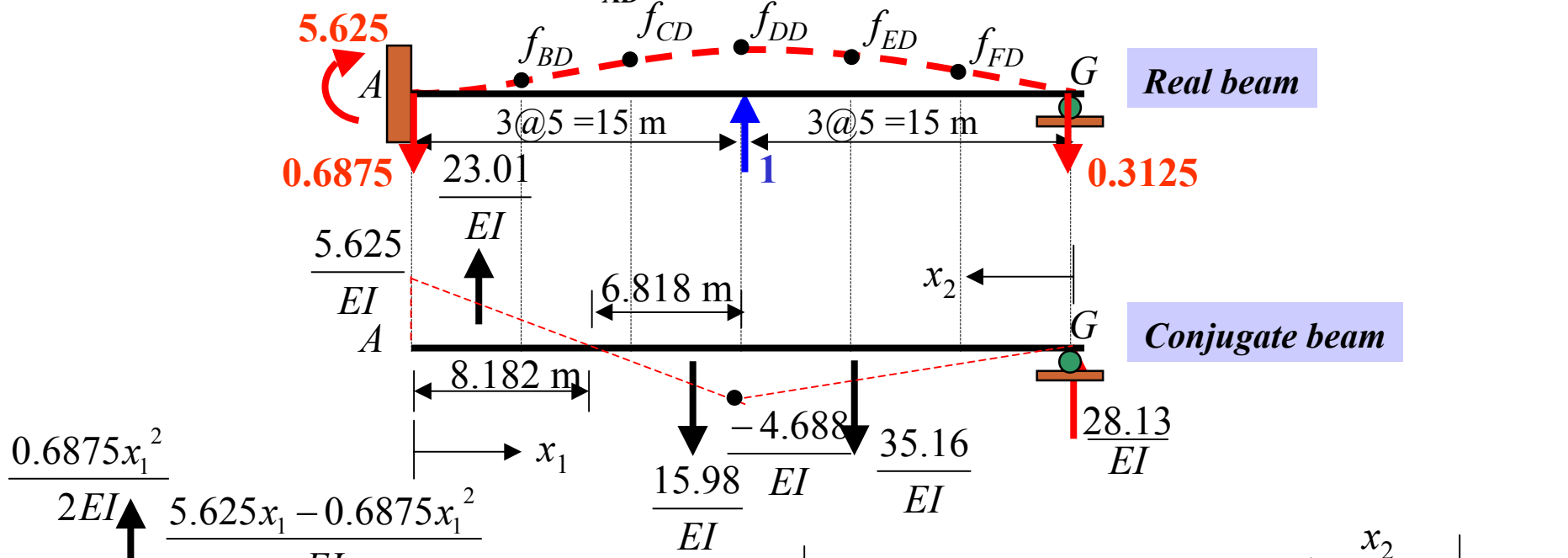
$$\frac{2812.5}{EI} + \frac{9000}{EI} R_G = 0$$

