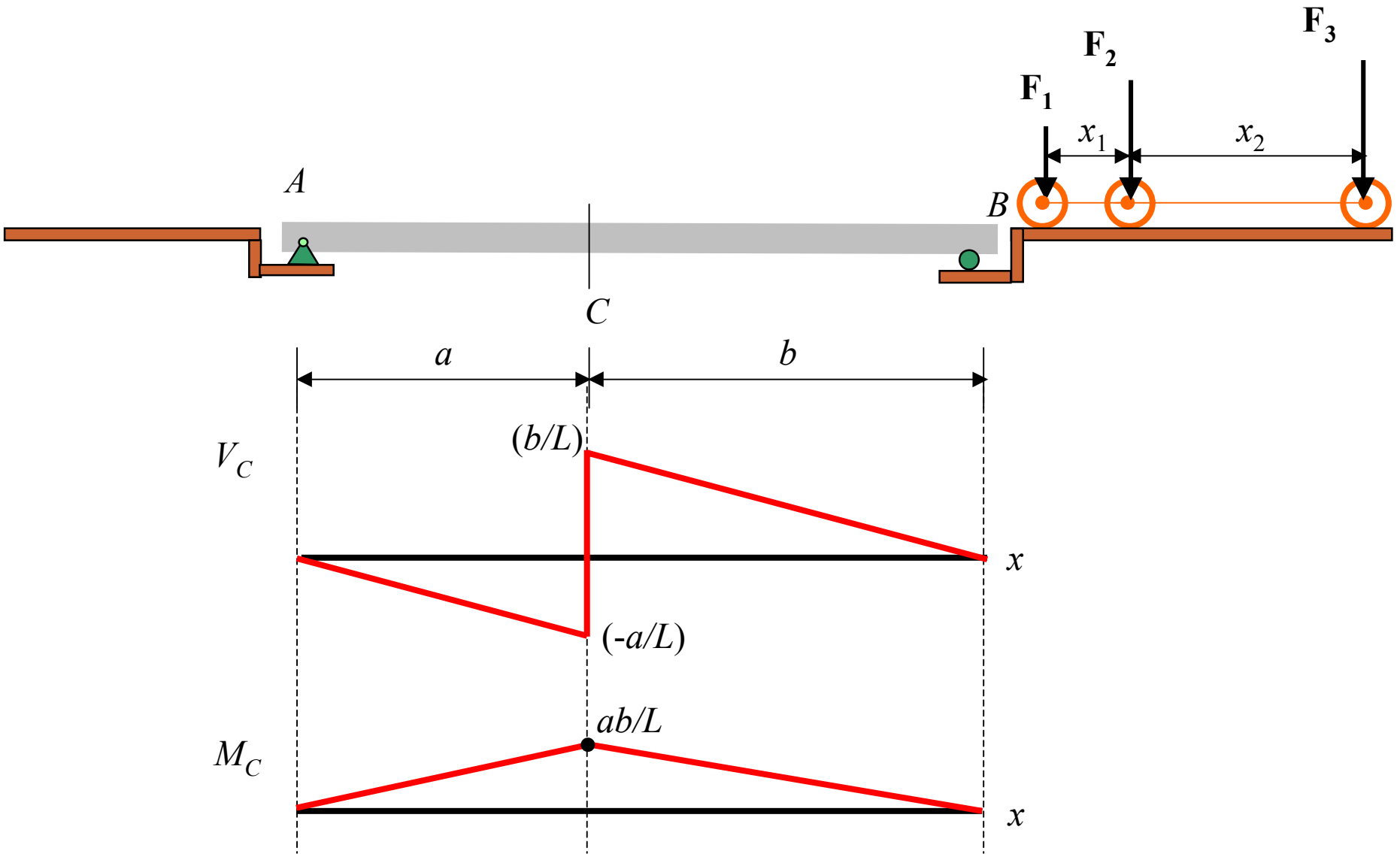
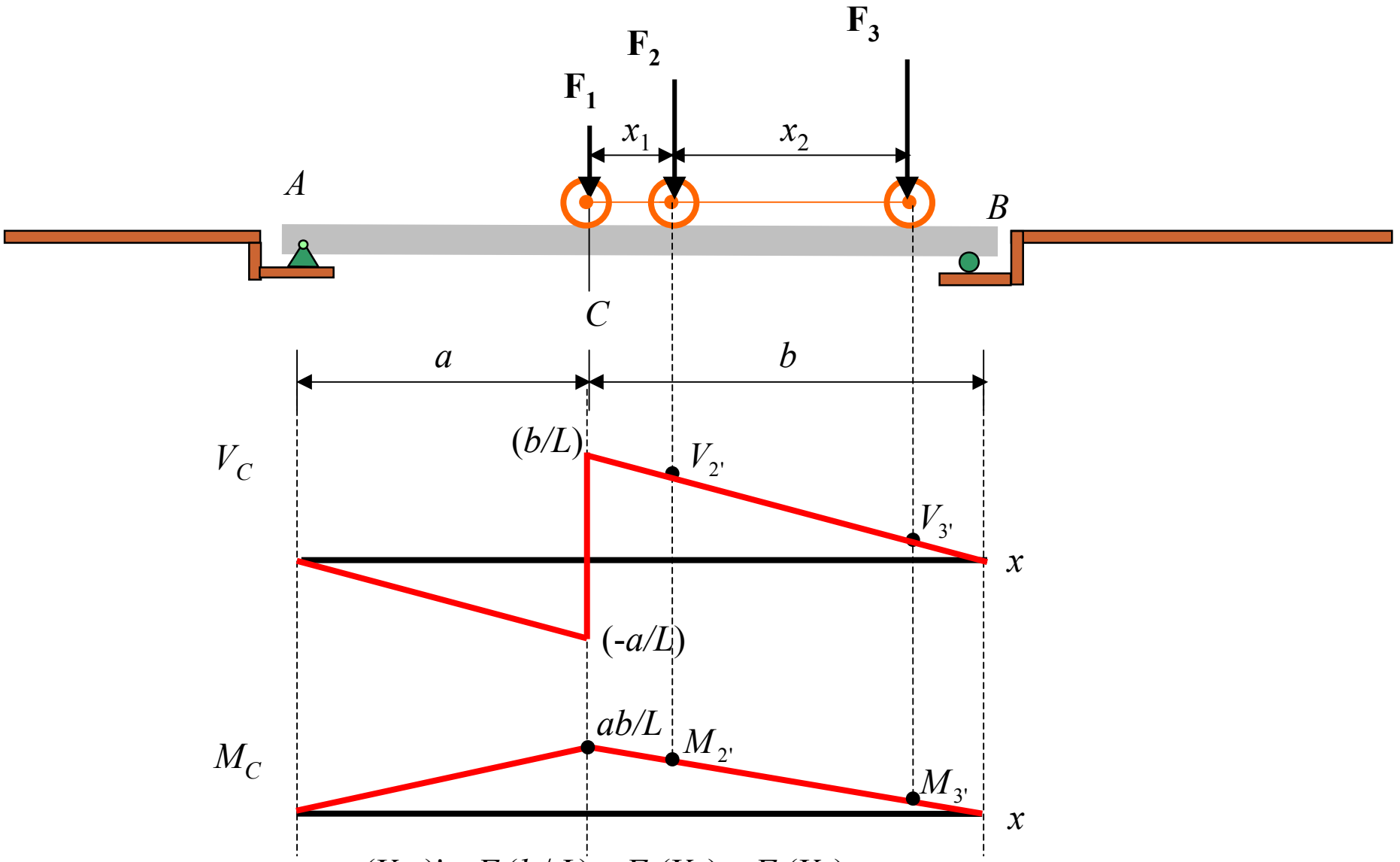
Maximum Influence at a Point Due to a Series of Concentrated Loads

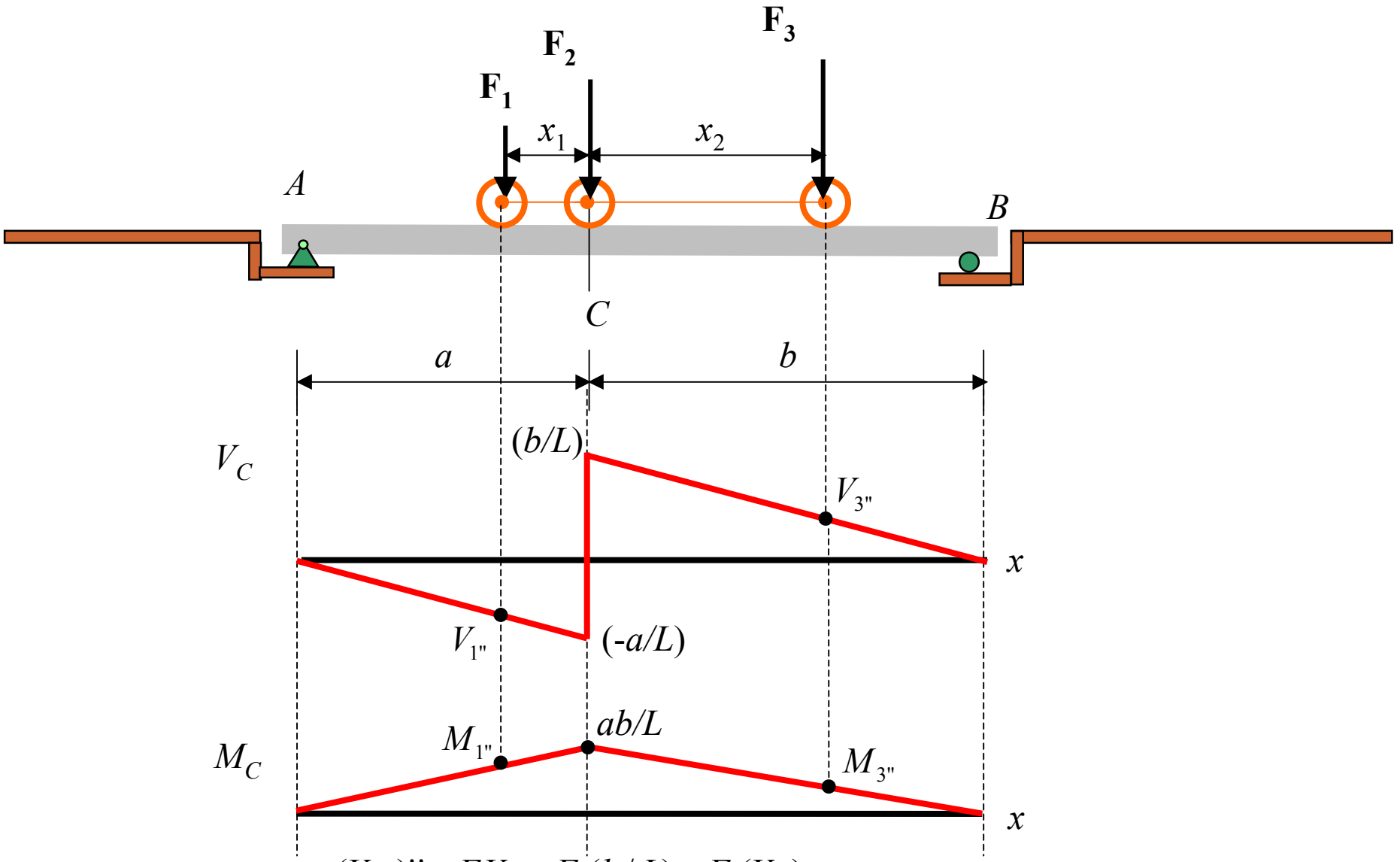




$$(V_{CR})' = F_1(b/L) + F_2(V_{2'}) + F_3(V_{3'})$$

$$(V_{CL})' = F_1(-a/L) + F_2(V_{2'}) + F_3(V_{3'})$$

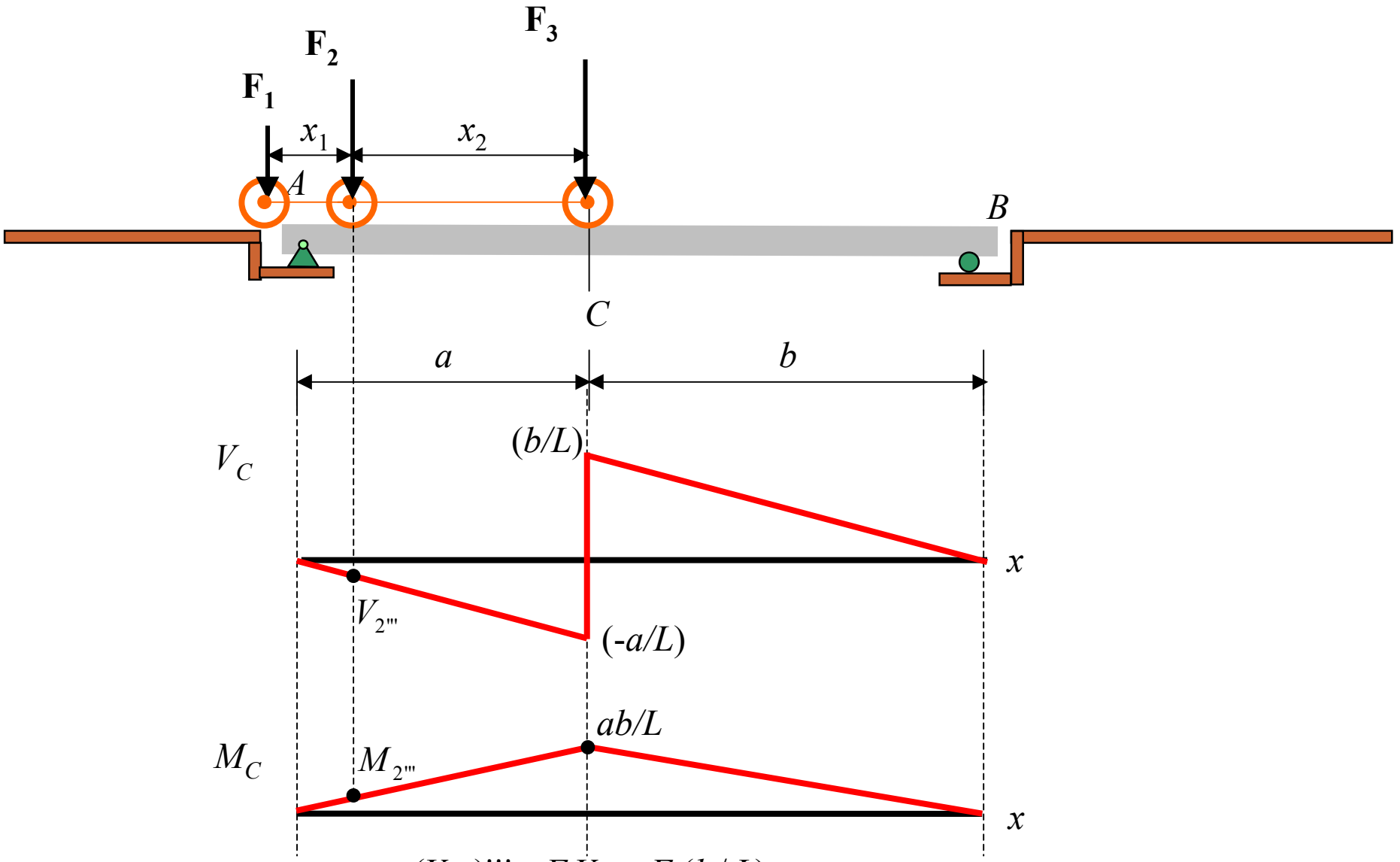
$$M' = F_1(ab/L) + F_2(M_{2'}) + F_3(M_{3'})$$



$$(V_{CR})'' = F_1 V_{1''} + F_2 (b/L) + F_3 (V_{3''})$$

$$(V_{CL})'' = F_1 V_{1''} + F_2 (-a/L) + F_3 (V_{3''})$$

$$M'' = F_1 M_{1''} + F_2 (ab/L) + F_3 (M_{3''})$$

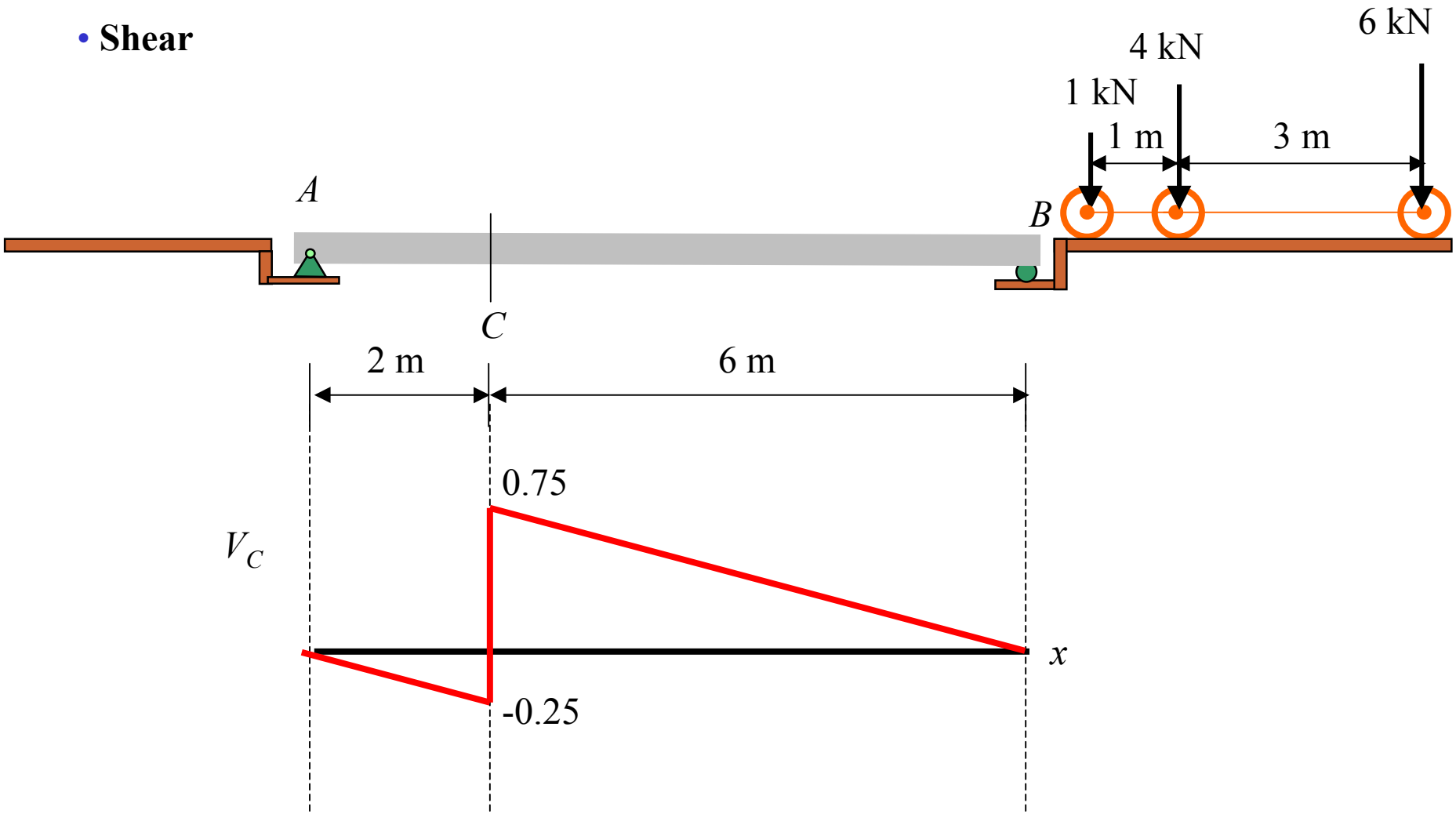


$$(V_{CR})''' = F_2 V_{2''} + F_3 (b/L)$$

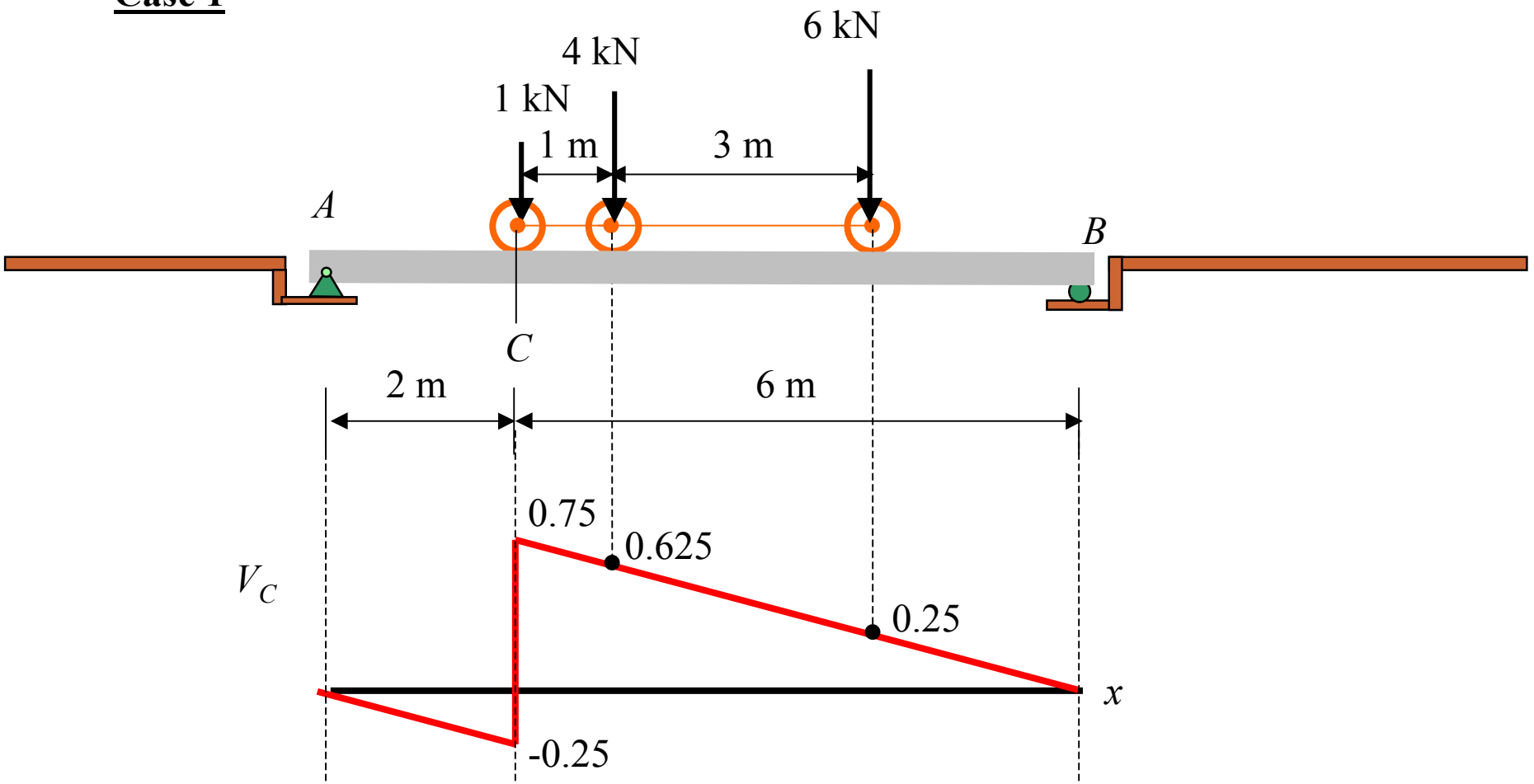
$$(V_{CL})''' = F_2 V_{2'''} + F_3 (-a/L)$$

$$M''' = F_2 M_{2'''} + F_3 (ab/L)$$

- **Shear**

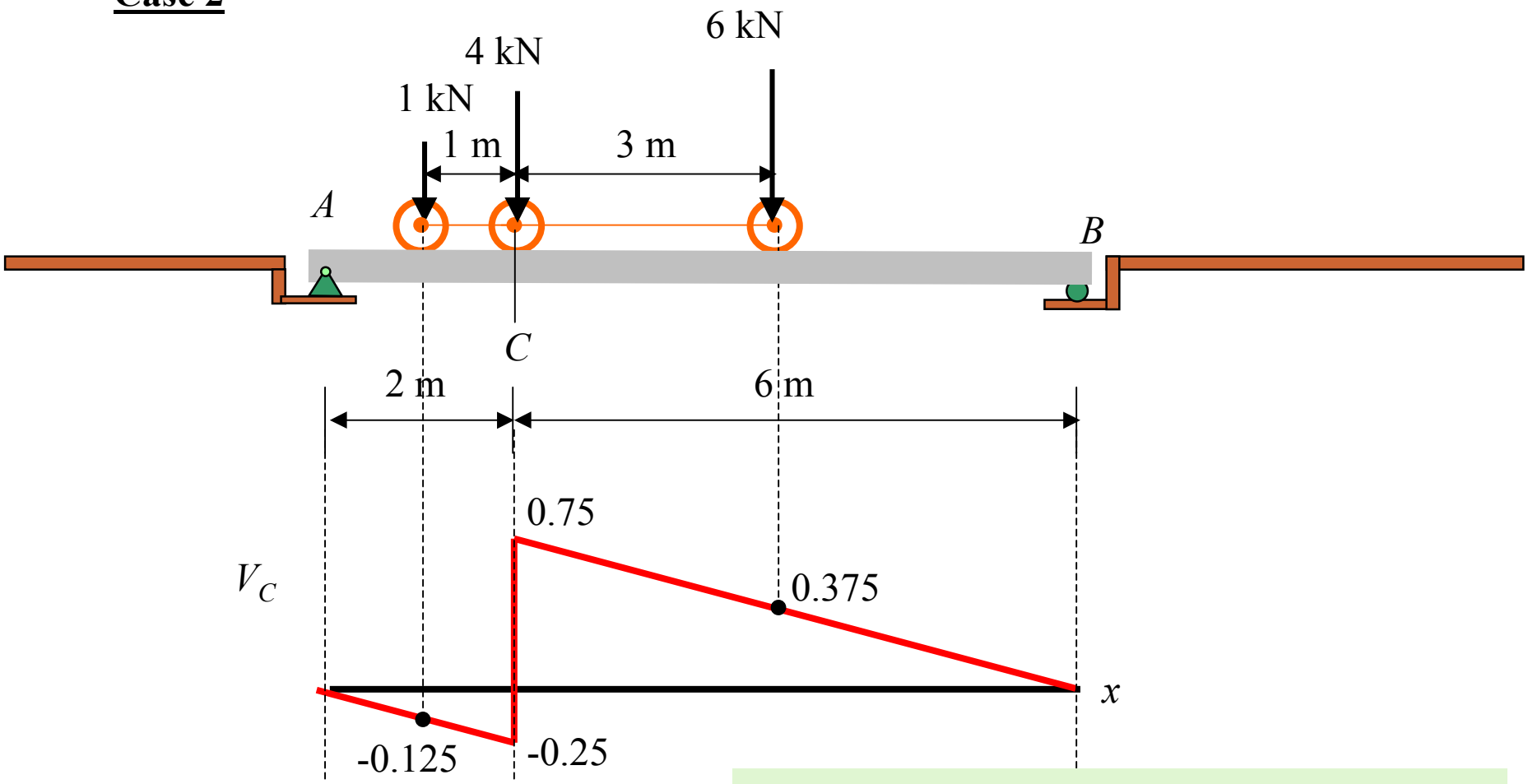


Case 1



$$(V_C)_1 = 1(0.75) + 4(0.625) + 6(0.25) = 4.75 \text{ kN}$$

Case 2



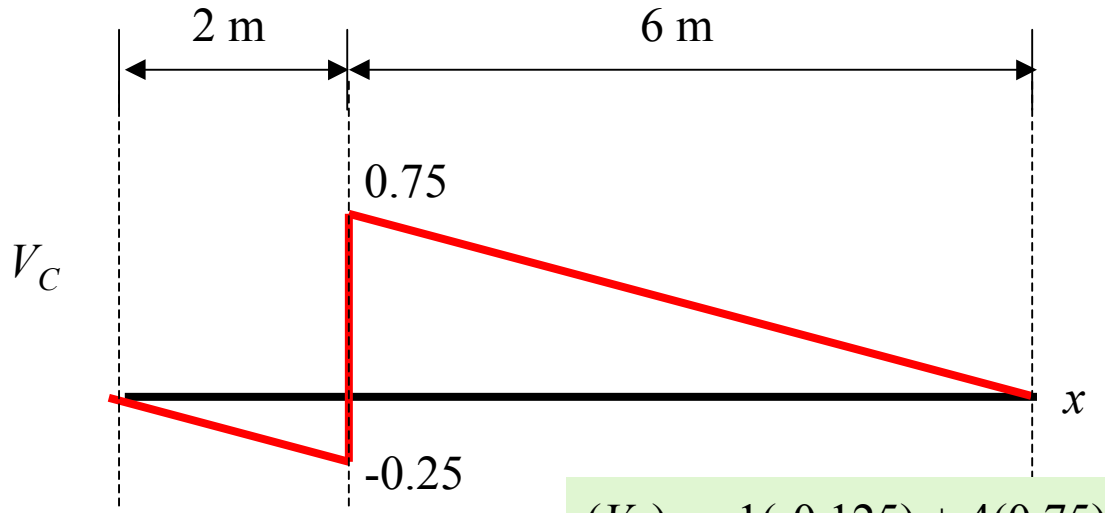
$(V_C)_1 = 1(0.75) + 4(0.625) + 6(0.25) = 4.75$ kN

$(V_C)_2 = 1(-0.125) + 4(0.75) + 6(0.375) = 5.125$ kN

$\Delta V_{1\text{-}2} = 5.125 - 4.75 = 0.375$ kN

Case 3





$(V_C)_2 = 1(-0.125) + 4(0.75) + 6(0.375) = 5.125$ kN

$(V_C)_3 = 6(0.75) = 4.5$ kN

$\Delta V_{2\text{-}3} = 4.5 - 5.125 = -0.625$ kN

$(V_C)_{max} = (V_C)_2 = 5.125$ kN

The critical position of the loads can be determined in a more direct manner by finding the change in shear, ΔV, which occurs when the load are moved from Case 1 to Case 2, then from Case 2 to Case 3, and so on.If the slope of the influence line is s, then $(y_2 - y_1) = s(x_2 - x_1)$, and therefore

$$\Delta V = Ps\,(x_2 - x_1)$$

Sloping Line

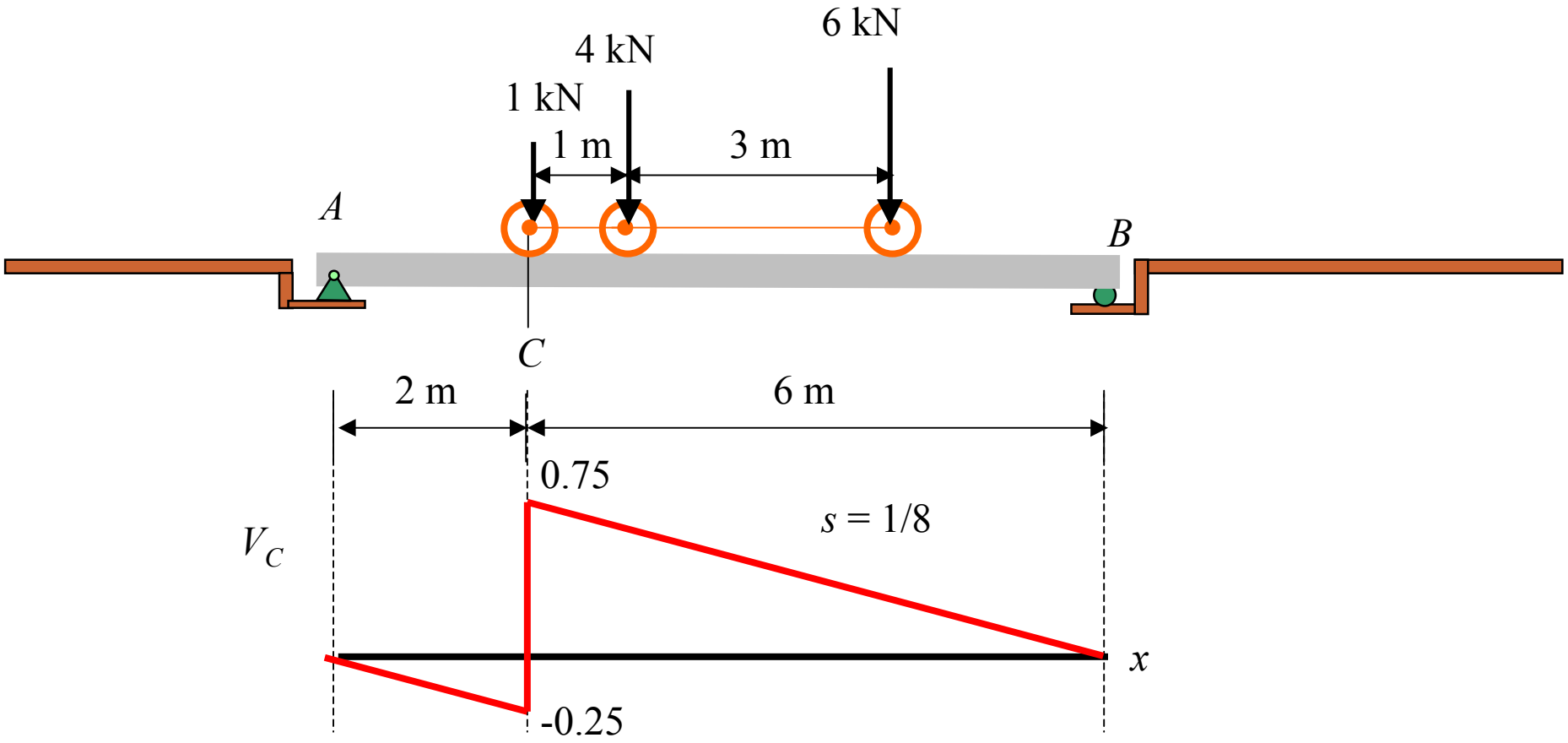
----------(6-1)

If the load moves past a point where there is a discontinuity or “jump” in the influence line, as point C, then the change in shear is simply

$$\Delta V = P\,(y_2 - y_1)$$

Jump

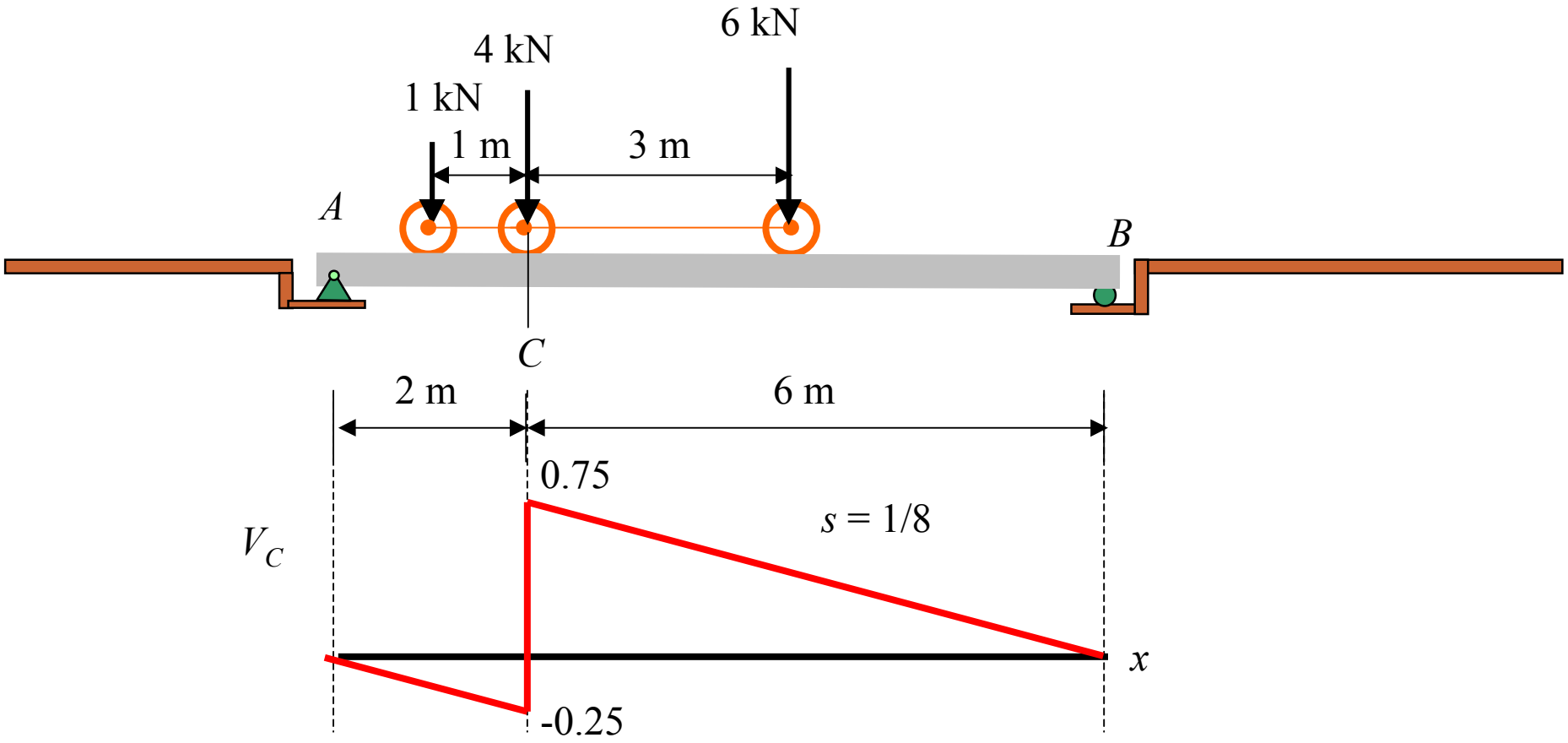
----------(6-2)



$$(V_C)_1 = 1(0.75) + 4(0.625) + 6(0.25) = 4.75 \text{ kN}$$

$$\Delta V_{1\text{-}2} = 1(-0.25 - 0.75) + 1(1/8)(1) + 4(1/8)(1) + 6(1/8)(1) = 0.375 \text{ kN}$$

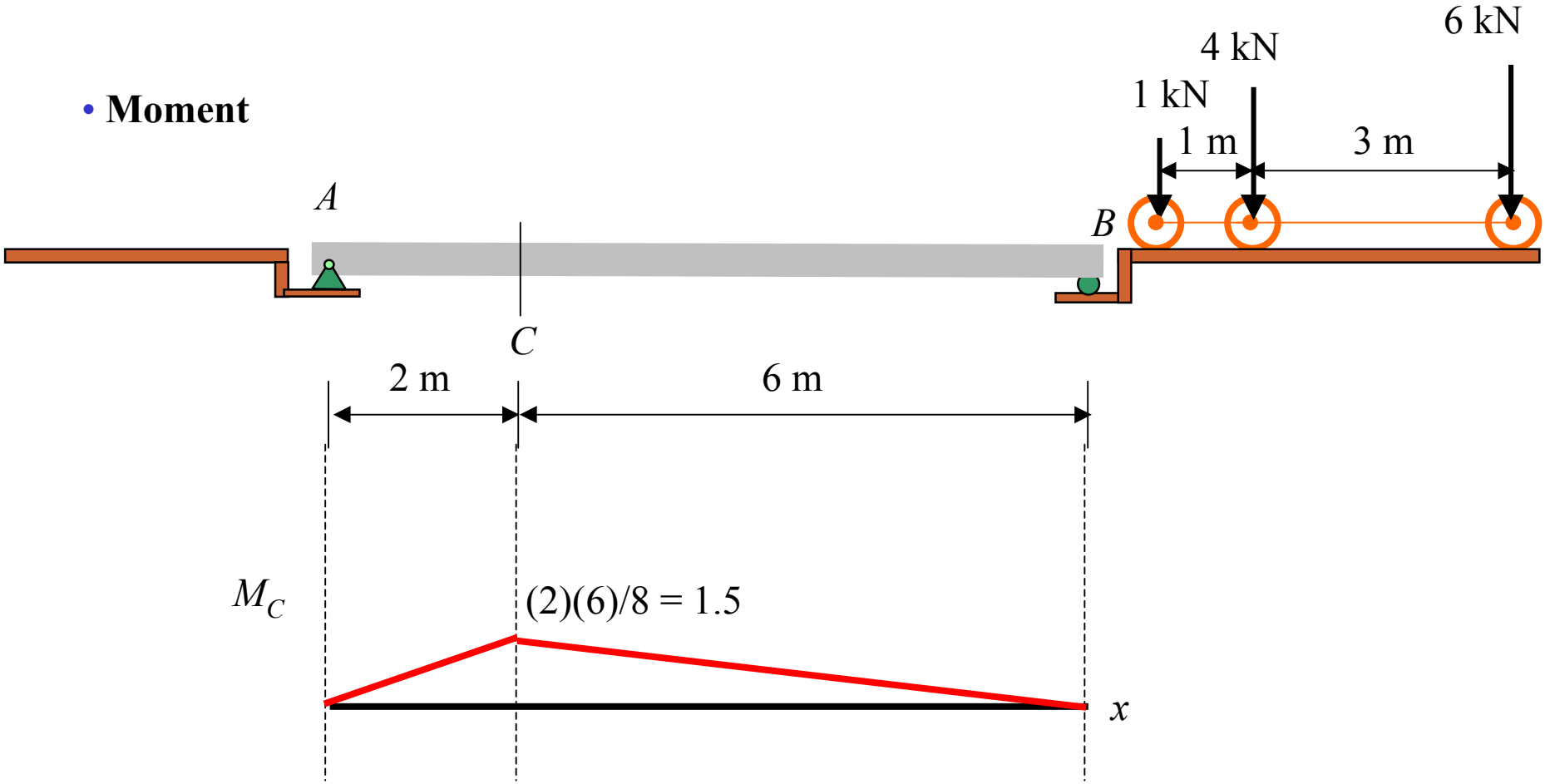
$$(V_C)_2 = (V_C)_1 + \Delta V_{1\text{-}2} = 4.75 + 0.375 = 5.125 \text{ kN}$$