$$R_G = -0.3125\, kN, \downarrow$$



x R_G = -0.3125 kN

• **Use the conjugate beam for find f_{XD}**



$x_1 = 5$ m -----> $f_{BD} = M'_1 = 56/EI$

$x_1 = 10$ m -----> $f_{CD} = M'_1 = 166.7/EI$

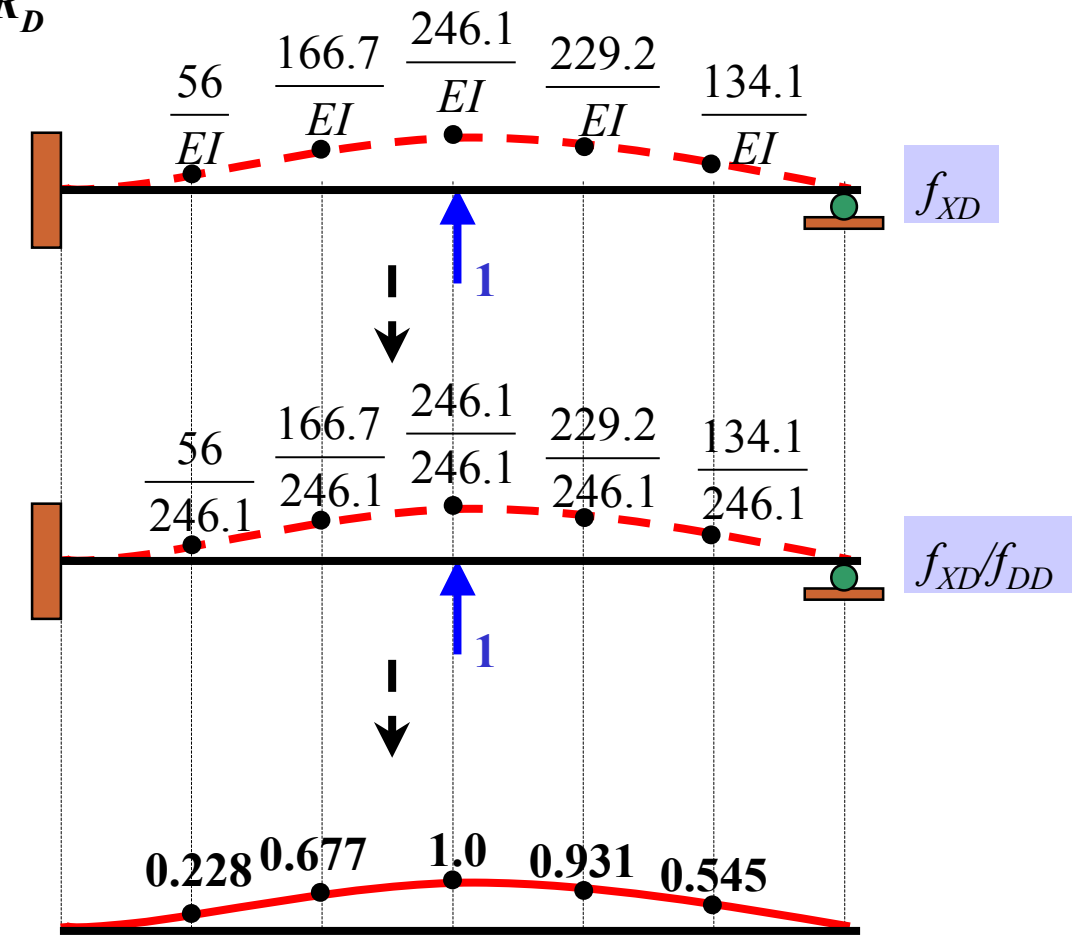
$x_1 = 15$ m -----> $f_{DD} = M'_1 = 246.1/EI$

$x_2 = 5$ m -----> $f_{FD} = M'_2 = 134.1/EI$

$x_2 = 10$ m -----> $f_{ED} = M'_2 = 229.1/EI$

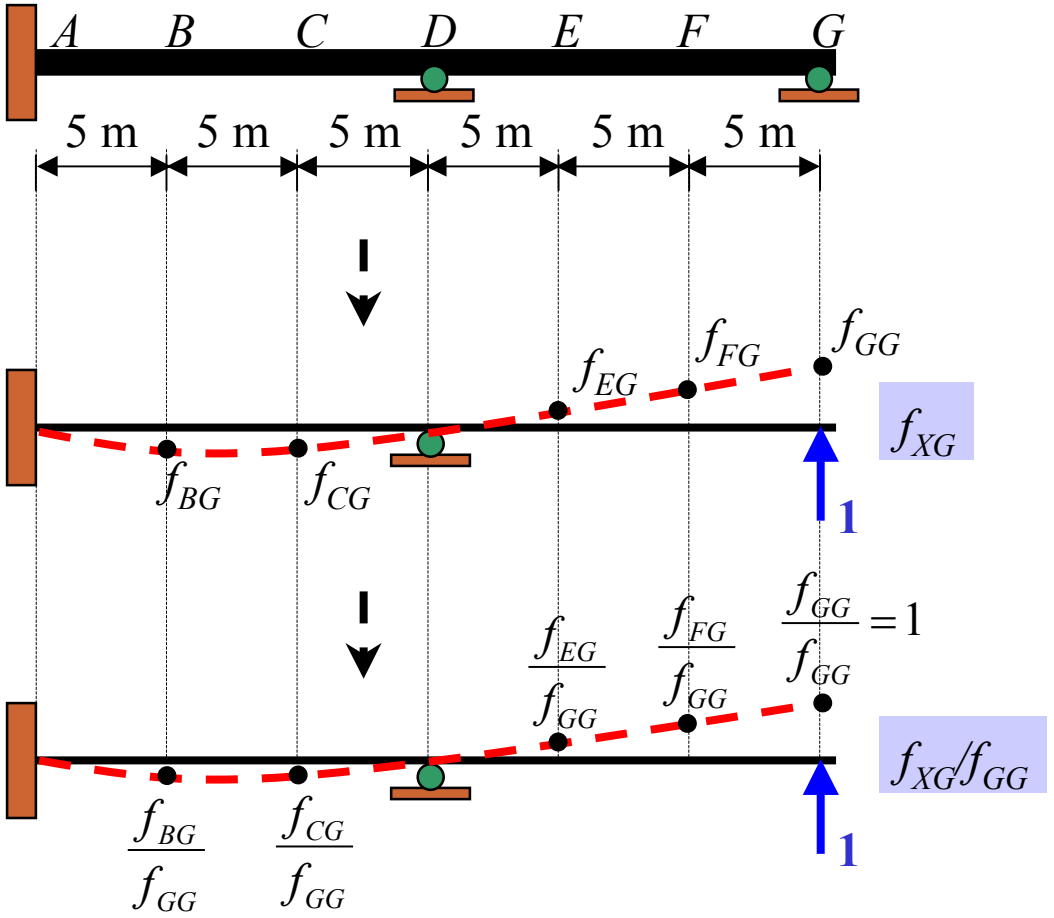
$x_2 = 15$ m -----> $f_{DD} = M'_2 = 246.1/EI$