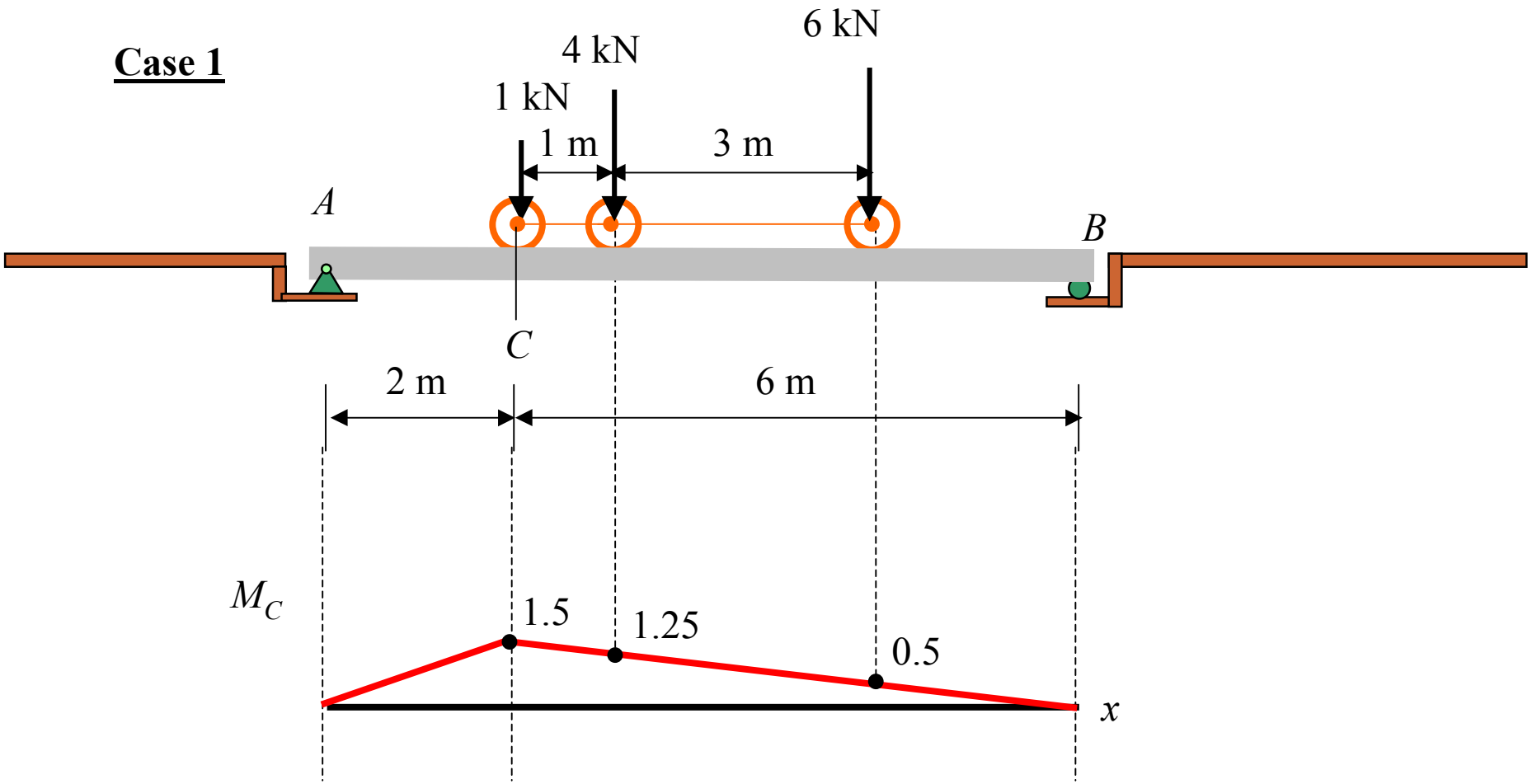
$$\Delta V_{2\text{-}3} = 1(1/8)(1) + 4(-0.25\text{-}0.75) + 4(1/8)(2) + 6(1/8)(3) = -0.625 \text{ kN}$$

$$(V_C)_3 = (V_C)_2 + \Delta V_{2\text{-}3} = 5.125 - 0.625 = 4.5 \text{ kN}$$

- **Moment**

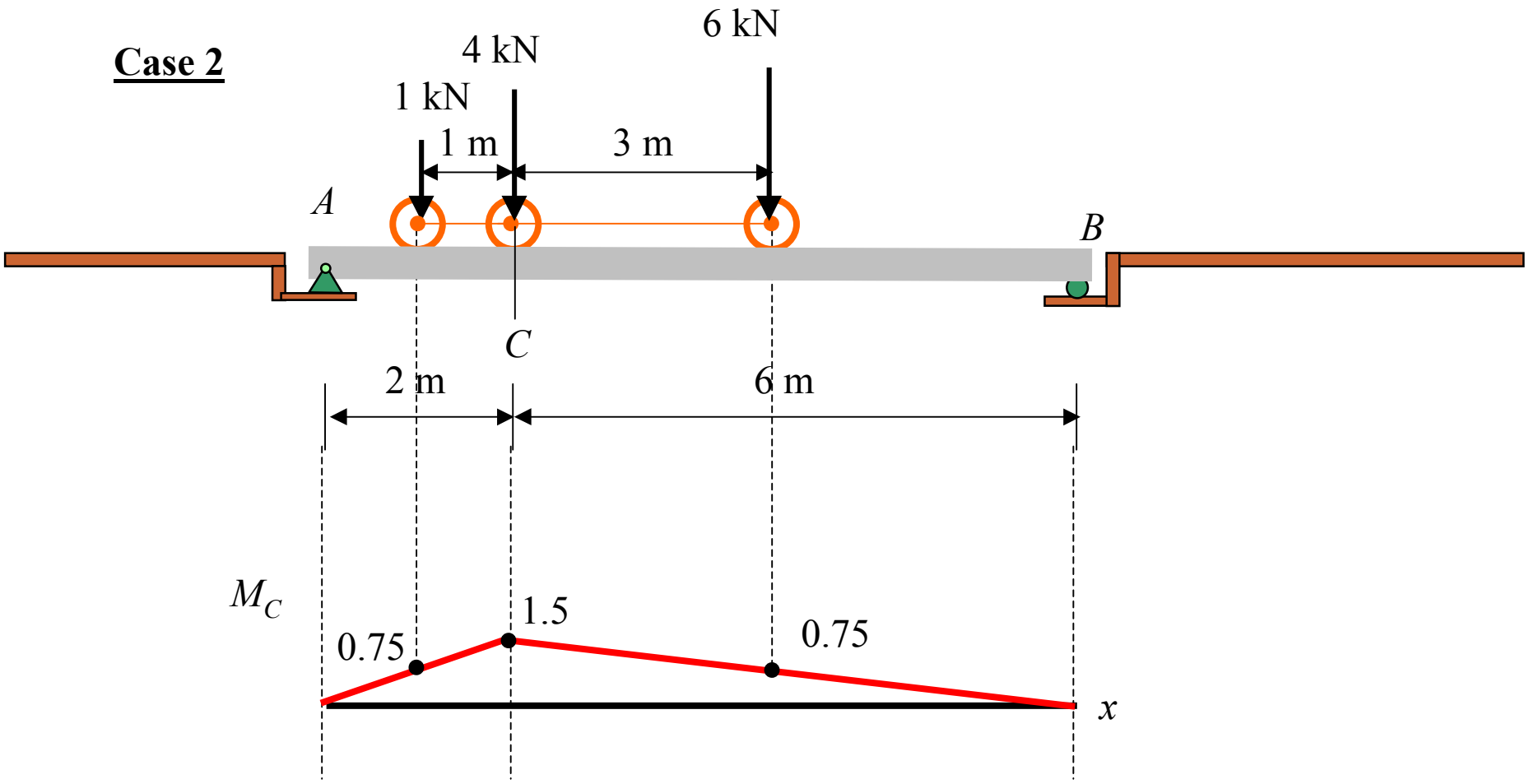


Case 1



$$(M_C)_1 = 1(1.5) + 4(1.25) + 6(0.5) = 9.5 \text{ kN•m}$$

Case 2

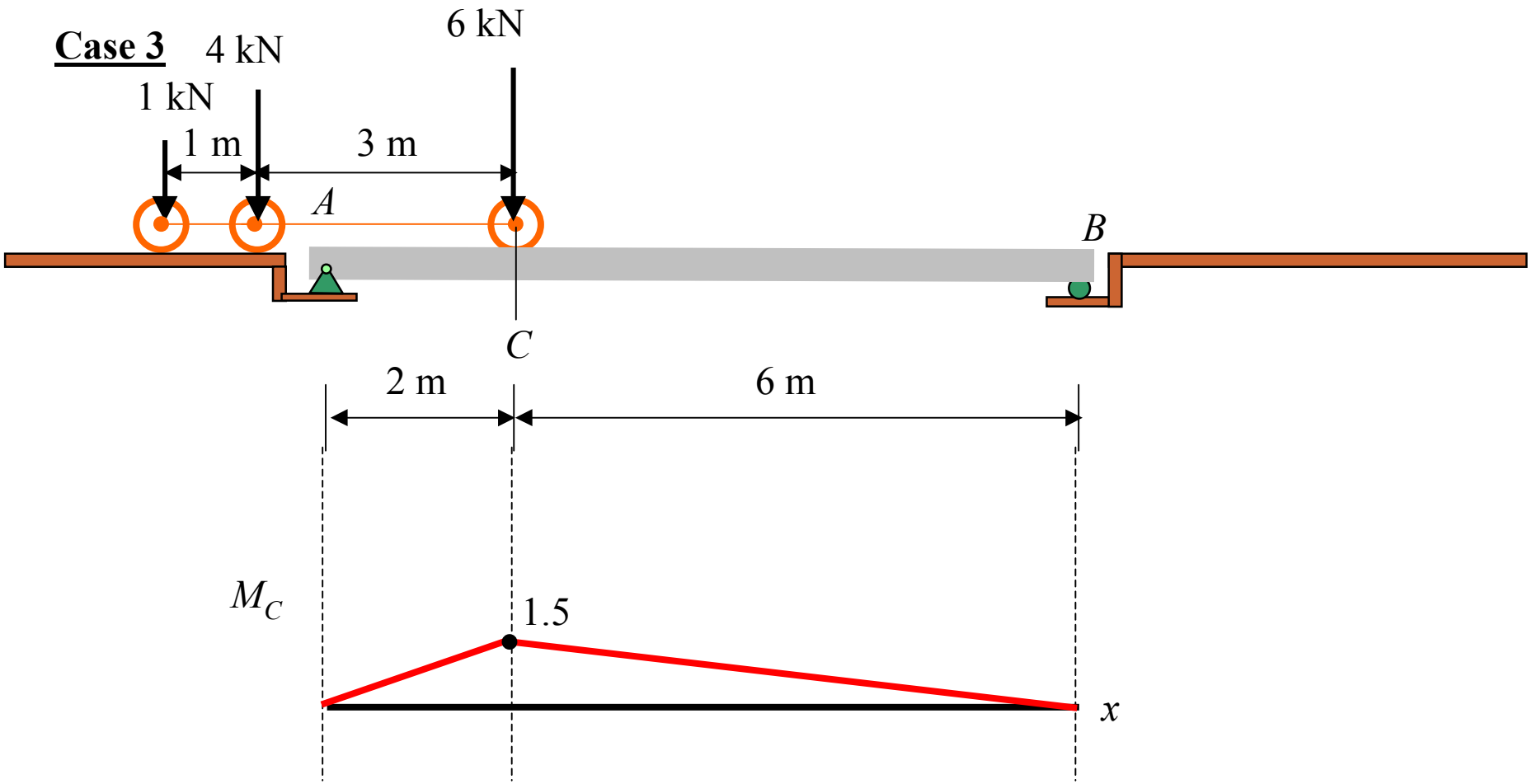


$(M_C)_1 = 1(1.5) + 4(1.25) + 6(0.5) = 9.5$ kN•m

$(M_C)_2 = 1(0.75) + 4(1.5) + 6(0.75) = 11.25$ kN•m

$\Delta M_{1\text{-}2} = 11.25 - 9.5 = 1.75$ kN•m

Case 3



$(M_C)_3 = 6(1.5) = 9$ kN•m

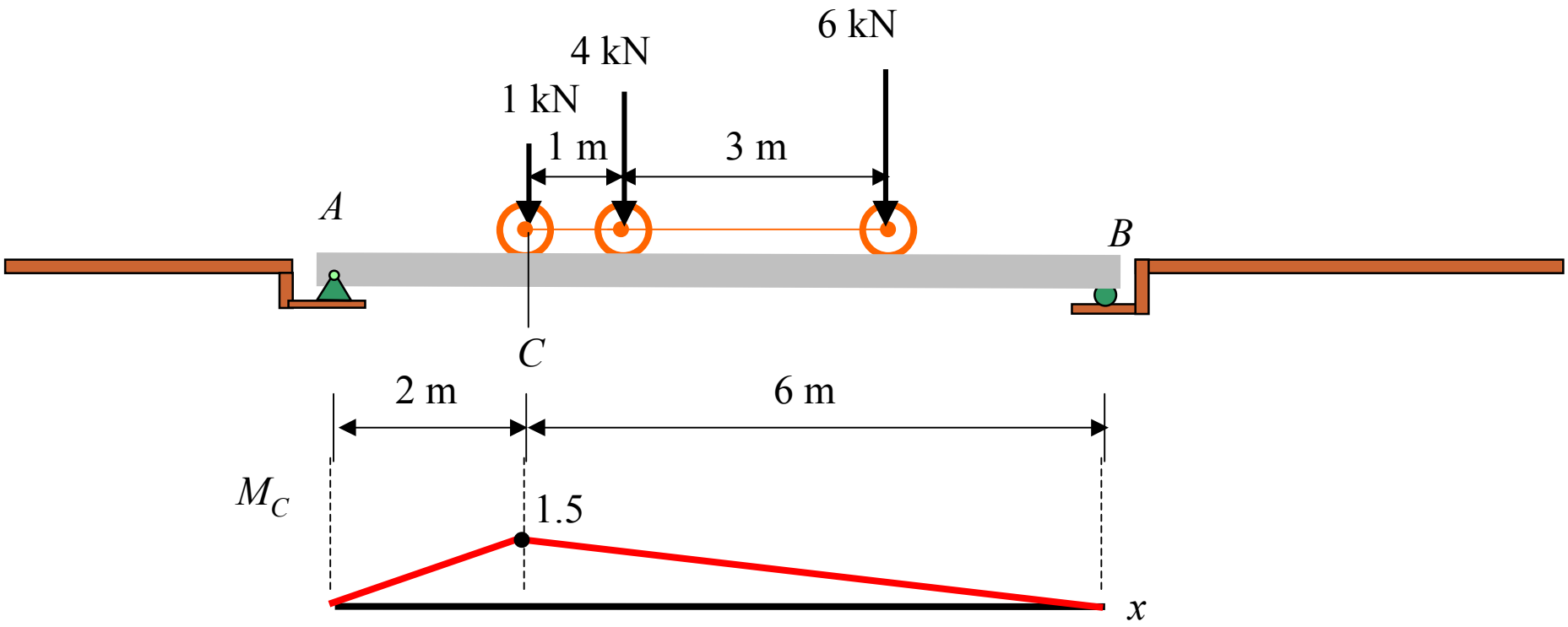
$\Delta M_{2\text{-}3} = 9 - 11.25 = -2.25$ kN•m

$(M_C)_{max} = (M_C)_2 = 11.25$ kN•m

We can also use the foregoing methods to determine the critical position of a series of concentrated forces so that they create the largest internal moment at a specific point in a structure. Of course, it is first necessary to draw the influence line for the moment at the point and determine the slopes s of its line segments. For a horizontal movement $(x_2 - x_1)$ of a concentrated force P, the change in moment, ΔM, is equivalent to the magnitude of the force times the change in the influence-line ordinate under the load, that is

$$\Delta M = Ps\,(x_2 - x_1) \qquad \text{----------(6-3)}$$

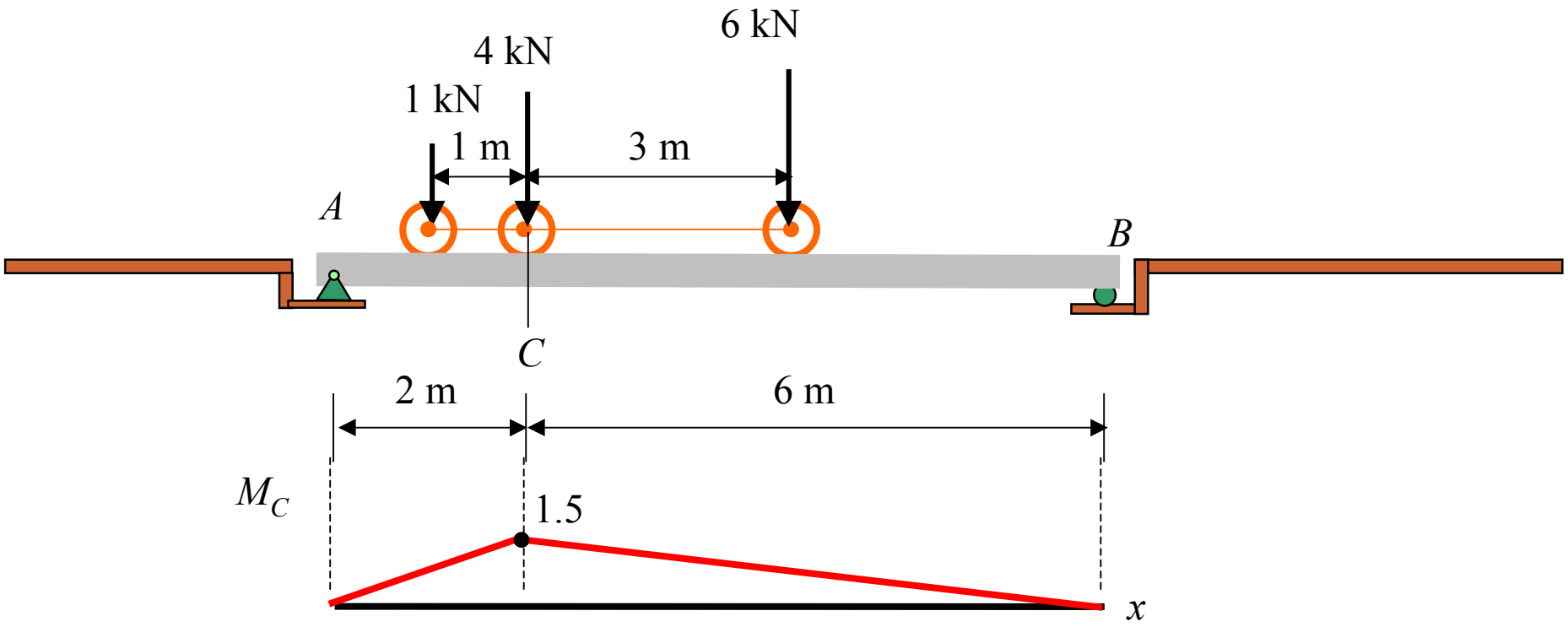
Sloping Line



$(M_C)_1 = 1(1.5) + 4(1.25) + 6(0.5) = 9.5$ kN•m

$$\Delta M_{1-2} = -1(\frac{1.5}{2})(1) + (4+6)(\frac{1.5}{6})(1) = 1.75\ kN \bullet m$$

$$(M_C)_2 = (M_C)_1 + \Delta M_{1-2} = 9.5 + 1.75 = 11.25\ kN \bullet m$$



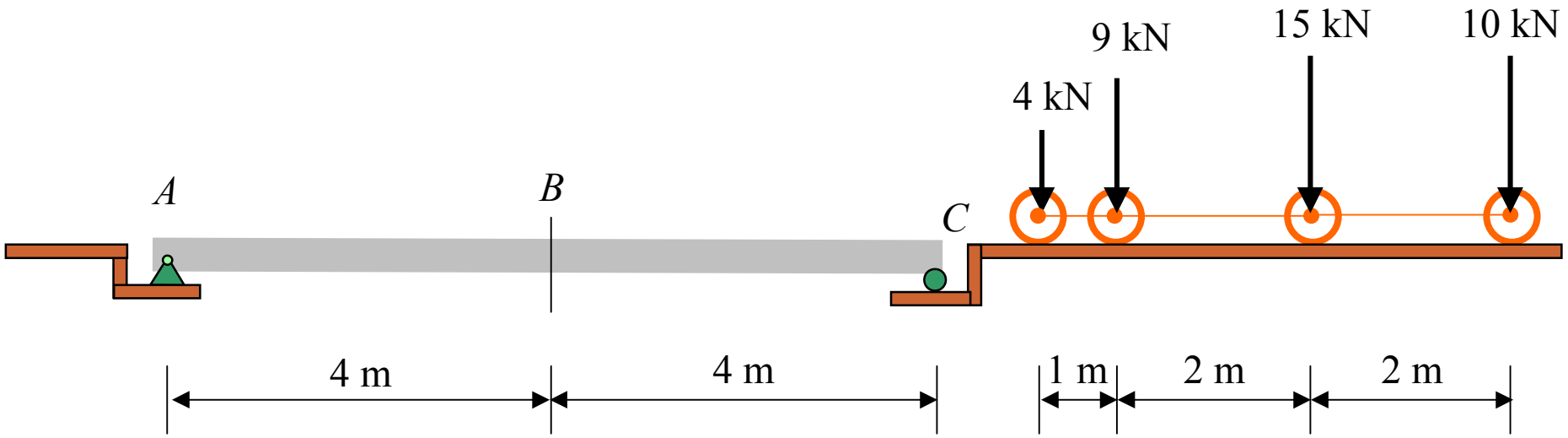
$(M_C)_2 = 11.25$ kN•m

$$\Delta M_{2-3} = -1(\frac{1.5}{2})(1) + 4(\frac{1.5}{2})(2) + 6(\frac{1.5}{6})(3) = -2.25\ kN \bullet m$$