• **Influence Line for R_D**

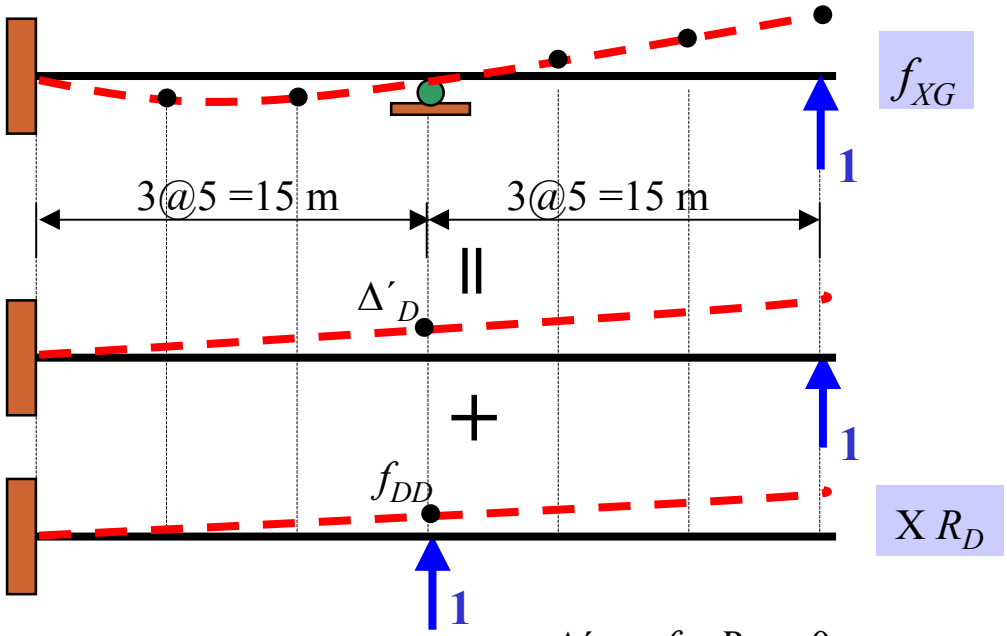


Influence Line for R_D

Influence line for R_G

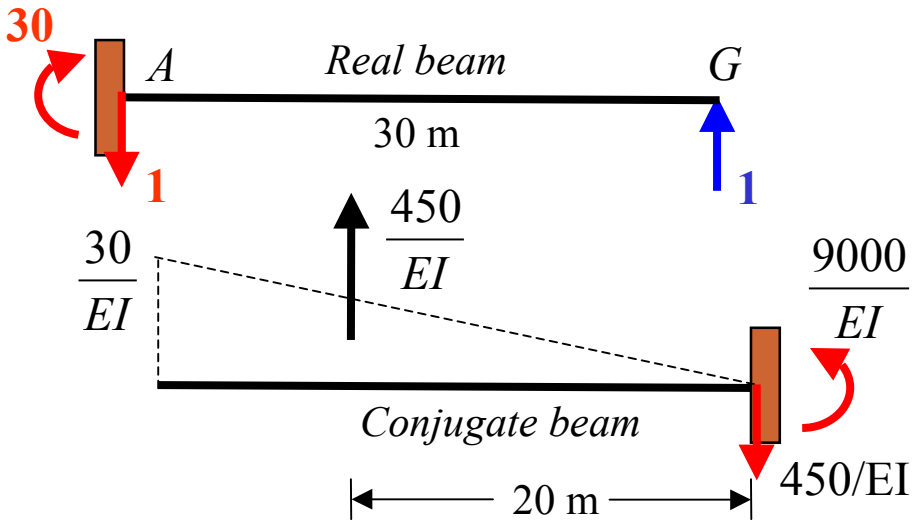


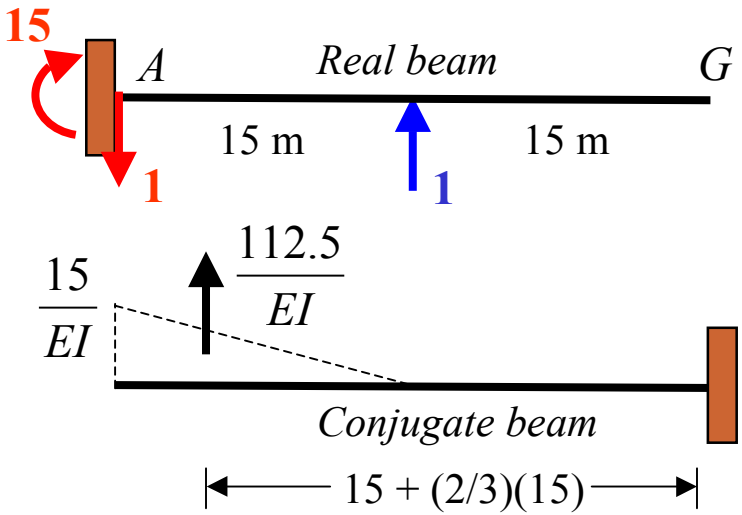
• **Use consistency deformations**

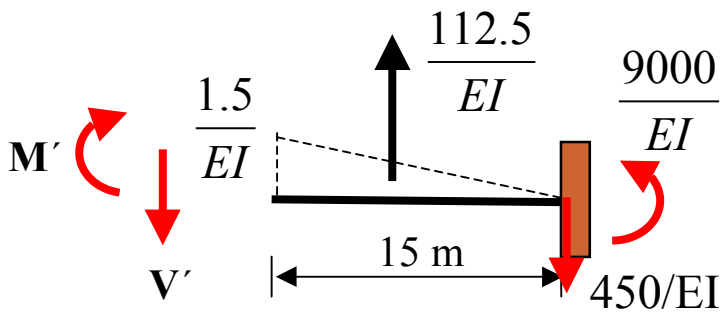


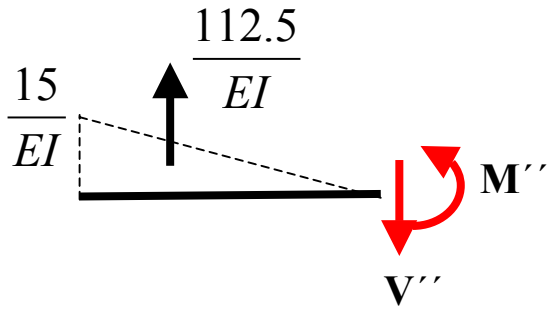
$$\Delta'_D + f_{DD}R_D = 0 \qquad \text{------(2)}$$

- Use conjugate beam for find Δ'_D and f_{DD}





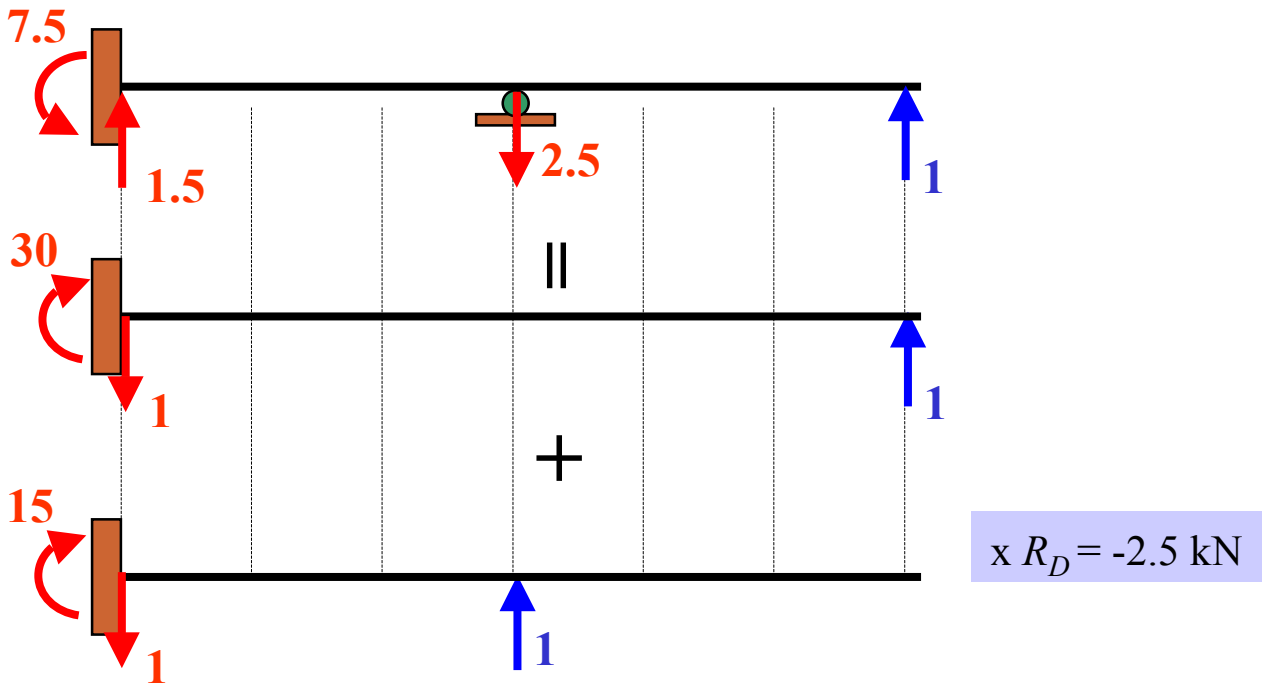




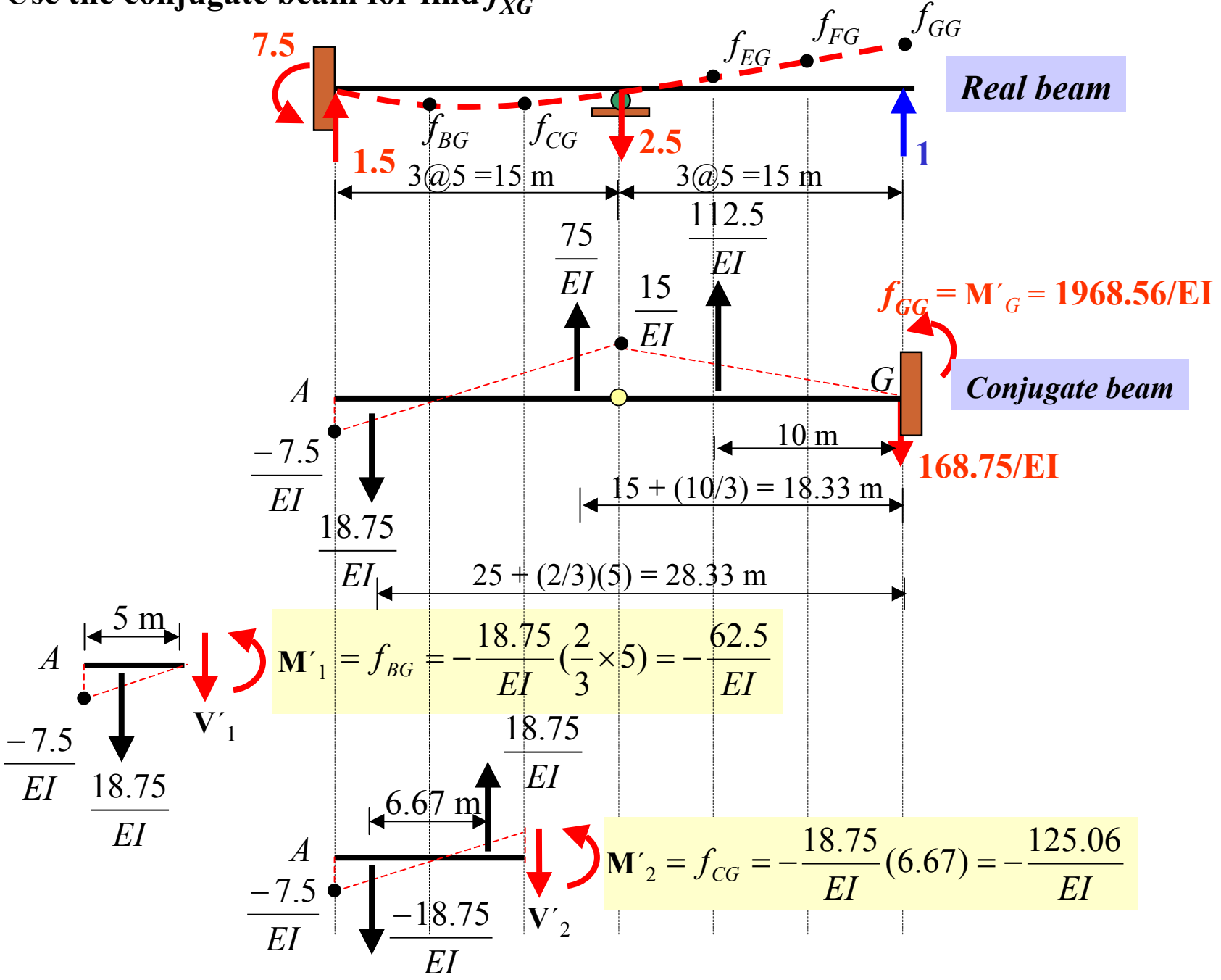
$$\Delta'_D = M' = \frac{112.5}{EI}\left(\frac{15}{3}\right) + \frac{9000}{EI} - \frac{450}{EI}(15) = \frac{2812.5}{EI}$$