$$(M_C)_3 = (M_C)_2 + \Delta M_{2-3} = 11.25 - 2.25 = 9\ kN \bullet m$$

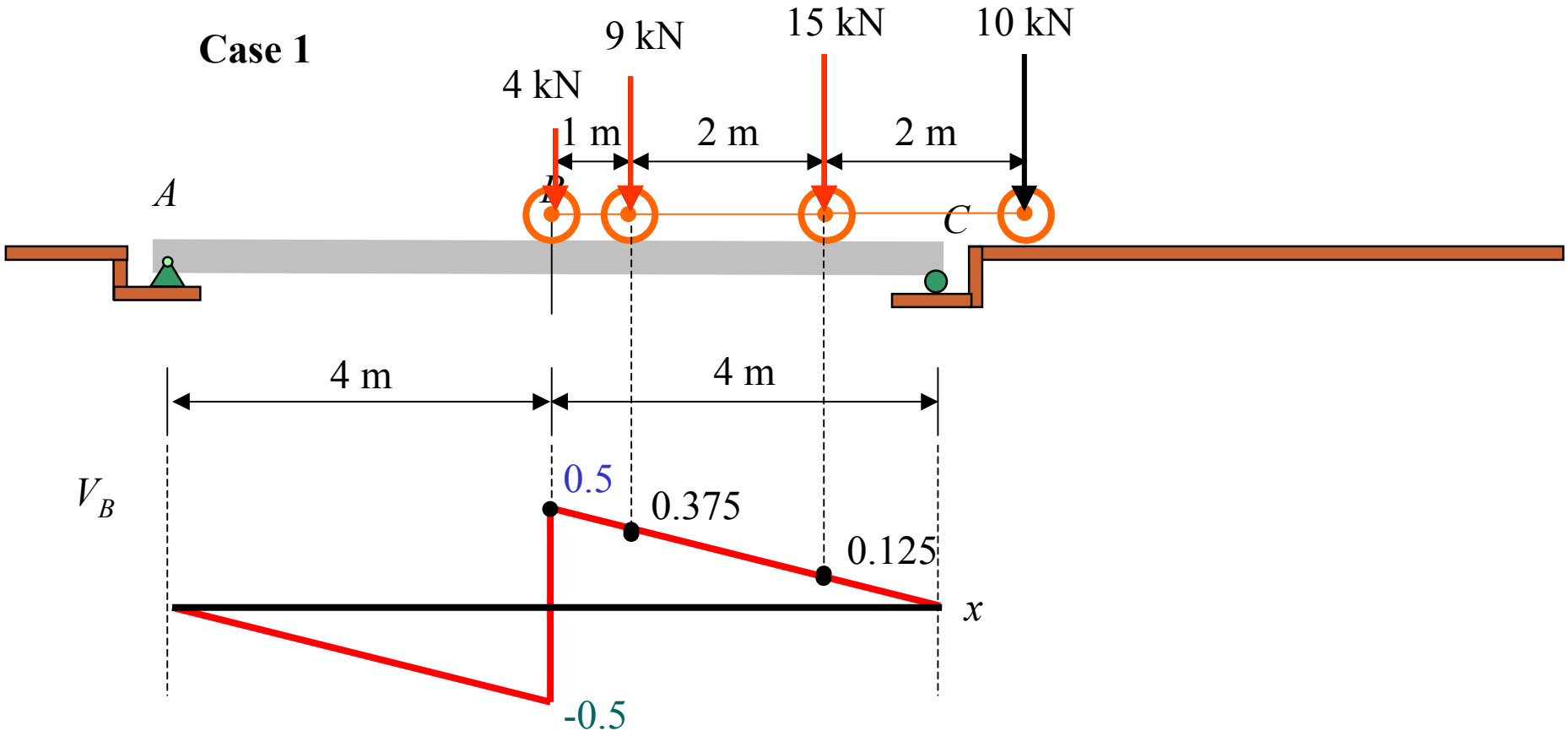
Example 6-9

Determine the maximum shear created at point *B* in the beam shown in the figure below due to the wheel loads of the moving truck.



SOLUTION

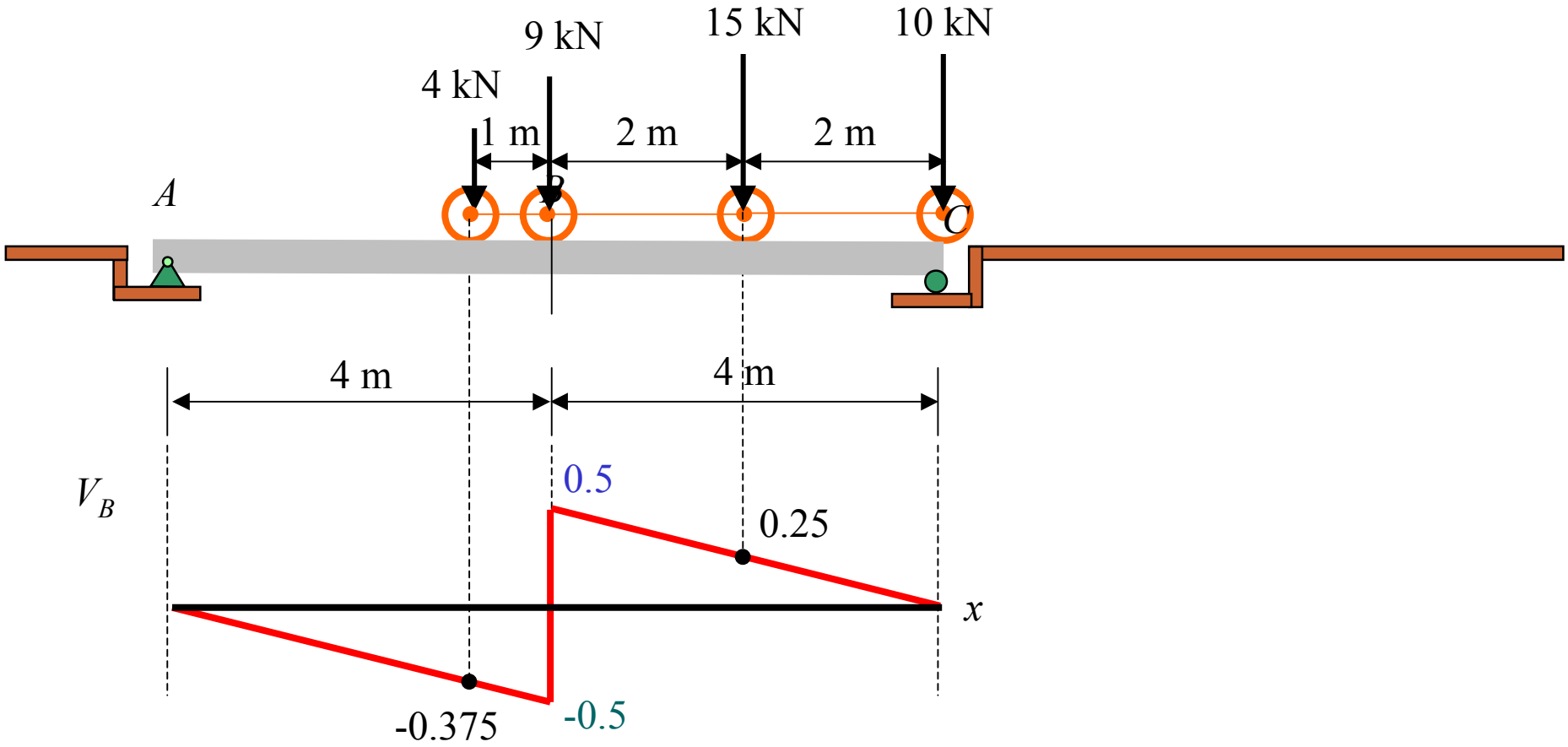
Case 1



$$V_{BR} = 4(0.5) + 9(0.375) + 15(0.125) = 7.25 \text{ kN}$$

$$V_{BL} = 4(-0.5) + 9(0.375) + 15(0.125) = 3.25 \text{ kN}$$

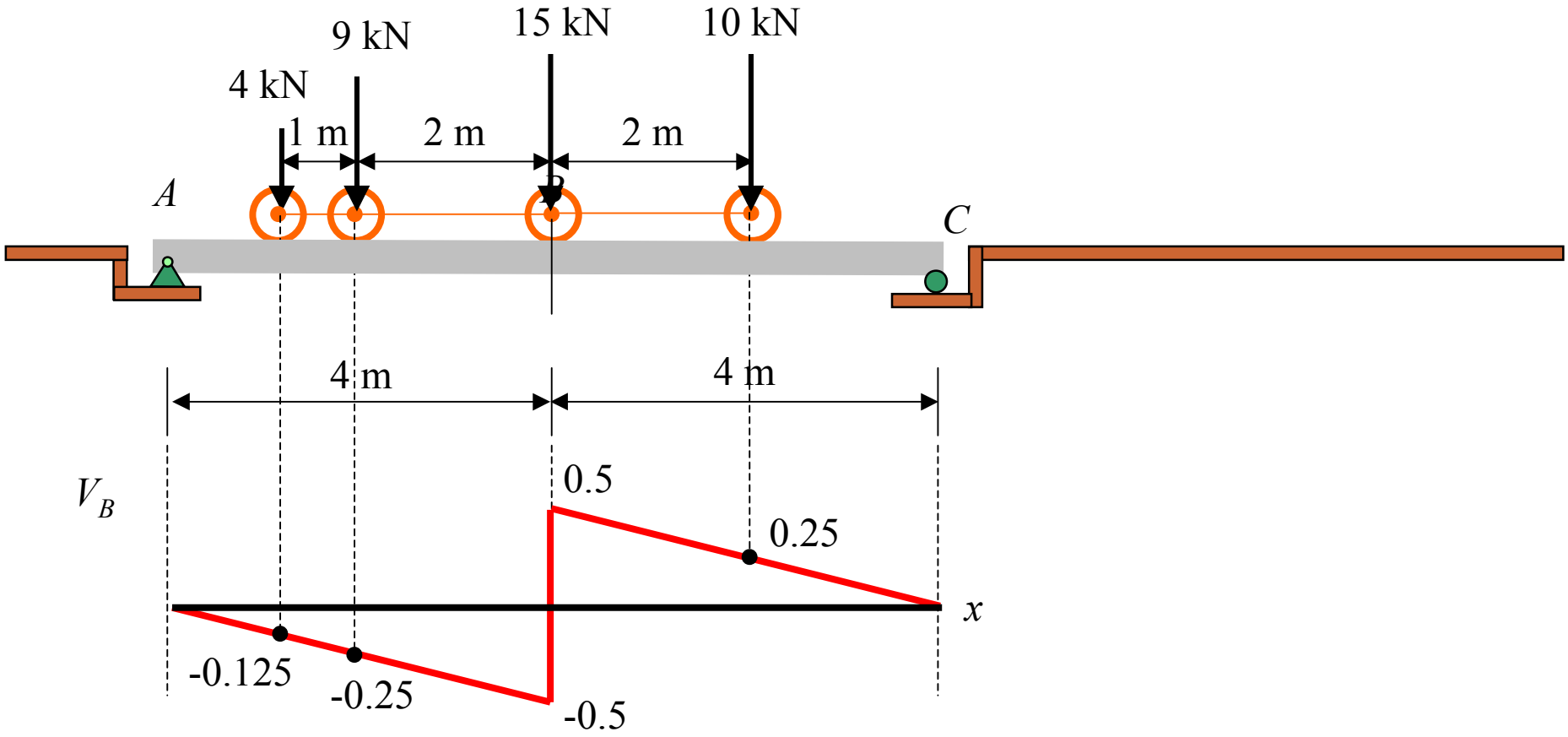
Case 2



$$V_{BR} = 4(-0.375) + 9(0.5) + 15(0.25) + 10(0) = 6.75 \text{ kN}$$

$$V_{BL} = 4(-0.375) + 9(-0.5) + 15(0.25) + 10(0) = -2.25 \text{ kN}$$

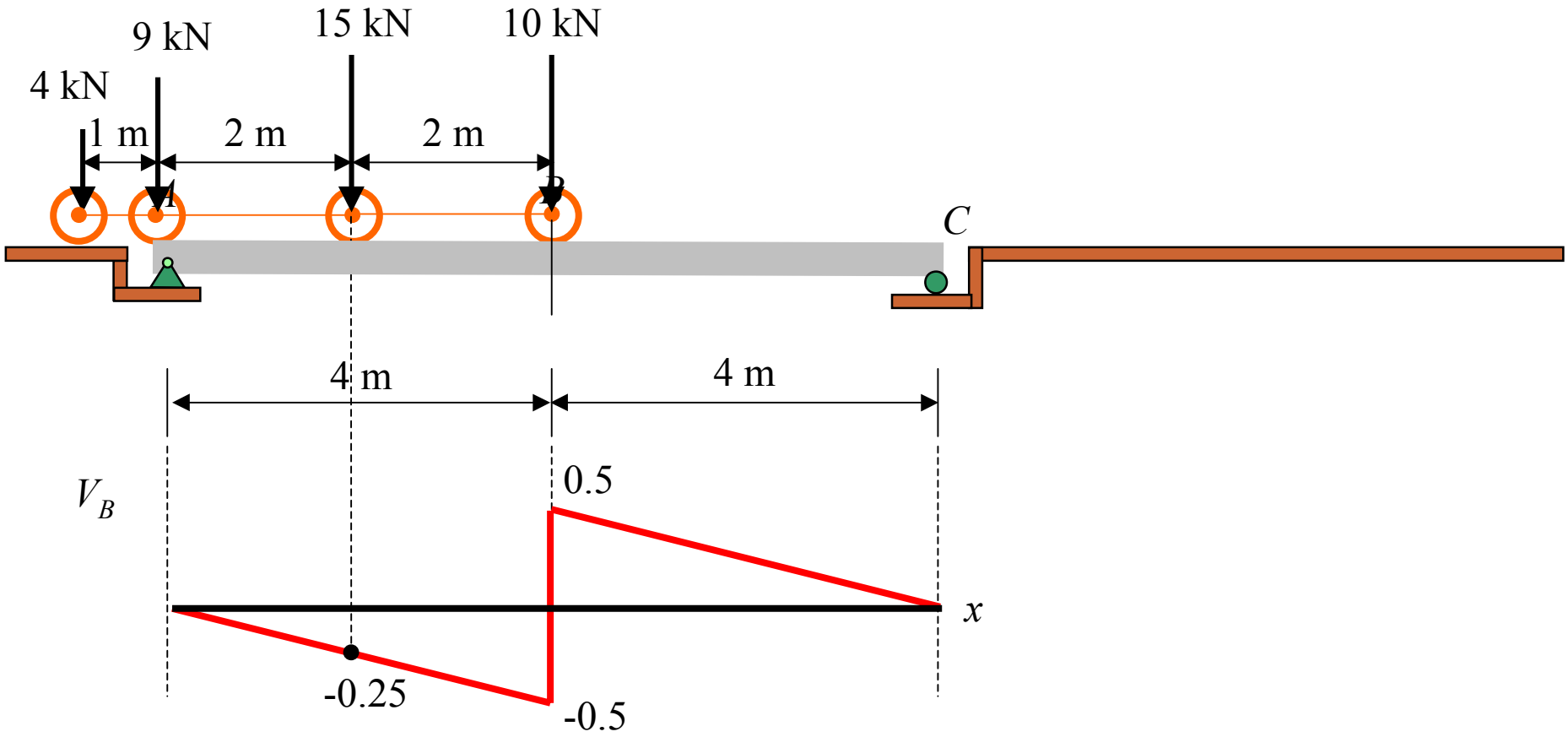
Case 3



$$V_{BR} = 4(-0.125) + 9(-0.25) + 15(0.5) + 10(0.25) = 7.25 \text{ kN}$$

$$V_{BL} = 4(-0.125) + 9(-0.25) + 15(-0.5) + 10(0.25) = -7.75 \text{ kN}$$

Case 4



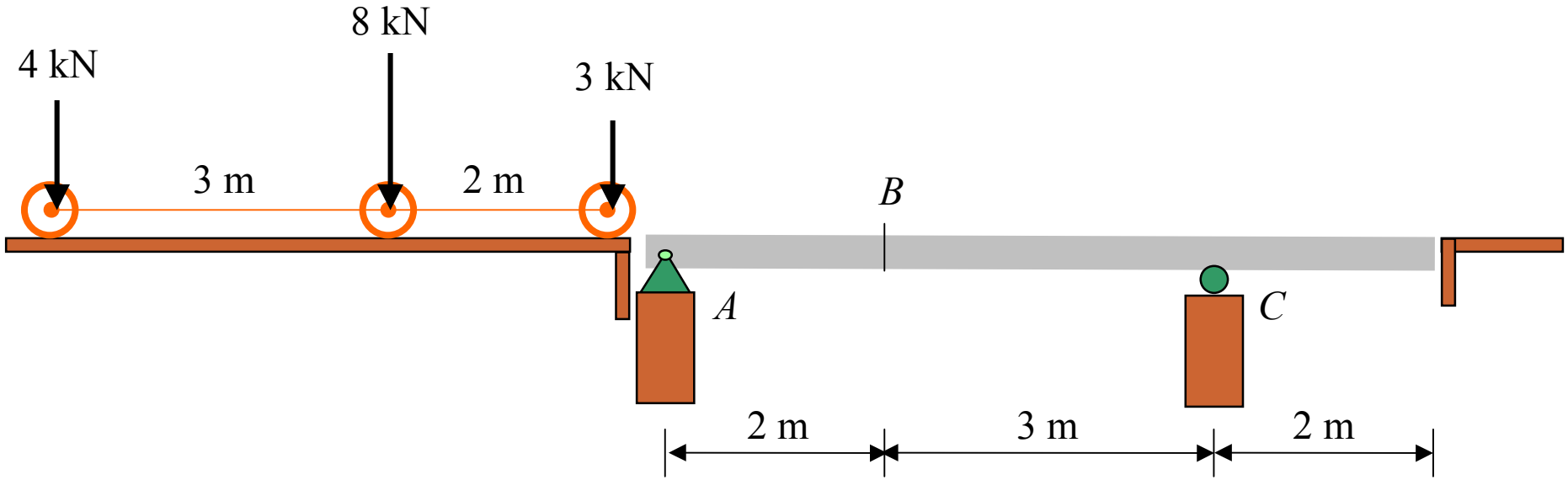
$V_{BR} = 9(0) + 15(-0.25) + 10(0.5) = 1.25$ kN

$V_{BL} = 9(0) + 15(-0.25) + 10(-0.5) = -8.75$ kN

The *maximum shear* created at point *B* is -8.75 kN

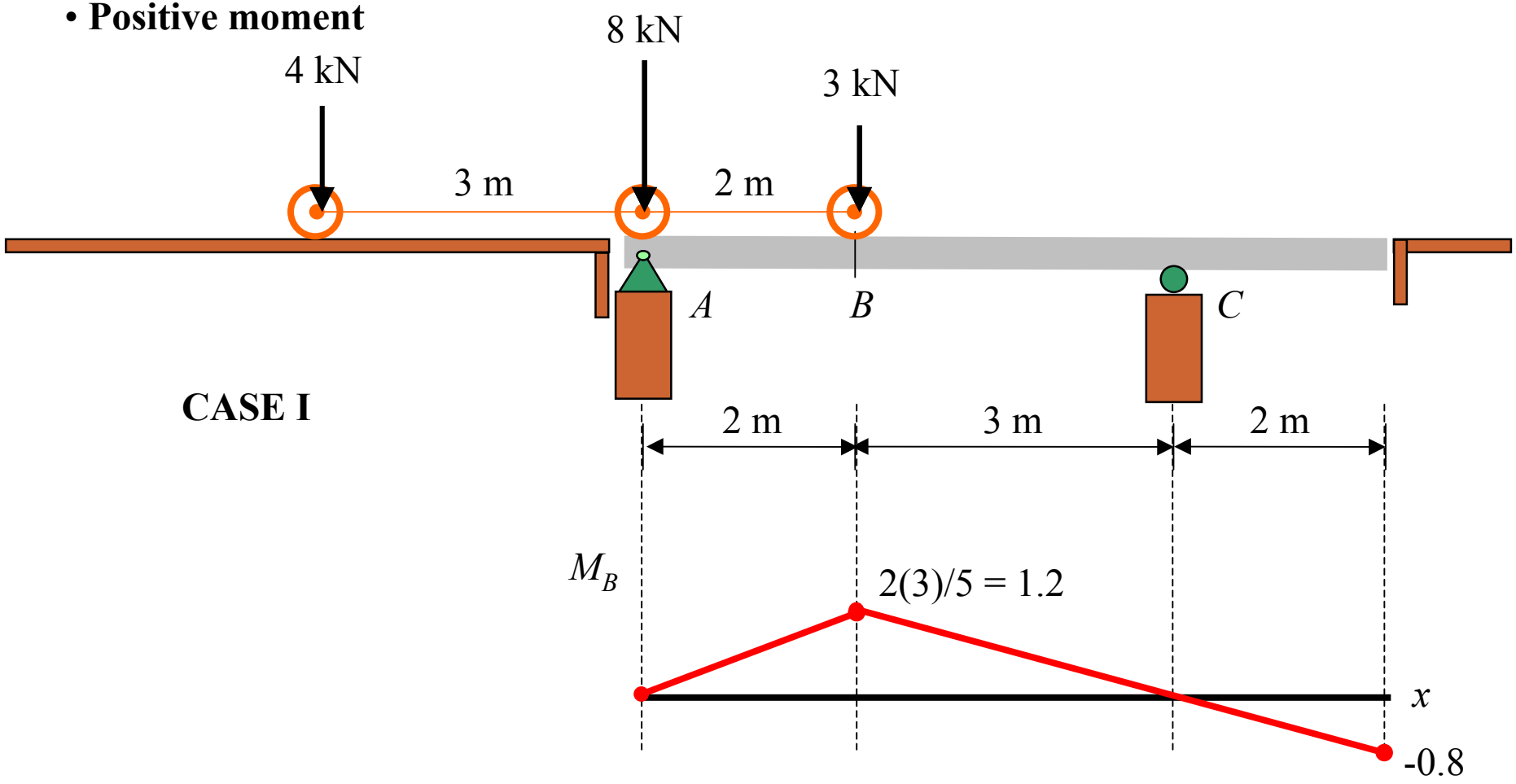
Example 6-10

Determine the maximum positive moment and negative moment created at point *B* in the beam shown in the figure below due to the wheel loads of the crane.

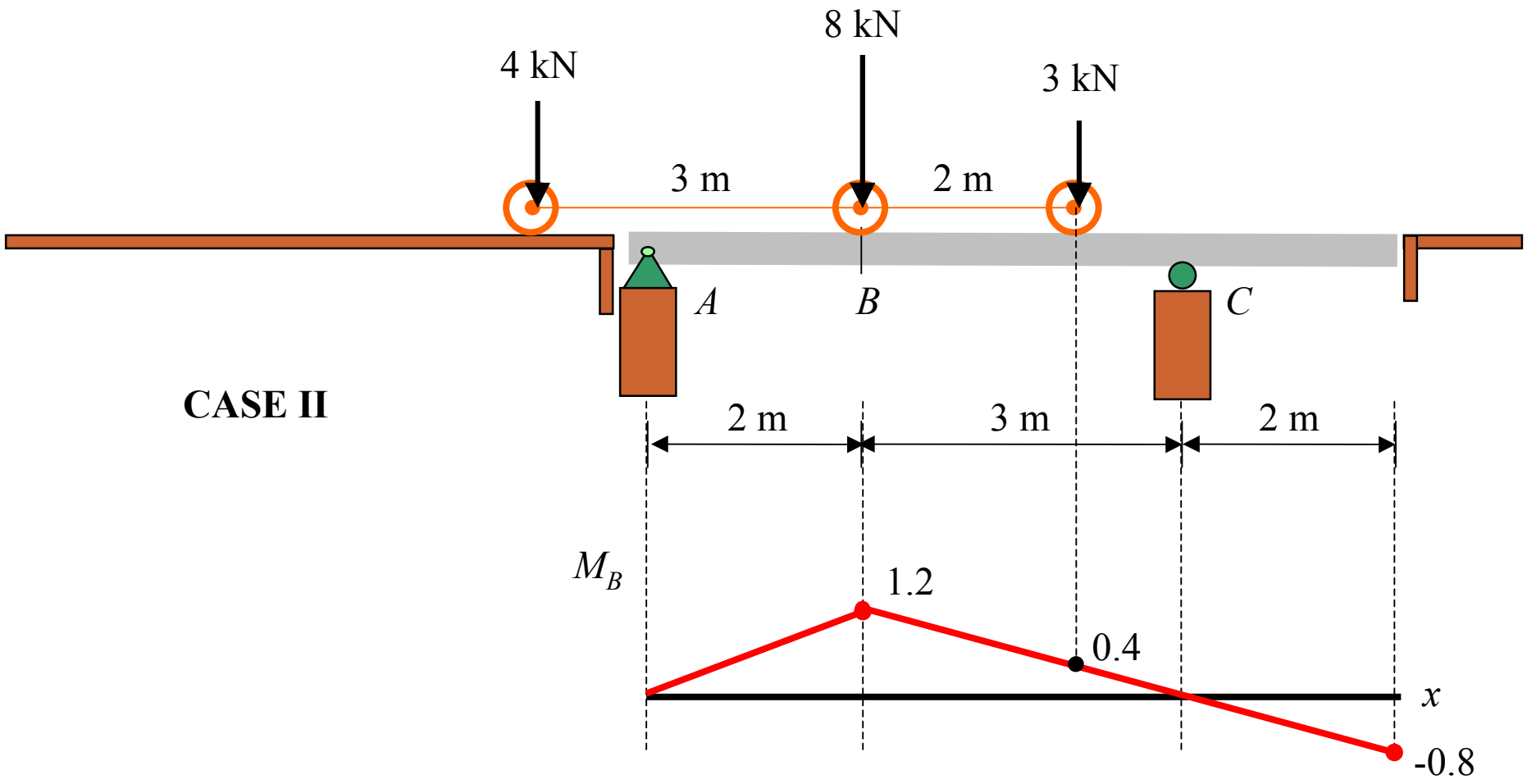


SOLUTION

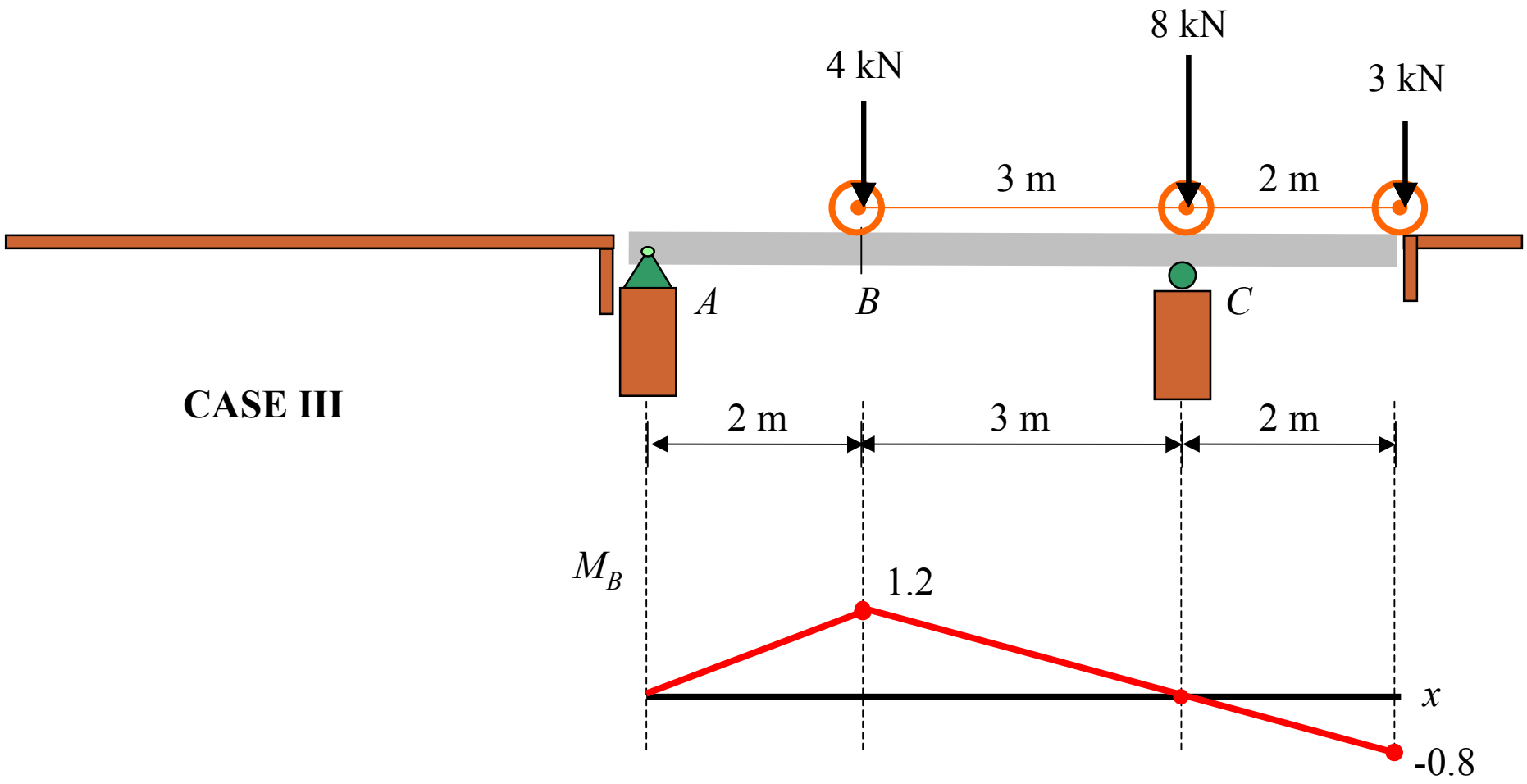
- **Positive moment**



$$M_B = 3(1.2) + 8(0) = 3.6 \text{ kN•m}$$



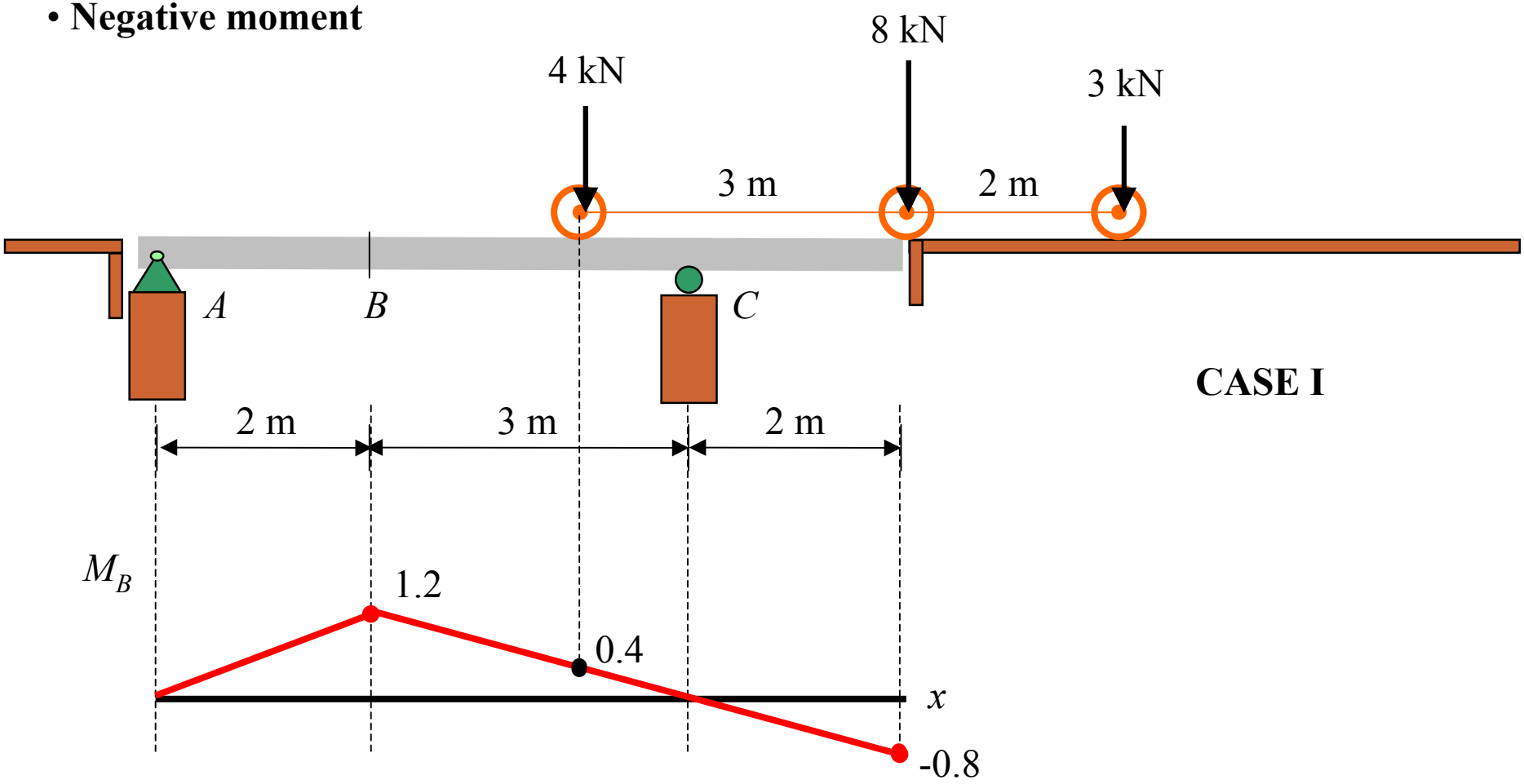
$M_B = 8(1.2) + 3(0.4) = 10.8$ kN•m



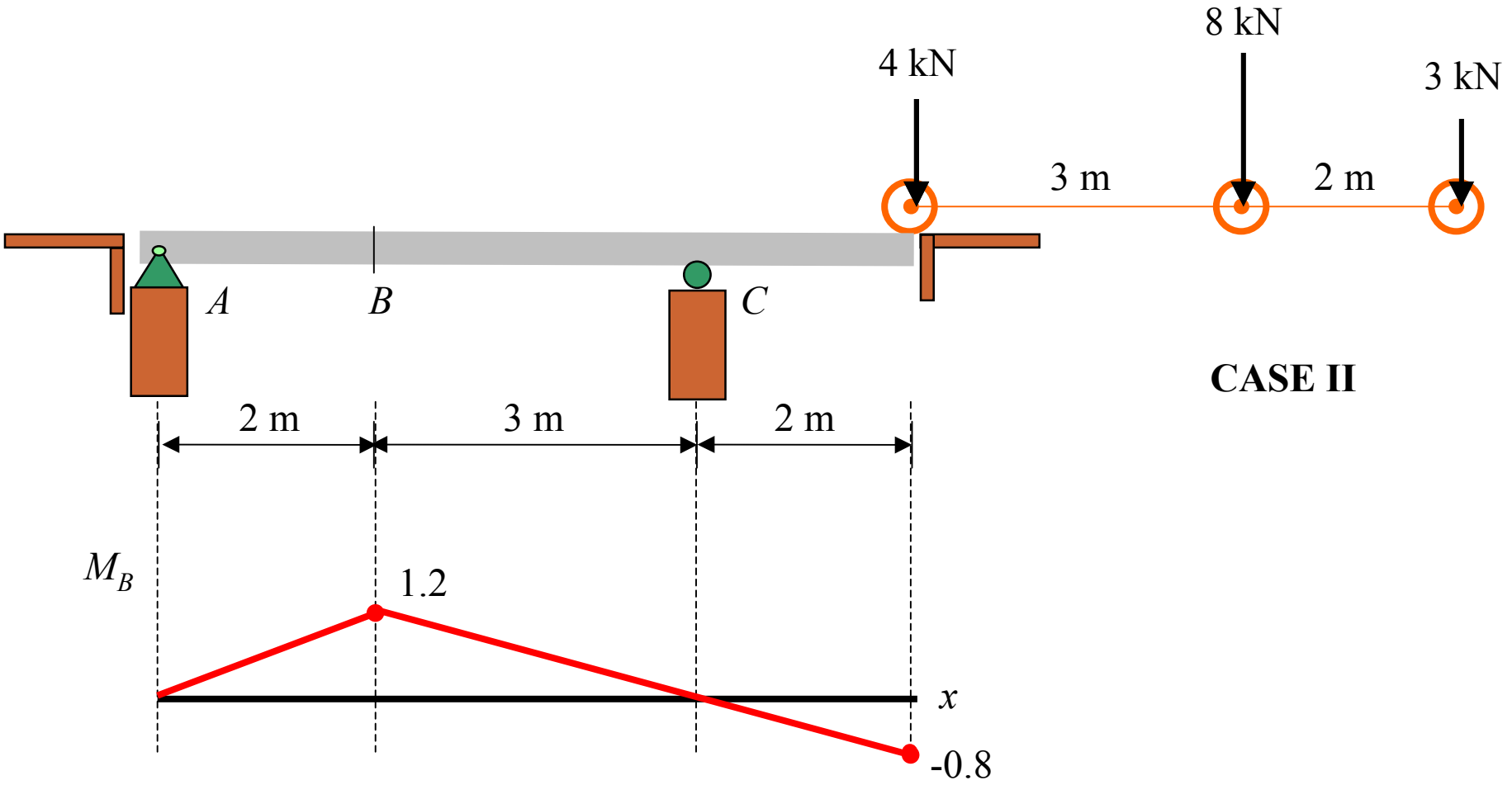
$$M_B = 4(1.2) + 8(0) + 3(-0.8) = 2.4 \text{ kN•m}$$