$$f_{DD} = M'' = \frac{112.5}{EI}\left(\frac{2}{3}\times 15\right) = \frac{1125}{EI}$$

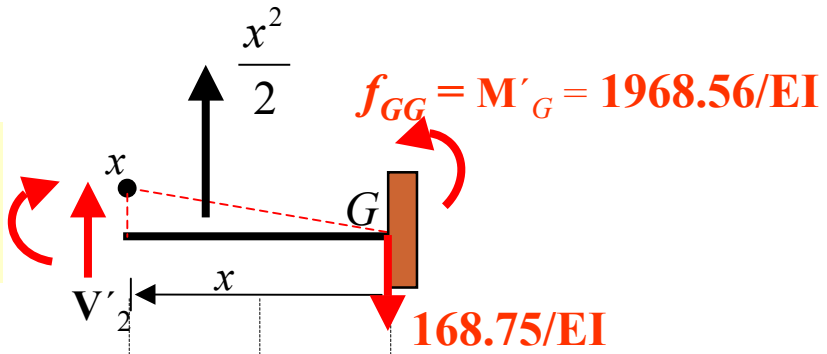
Substitute Δ'_D and f_{DD} in (2) : $\frac{2812.5}{EI} + \frac{1125}{EI} R_D = 0, R_D = -2.5kN = 2.5kN, \downarrow$



• **Use the conjugate beam for find f_{XG}**

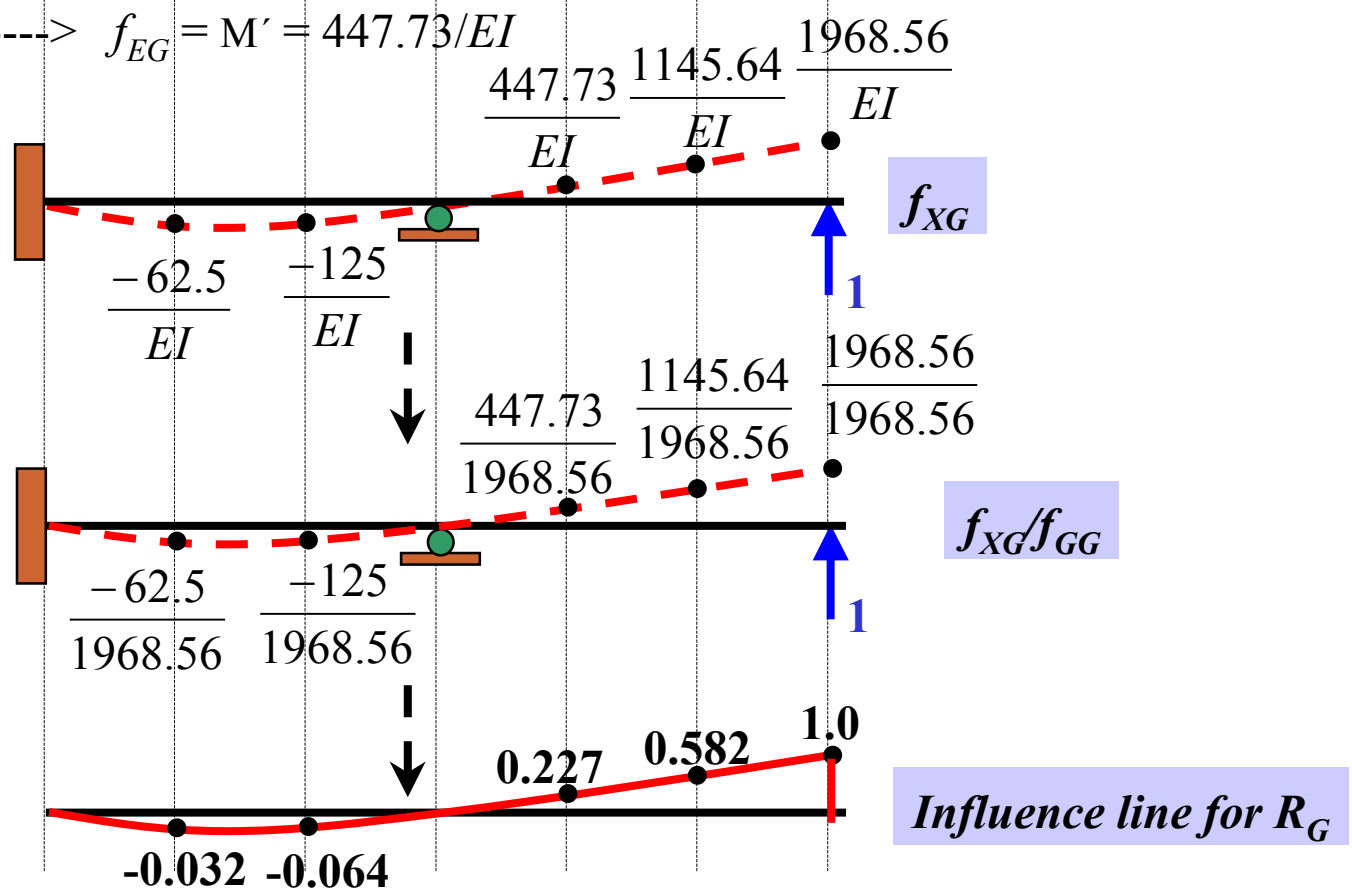


$$\frac{x_3^2}{2}\left(\frac{x_3}{3}\right)+\frac{1968.56}{EI}-\frac{168.75}{EI}x_3=\mathbf{M'}$$