The *maximum positive moment* created at point *B* is 10.8 kN•m

• **Negative moment**



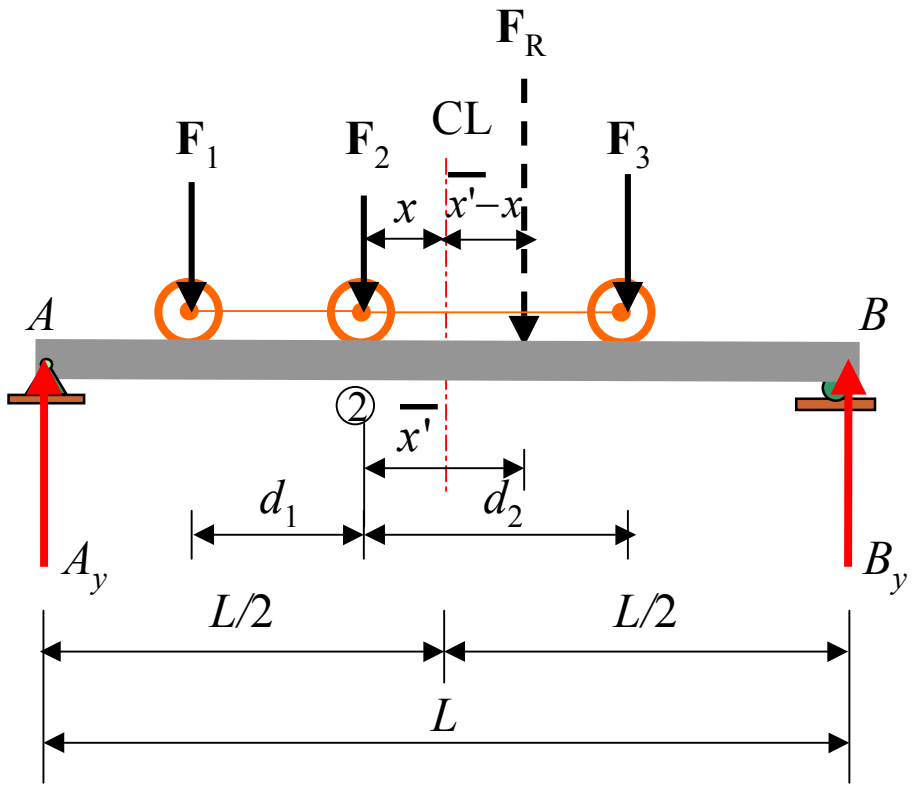
$M_B = 8(-0.8) + 4(0.4) = -4.8$ kN•m



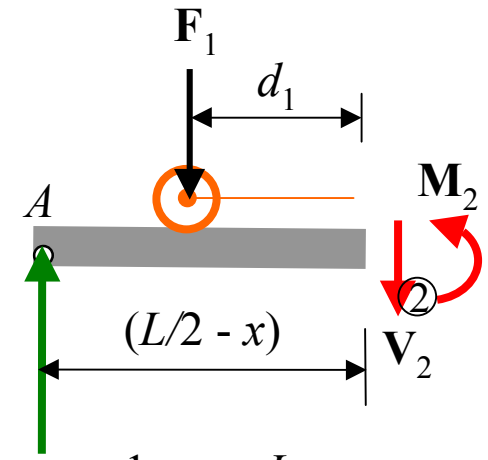
$M_B = 4(-0.8) = -3.2$ kN•m

The *maximum negative moment* created at point *B* is - 4.8 kN•m

Absolute Maximum Shear and Moment



$$\Sigma M_B = 0: \qquad A_y = \frac{1}{L}(F_R)[\frac{L}{2} - (\bar{x}' - x)]$$

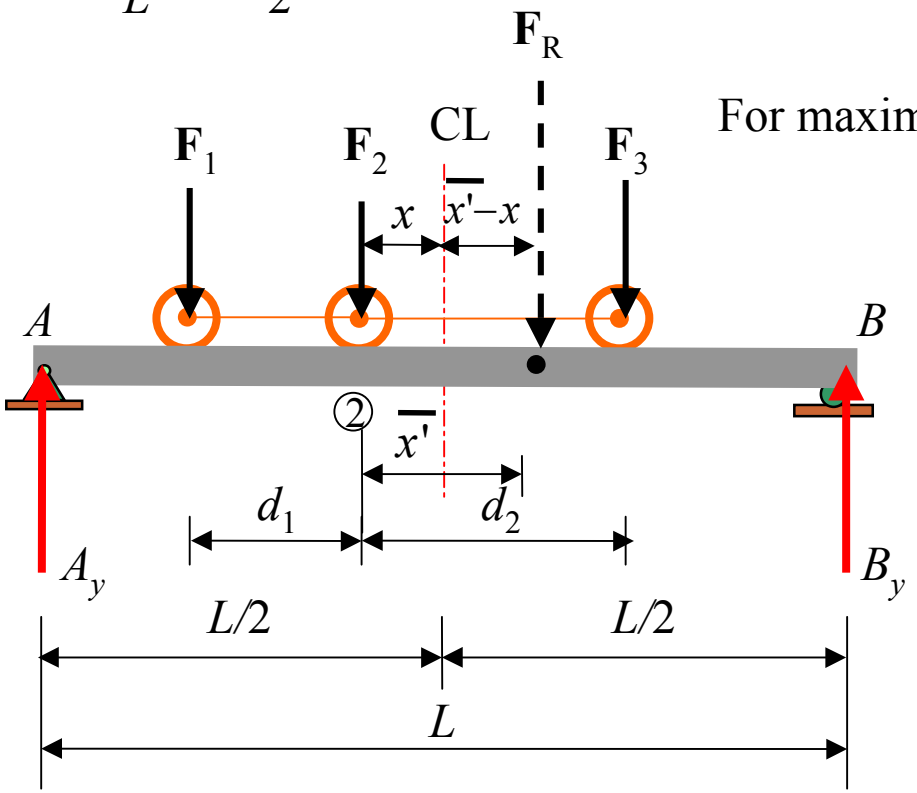


$$A_y = \frac{1}{L}(F_R)[\frac{L}{2} - (\bar{x}' - x)]$$

$$\Sigma M_2 = 0: \qquad M_2 = A_y(\frac{L}{2} - x) - F_1 d_1$$

$$= \frac{1}{L}(F_R)[\frac{L}{2} - (\bar{x}' - x)](\frac{L}{2} - x) - F_1 d_1$$

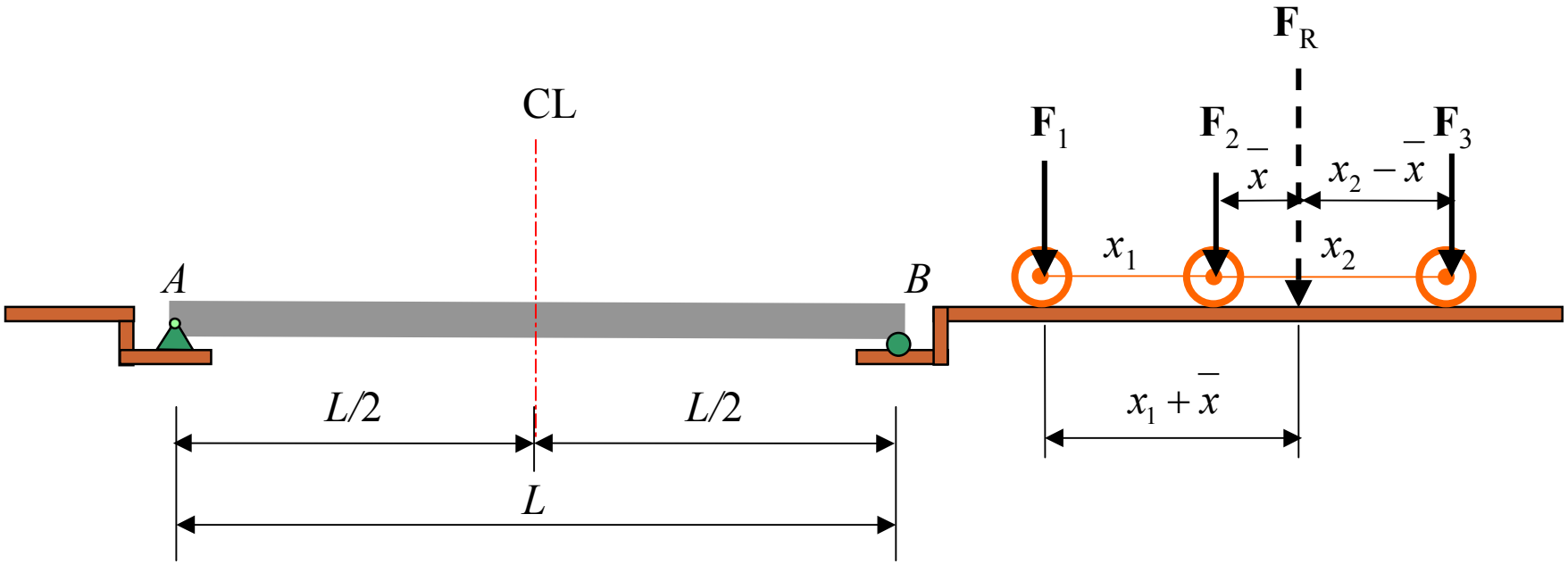
$$M_2 = \frac{F_R L}{4} - \frac{F_R \bar{x}'}{2} - \frac{F_R x^2}{L} + \frac{F_R x \bar{x}'}{L} - F_1 d_1$$

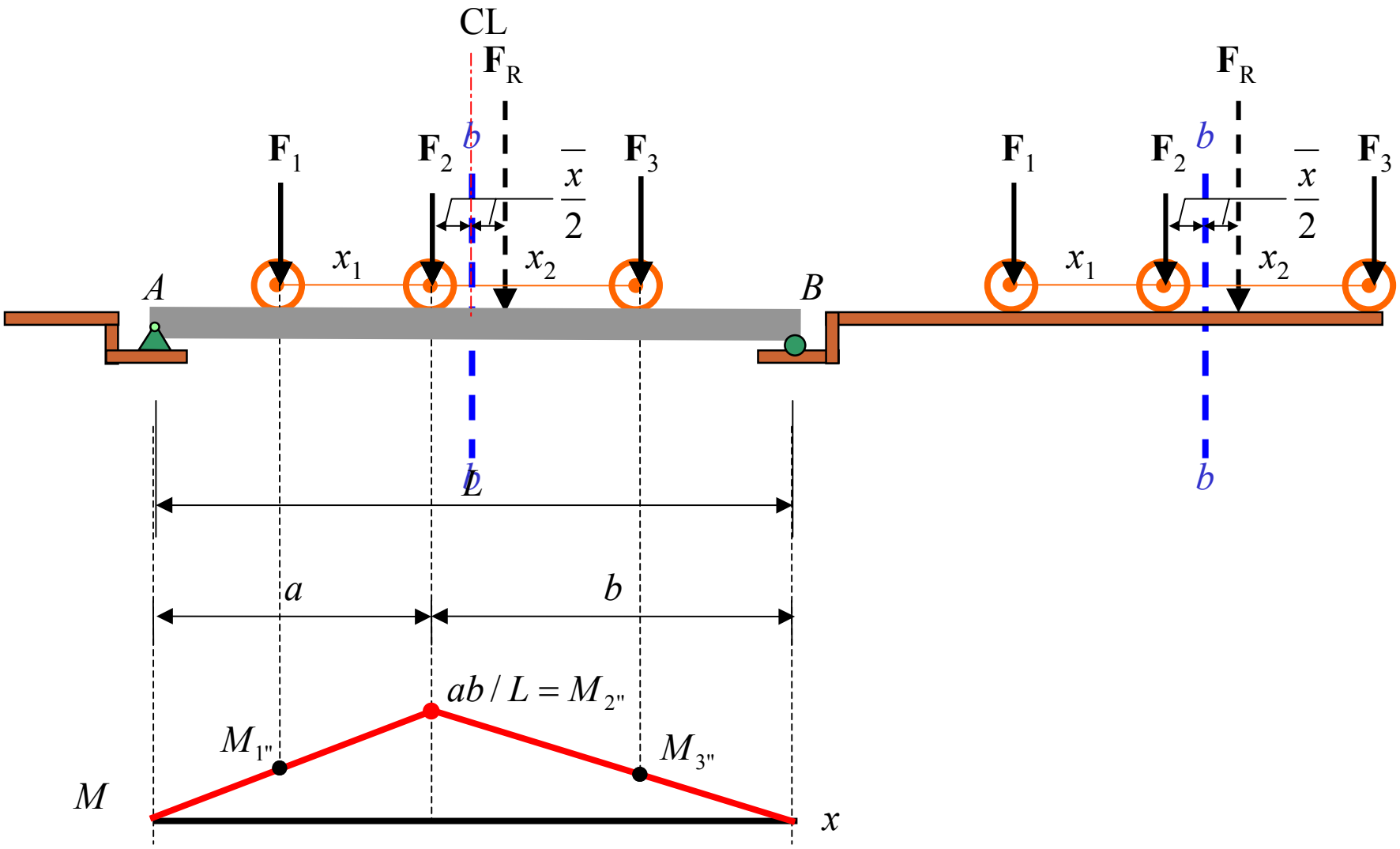


For maximum M_2 we require

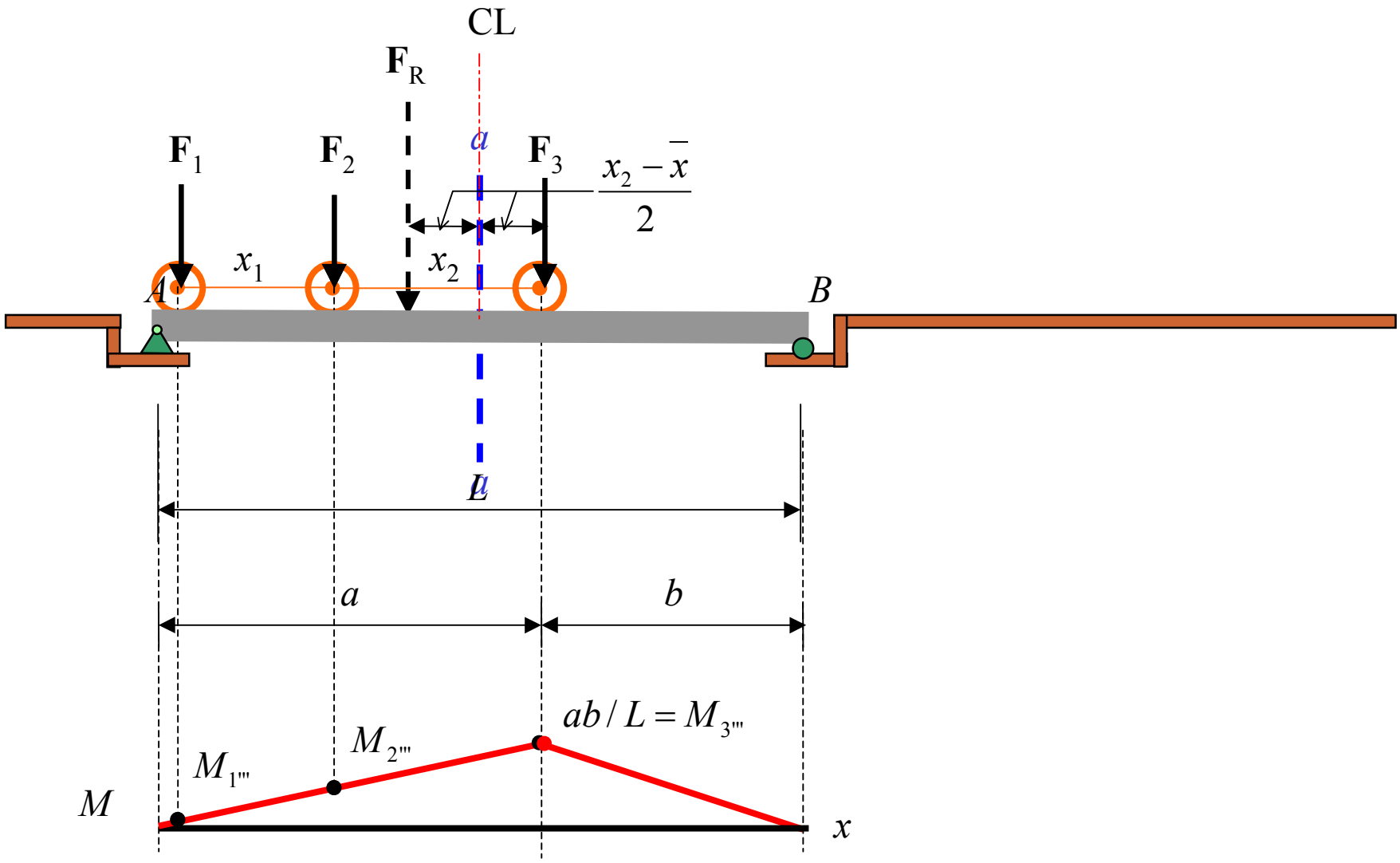
$$\frac{dM_2}{dx} = -\frac{2F_R x}{L} + \frac{F_R \bar{x}'}{L} = 0$$

$$x = \frac{\bar{x}'}{2}$$


CL
A
B
L/2
L/2
L
F1
F2
F3
FR
x1
x2
x̄
x2 − x̄
x1 + x̄



$$M_{S1} = F_1 M_{1"} + F_2 M_{2"} + F_3 M_{3"}$$

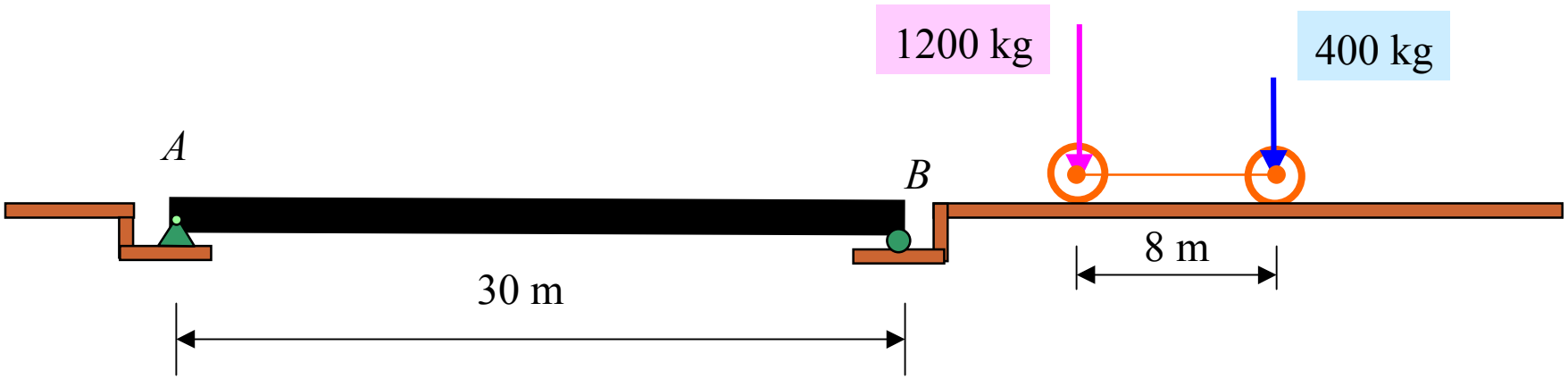


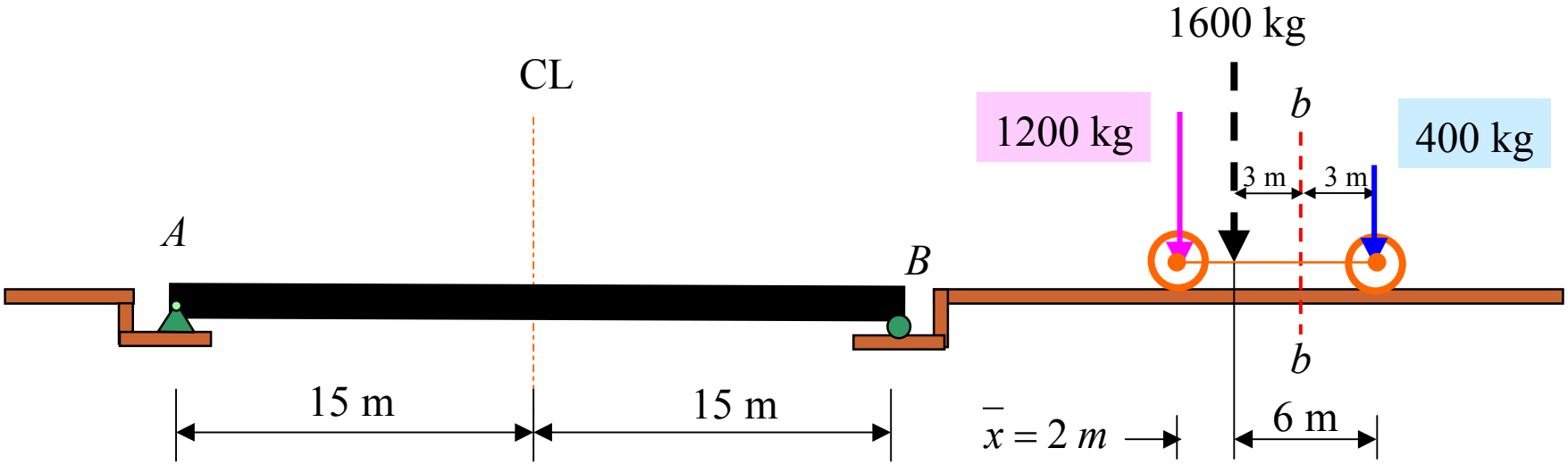
$$M_{S2} = F_1 M_{1'''} + F_2 M_{2'''} + F_3 M_{3'''}$$

The *absolute maximum moment* is comparison by M_{S1} and M_{S2}.

Example 6-11

Determine the absolute maximum moment on the simply supported beam cased by the wheel loads.

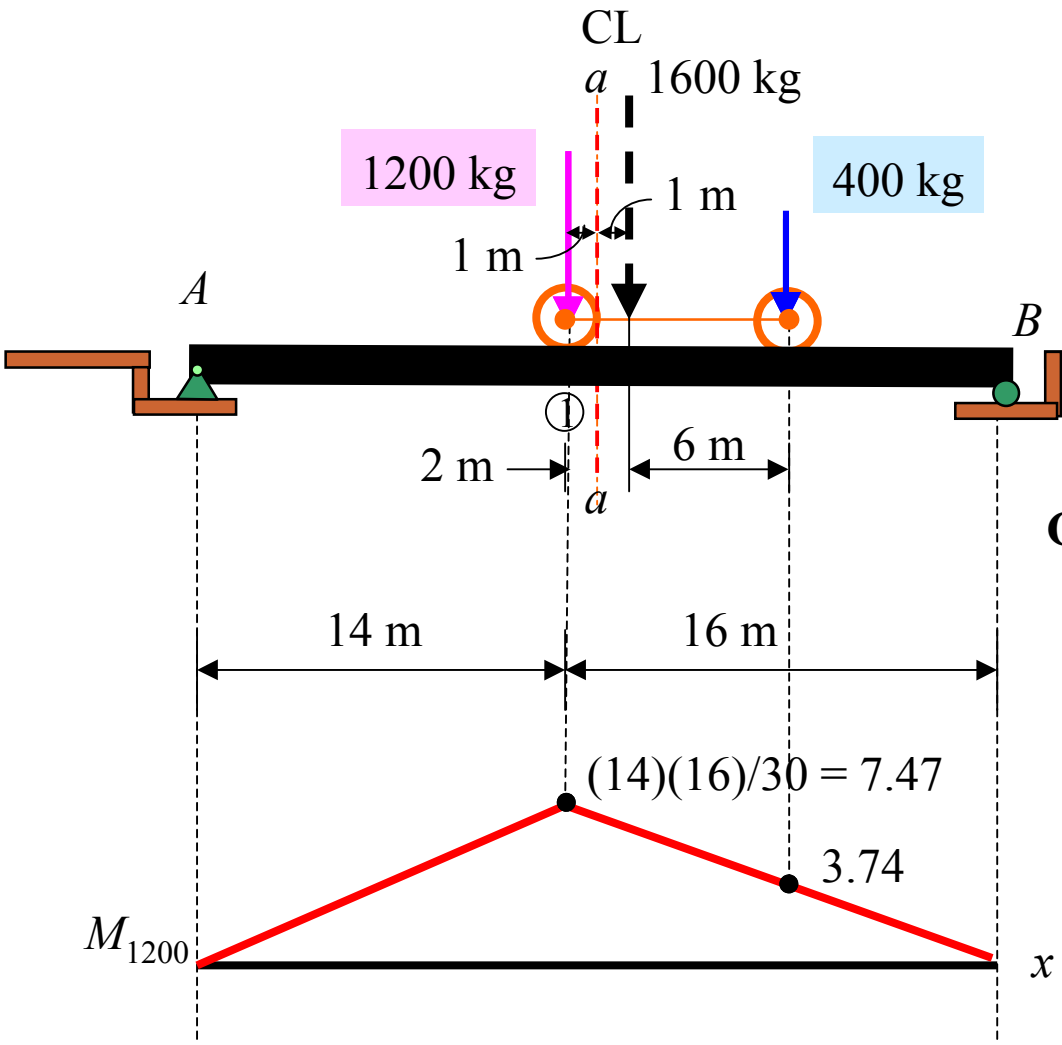




$$\Sigma M_{@1200\ \text{kg}} = 0:$$

$$1600\bar{x} = 1200(0) + 400(8)$$

$$\bar{x} = 2\ m$$



$M_S =$ (1200)(7.47) + (400)(3.74)

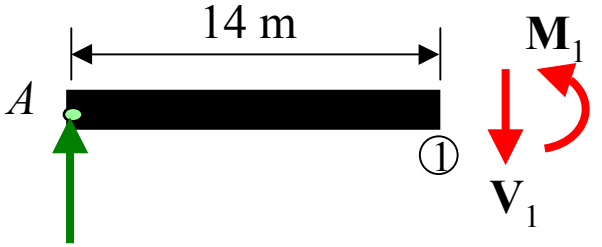
$= 10\,460$ kg•m

Or using equilibrium conditions:

Global: $+\circlearrowleft \Sigma M_B = 0$:

$$1600(14) - A_y(30) = 0$$

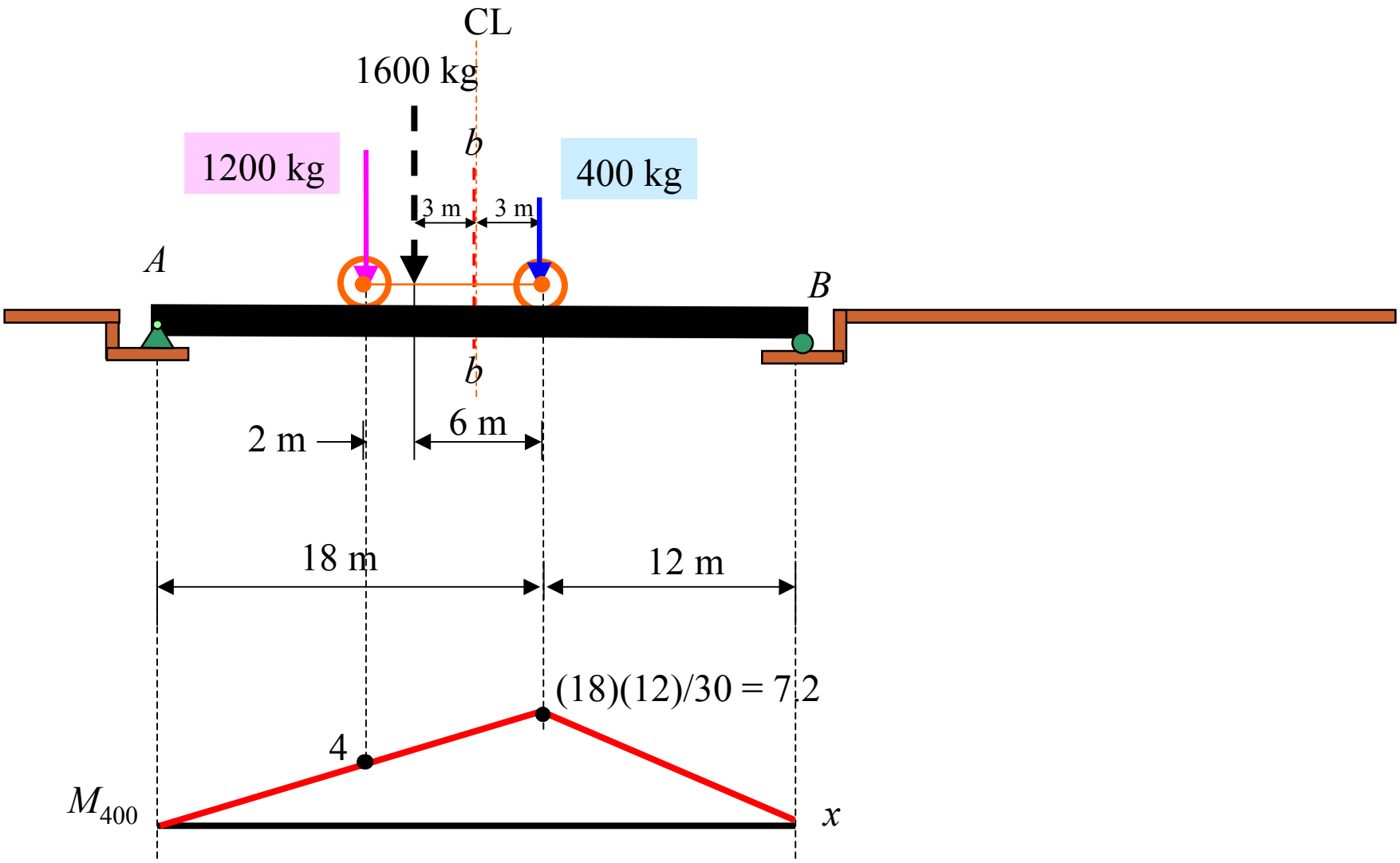
$$A_y = 746.67 \text{ kg}$$



$A_y = 746.7$ kg

$+\circlearrowleft \Sigma M_1 = 0$:

$M_1 = 10\,460$ kg•m

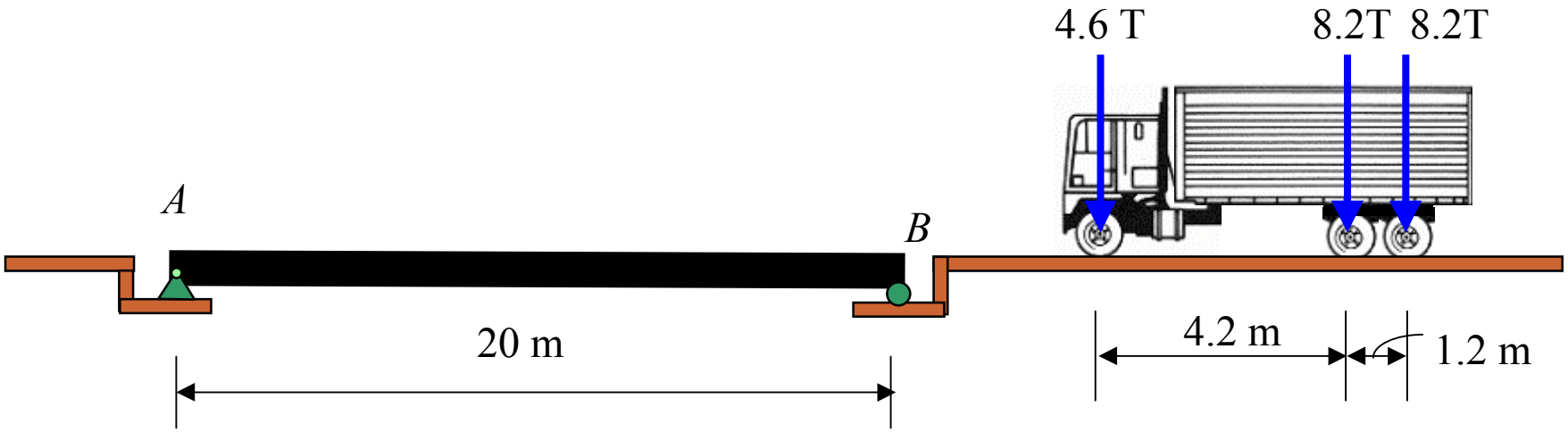


M_S = (1200)(4) + (400)(7.2) = 7680 kg•m

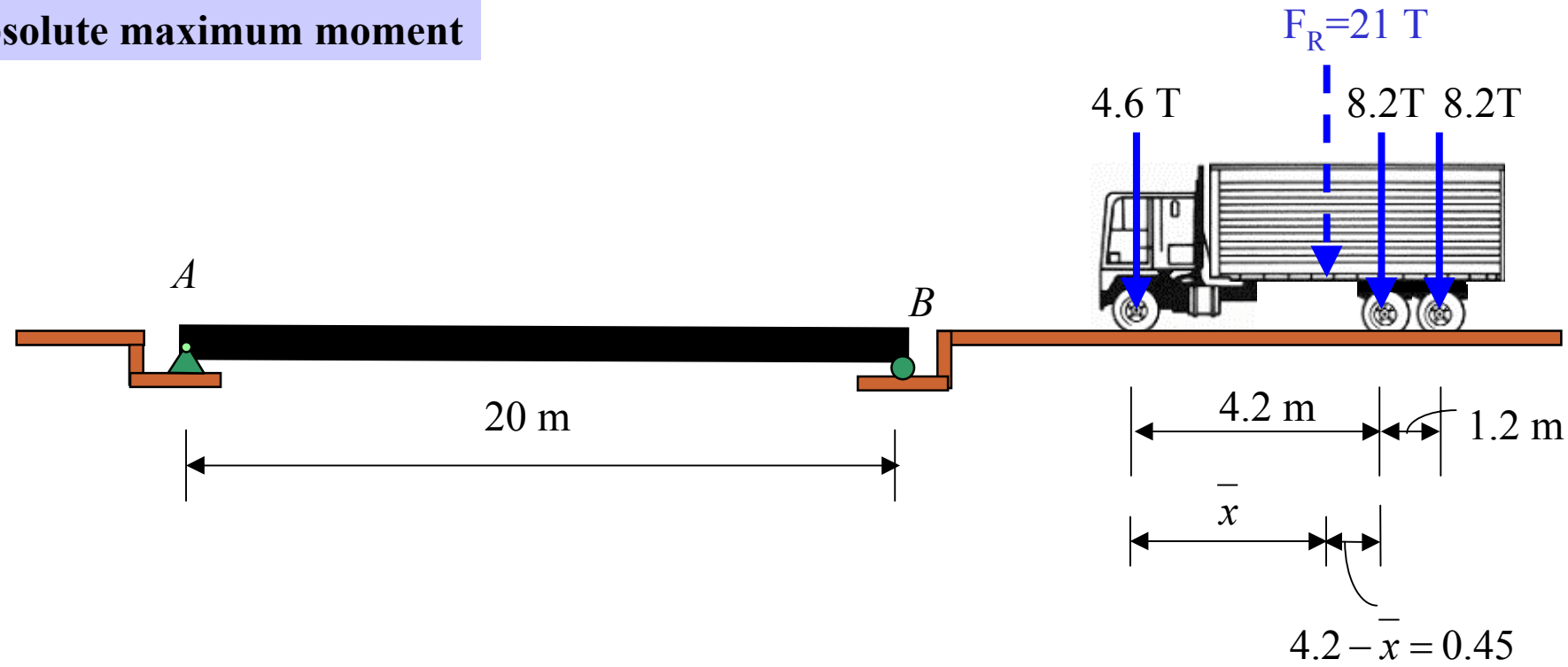
By comparison, M_{max} = 10 460 kg•m

Example 6-12

Determine the absolute maximum moment and maximum shear on the simply supported beam cased by the wheel loads.



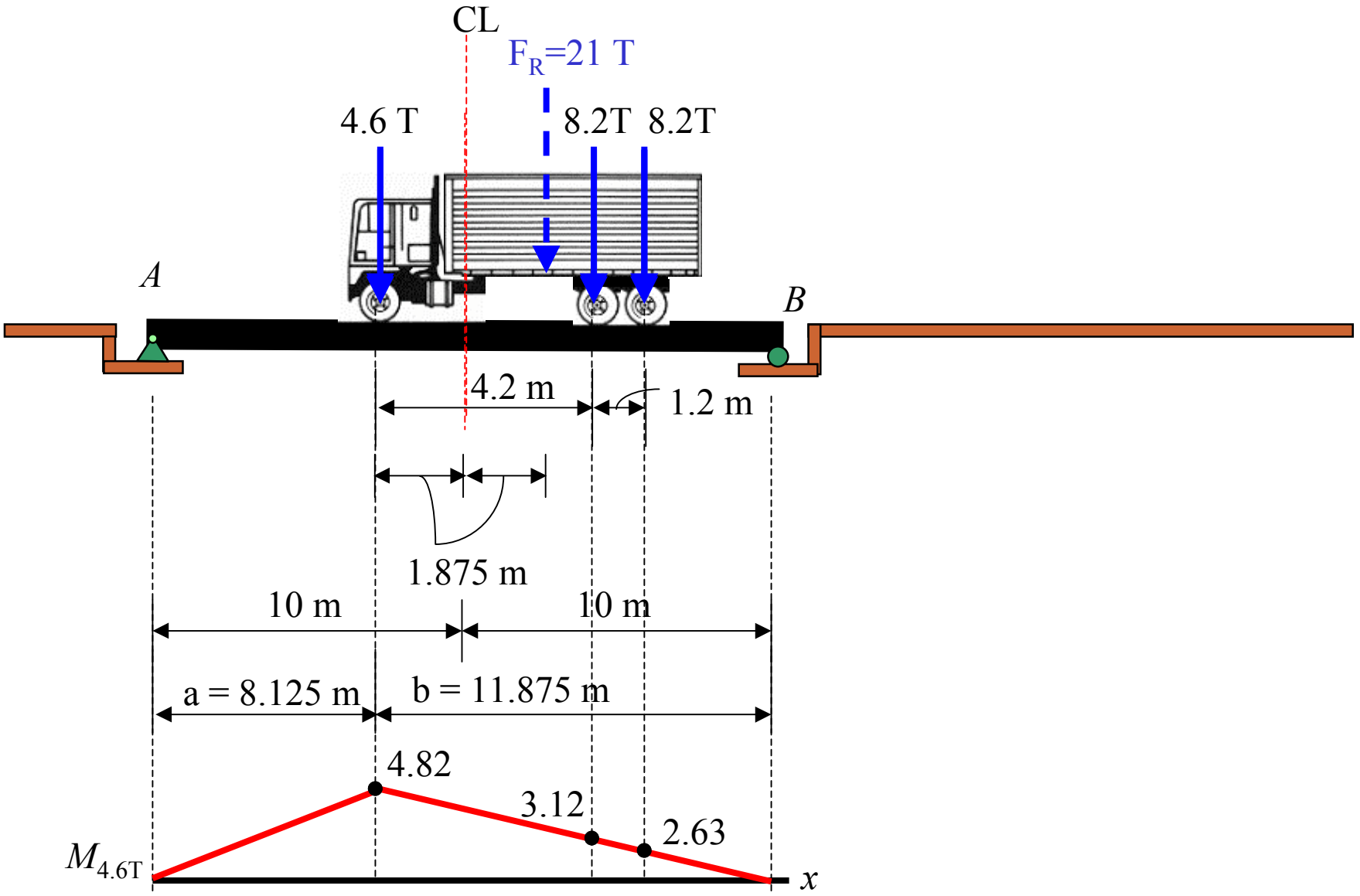
Absolute maximum moment



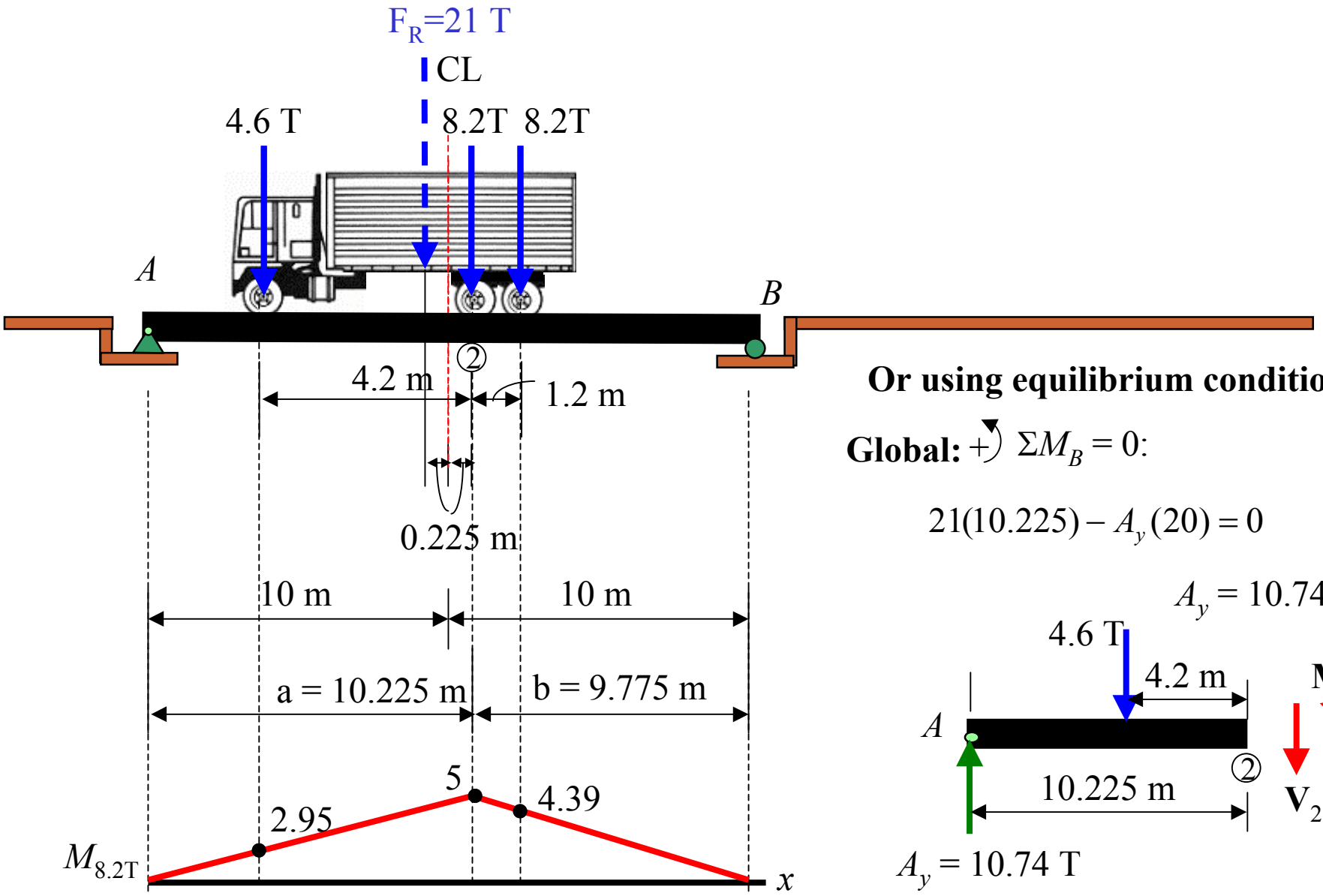
$$\Sigma M_{@\,4.6\,\mathrm{T}} = 0:$$

$$21\bar{x} = 4.6(0) + 8.2(4.2) + 8.2(5.4)$$

$$\bar{x} = 3.75\ m$$



$$M_S = \quad (4.6)(4.82) + (8.2)(3.12) + (8.2)(2.63) \quad = 69.32 \text{ T•m}$$



Or using equilibrium conditions:

Global: $+\circlearrowleft \Sigma M_B = 0$:

$$21(10.225) - A_y(20) = 0$$

$$A_y = 10.74 \text{ T}$$

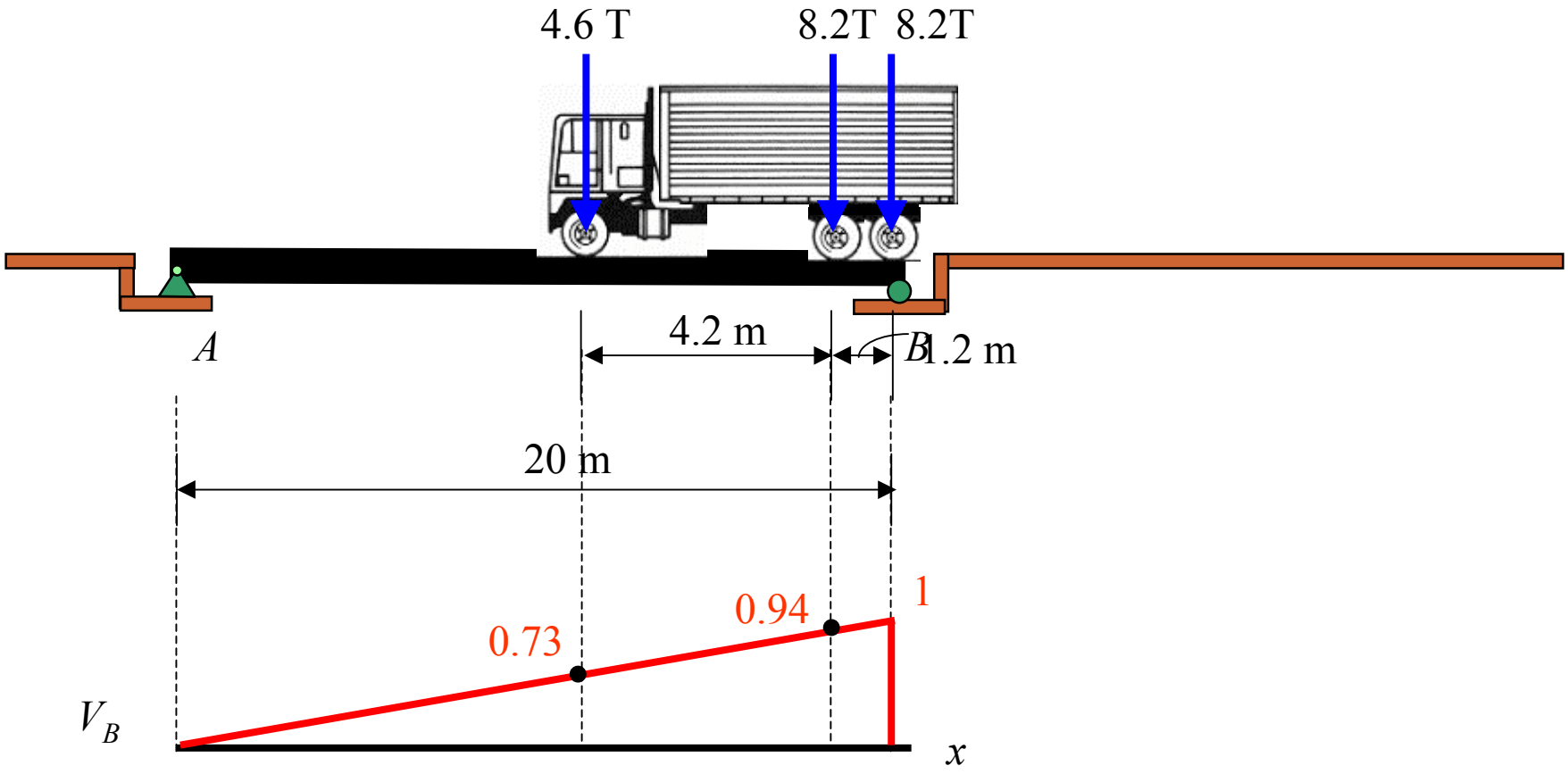
$$M_S = (4.6)(2.95) + (8.2)(5) + (8.2)(4.39) = 90.57 \text{ T•m}$$

By comparison, $M_{max} = 90.57$ T•m

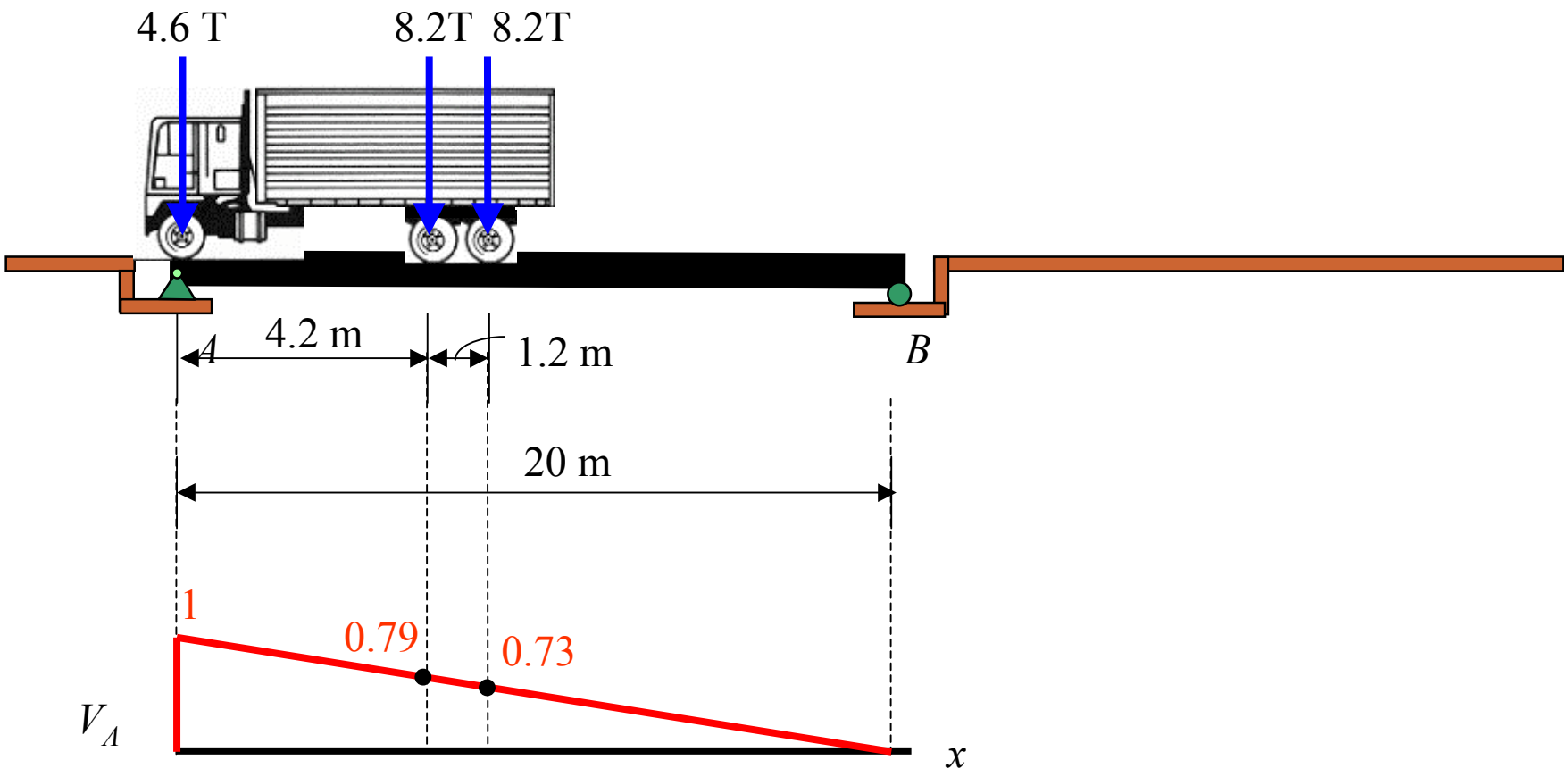
$+\circlearrowleft \Sigma M_2 = 0$:

$M_2 = 90.50$ T•m

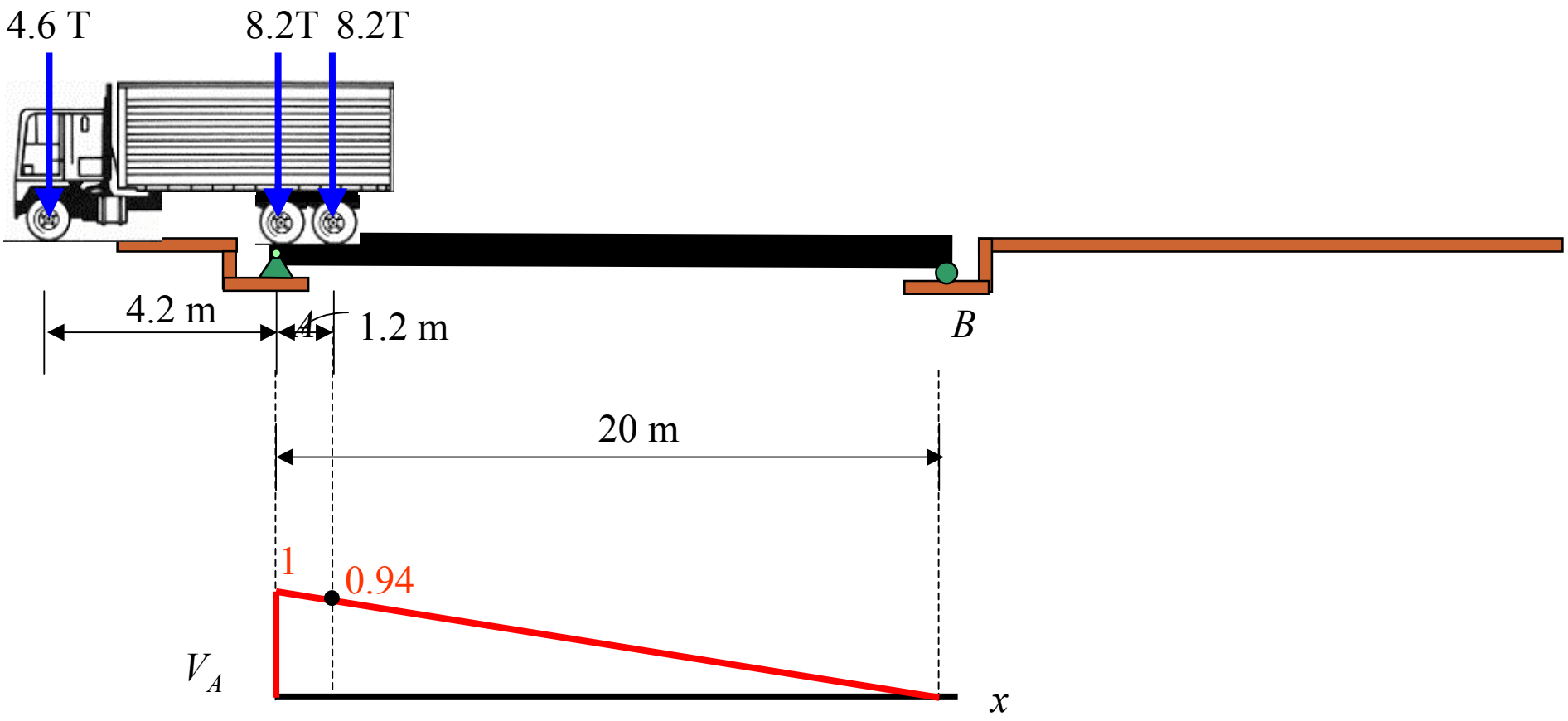
Absolute maximum shear



$$(V_{max})_1 = 4.6(0.73) + 8.2(0.94) + 8.2(1.0) = 19.27 \text{ T}$$



$$(V_{max})_2 = 4.6(1) + 8.2(0.79) + 8.2(0.73) = 17.06 \text{ T}$$

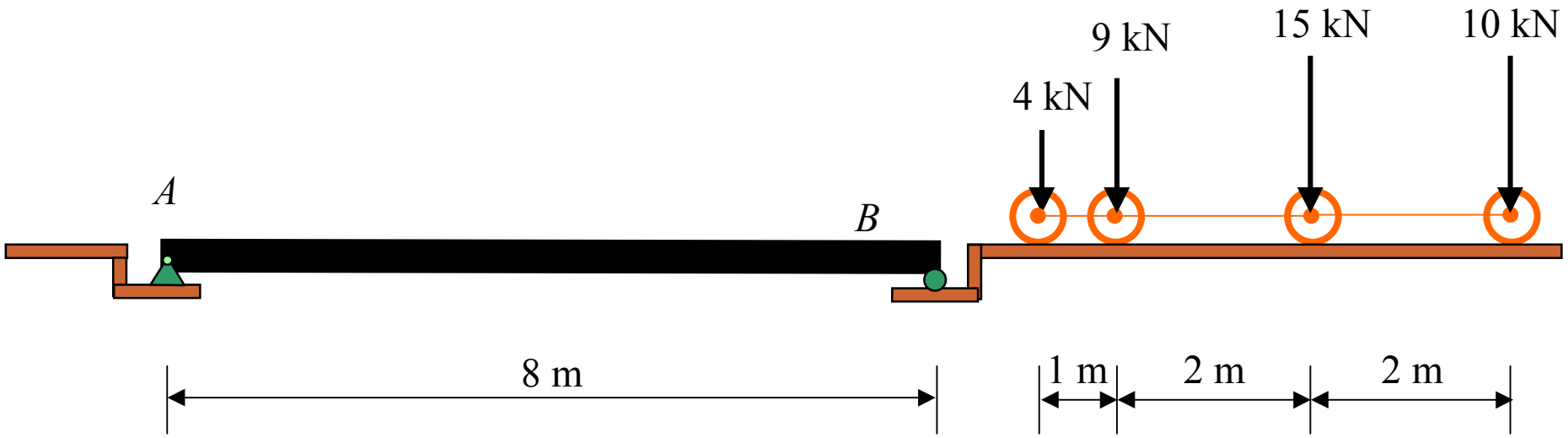


$(V_{max})_3 = 8.2(1.0) + 8.2(0.94) = 15.91$ T

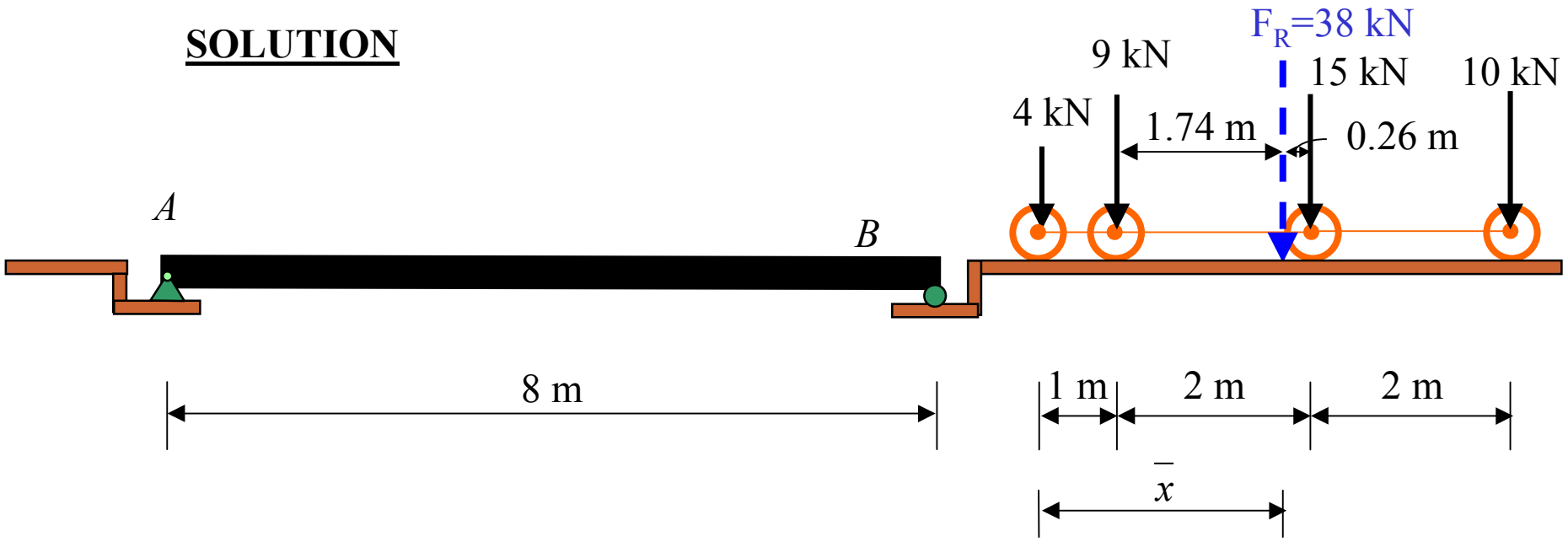
By comparison, $V_{max} = 19.27$ T ⇦

Example 6-13

Determine the absolute maximum moment on the simply supported beam cased by the wheel loads.



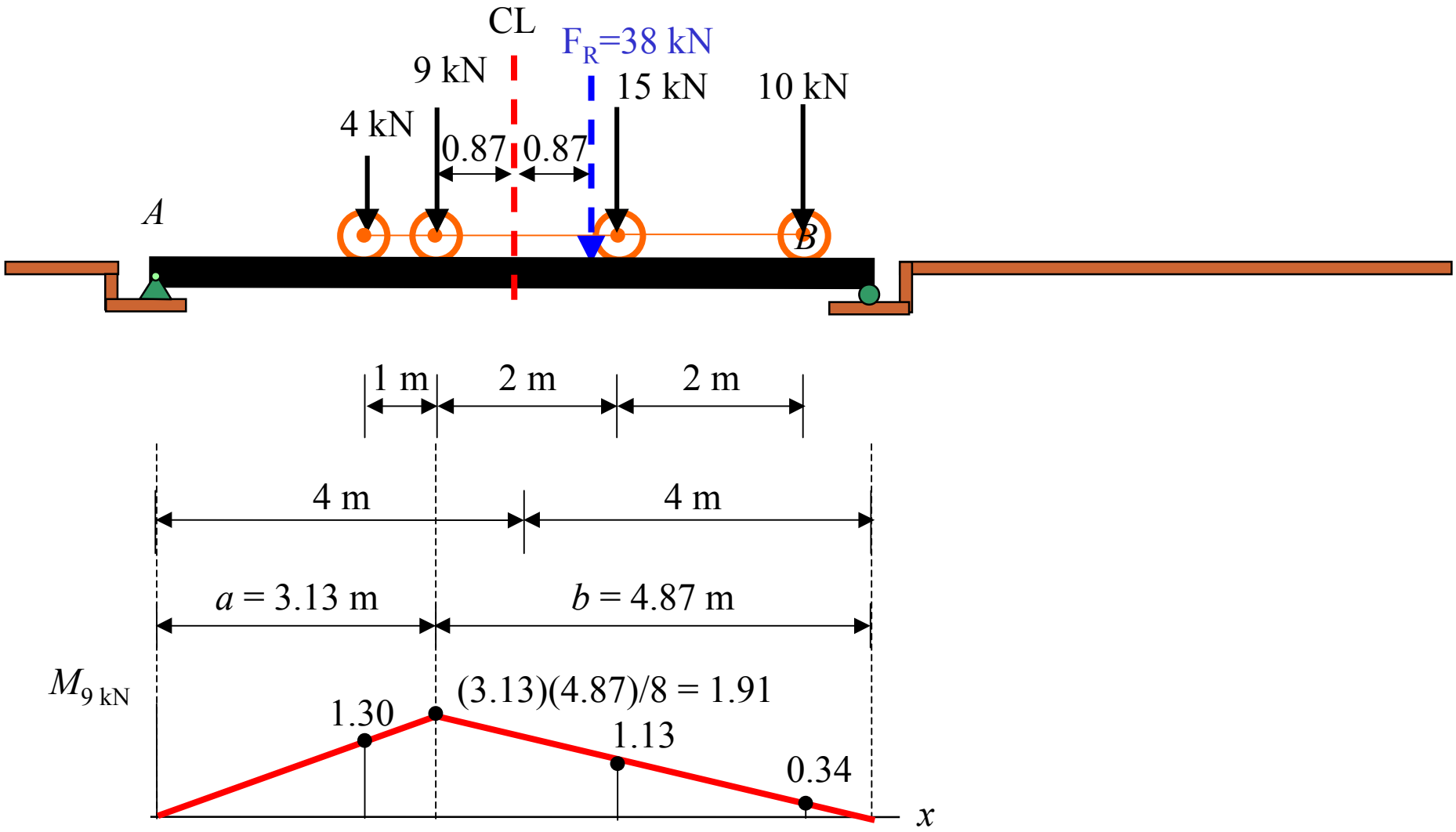
SOLUTION



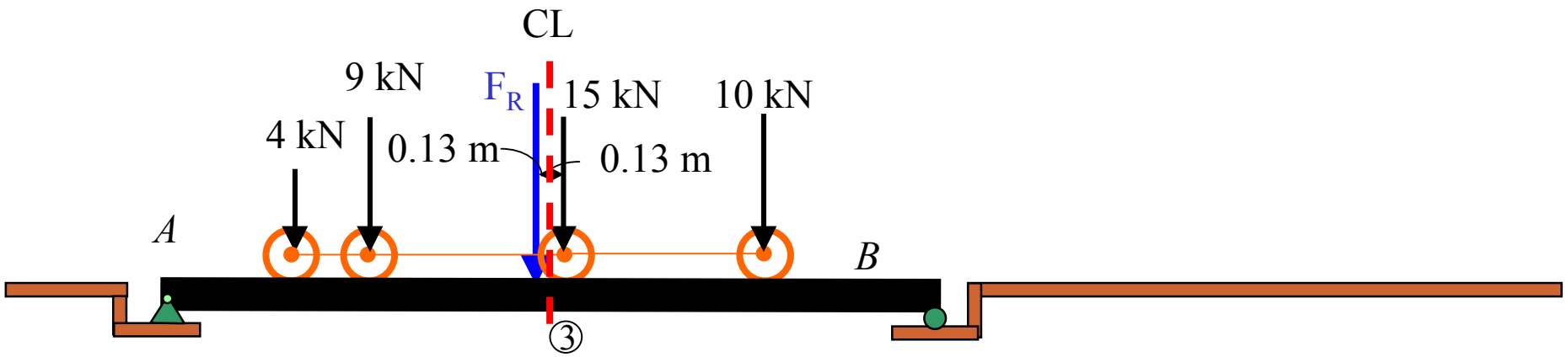
$$\Sigma M_{@\,4\text{ kN}} = 0:$$

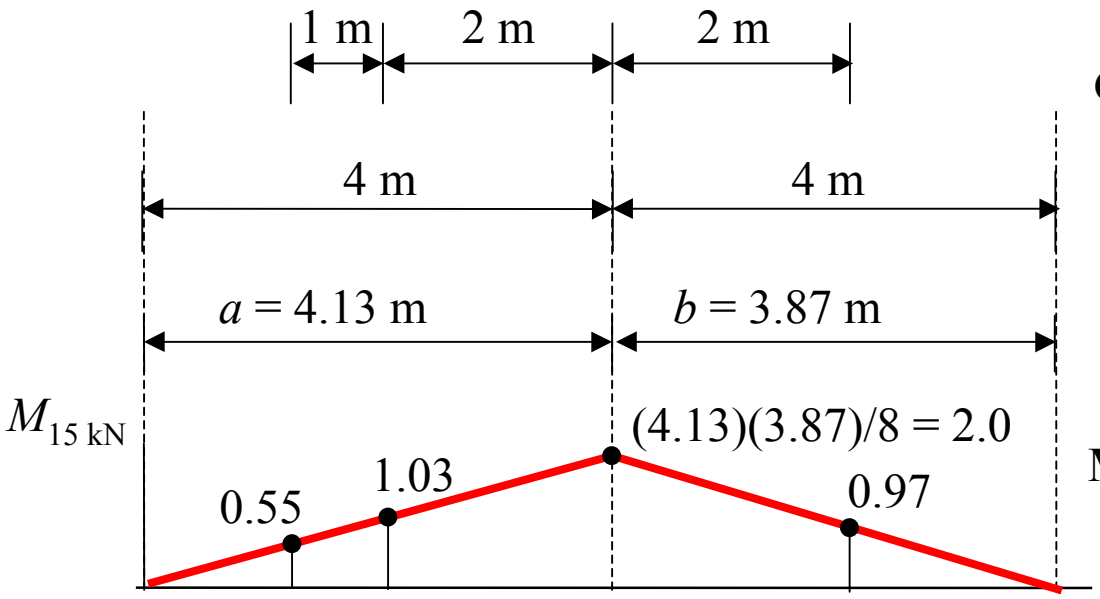
$$38\bar{x} = 4(0) + 9(1) + 15(3) + 10(5)$$

$$\bar{x} = 2.74\ m$$



$M_S = 4(1.30) + 9(1.91) + 15(1.13) + 10(0.34) = 42.74$ kN•m



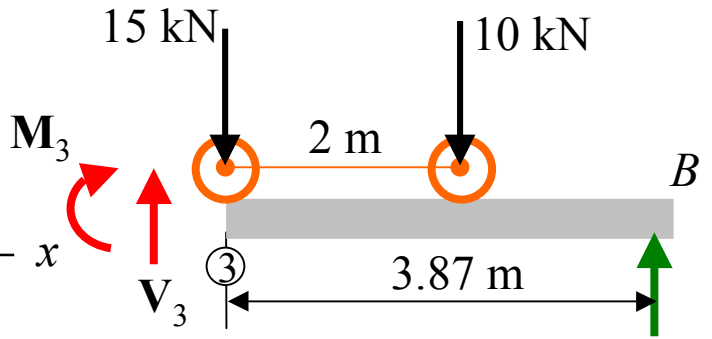


Or using equilibrium conditions:

Global: $+\circlearrowleft \Sigma M_A = 0$:

$$-38(3.87) + B_y(8) = 0$$

$$B_y = 18.38 \text{ kN}$$



$+\circlearrowleft \Sigma M_3 = 0$:

$$-M_3 - 10(2) + 18.38(3.87) = 0$$

$$M_3 = 51.13 \text{ kN•m}$$

$$M_S = 4(0.55) + 9(1.03) + 15(2.0) + 10(0.97) = 51.17 \text{ kN•m}$$

By comparison, $M_{max} = 51.17$ kN•m