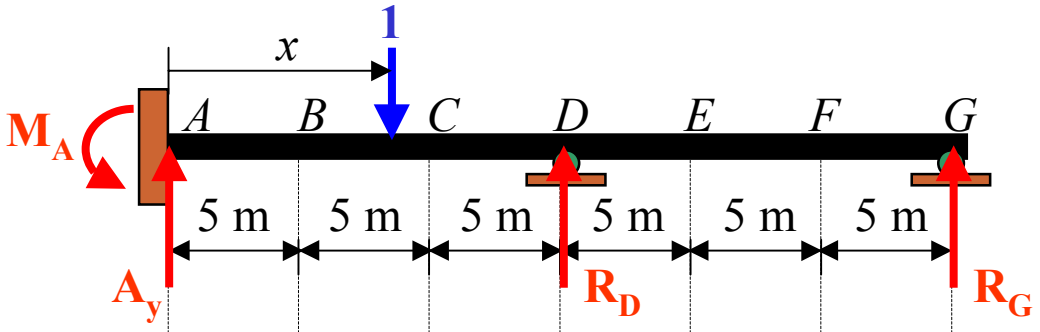


x = 5 m -----> f_{FG} = M′ = 1145.64/EI

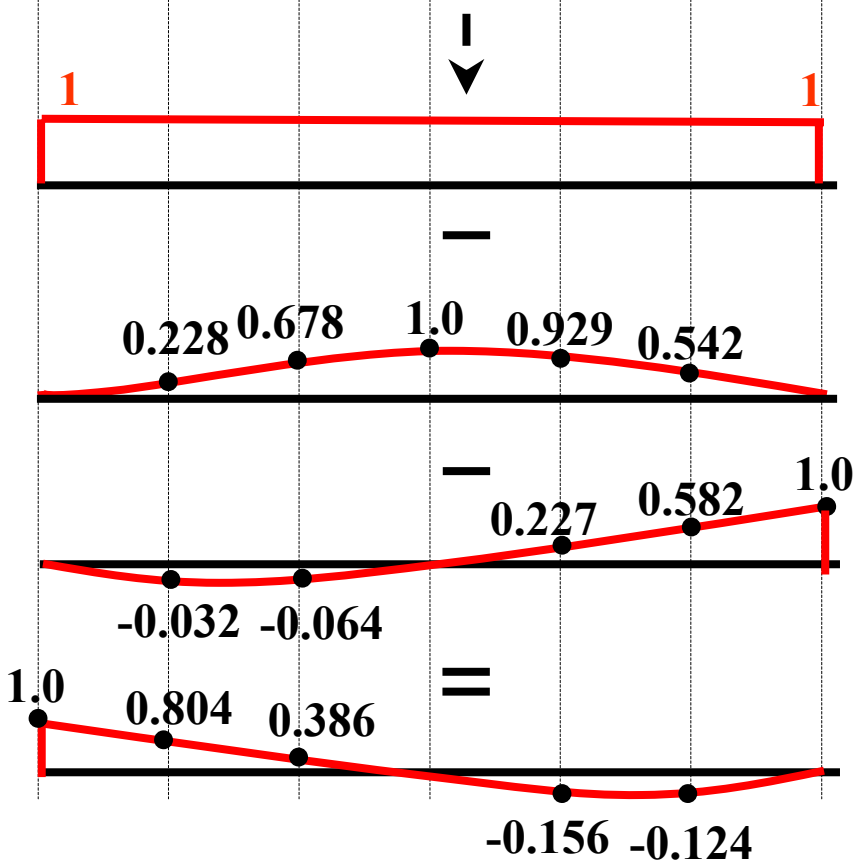
x = 10 m -----> f_{EG} = M′ = 447.73/EI



Using equilibrium condition for the influence line for A_y



$$+\uparrow \quad \Sigma F_y = 0: \quad R_A = 1 - R_D - R_C$$



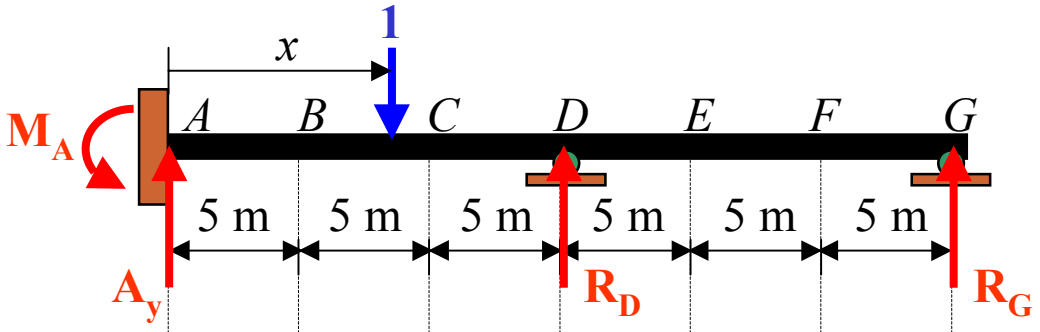
Unit load

Influence Line for R_D

Influence line for R_G

Influence line for A_y

Using equilibrium condition for the influence line for M_A



$$\curvearrowleft + \quad \Sigma M_A = 0: \quad 1x - 15R_D - 30R_C$$

