INTEGRAL

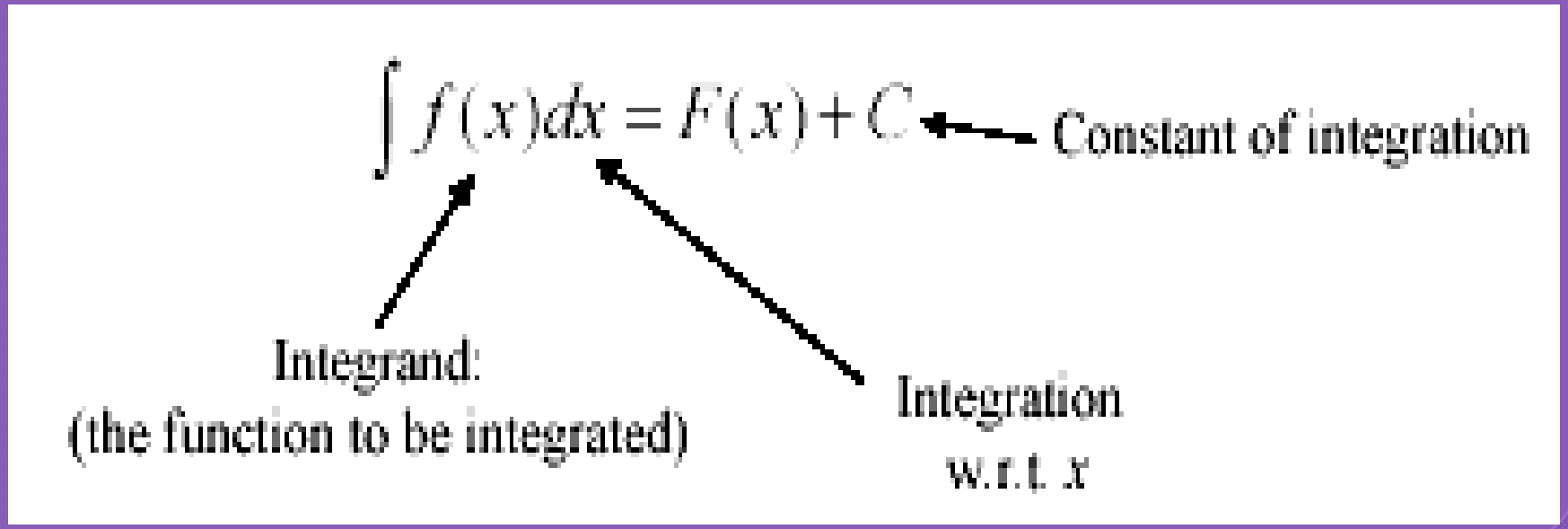
INDEFINITE INTEGRAL
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Pendahuluan

- Integral dikenal dua macam pengertian yaitu :
  a. Integral tak tentu (*Indefinite Integral*)
  b. Integral tentu (*Definite Intergral*)

Integral Tak Tentu (*Indefinite Integral*)

- Suatu konsep yang berhubungan dengan proses penemuan suatu fungsi asal, apabila turunan atau derivatif dari fungsinya diketahui
- Bentuk umum integral tak tentu:



Aturan Integral Tak Tentu

- **Rule 1**: Power rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

- **Rule 1'**: generalised "power rule"/ substitution rule:

$$\int [f(x)]^n \, f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad n \neq -1$$

Contoh

$$\int 3\,dx = 3x + c$$

$$\int x\,dx = \frac{1}{2}x^2 + c$$

$$\int (2x+2)^3 (2)\,dx = \frac{(2x+2)^4}{4} + C$$

Latihan

$$-\int x^3 dx =$$

$$-\int \sqrt{x^3}\,dx =$$

$$-\int \frac{1}{x^4}\,dx =$$

Aturan Integral Tak Tentu

- **Rule 2** : Integral of a constant multiple :

$$\int kf(x)dx = k\int f(x)dx$$

- **Contoh :**

$$-\int 1dx = \frac{1}{0+1}x^{0+1} + c = x + c$$

$$-\int 6dx = 6\frac{1}{0+1}x^{0+1} + c = 6x + c$$

Aturan Integral Tak Tentu

- **Rule 3** : Integral of a sum:

$$\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$$

- **Contoh** :

$$\int (3+2x)dx = \int 3dx + \int 2xdx = 3x + x^2 + c$$

$$\int 3xdx = 3\int xdx = 3\left(\frac{1}{2}x^2\right) = \frac{3}{2}x^2$$

$$\int (3x+2x^2)dx = \int 3xdx + \int 2x^2dx = \frac{3}{2}x^2 + \frac{2}{3}x^3 + c$$

Latihan

$$-\int (x^3 + x + 1)\,dx =$$

$$-\int (e^x + \frac{1}{x})\,dx =$$

Aturan Integral Tak Tentu

- **Rule 4** : Exponential rule:

$$\int e^x dx = e^x + C$$

- **Rule 4'**: Generalised Exponential rule:

$$\int e^{f(x)} f'(x) dx = e^{f(x)} + C$$

Contoh

$$\int e^4 dx = e^4 + c$$

$$\int e^{2x+5} dx = \frac{e^{2x+5}}{2} + c = \frac{1}{2} e^{2x+5} + c$$

Latihan

$$-\int e^4 dx =$$

$$-\int e^{x+1}) dx =$$

Aturan Integral Tak Tentu

- **Rule 5** : "aturan logaritma:

$$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$$

- **Rule 5'**: sering ditulis :

$$\int \frac{f'(x)}{f(x)} dx = \ln[f(x)] + C, \quad f(x) > 0$$

Contoh

$$\int \frac{1}{x} dx = \ln x + C$$

$$\int 2x^{-1} dx = 2\int \frac{1}{x} dx = 2.\ln x + c$$

Aturan Integral Tak Tentu

- Rule 6 : Integral Perkalian

$$\int kf(x)dx = k\int f(x)dx$$

- Contoh:

$$\int 2x^2 dx = 2\int x^2 dx = 2(\frac{x^3}{3} + c) = \frac{2}{3}x^3 + c$$

$$\int -f(x)dx = -\int f(x)dx$$

- Latihan: $\int (5e^x - \frac{1}{x^2} + \frac{3}{x})dx =$

Aturan Integral Tak Tentu

- Rule 7: Aturan Subtitusi

$$\int f(u)\frac{du}{dx}dx = \int f(u)du = f(u) + c$$

$$\frac{d}{dx} f(u) = \frac{d}{du} f(u)\frac{du}{dx} = f'(u)\frac{du}{dx}$$

Contoh

Cara Langsung :

$$cari \int 2x(x^2+1)dx$$

$$\int 2x(x^2+1)dx = \int (2x^3+2x)dx = \frac{x^4}{2} + x^2 + c$$

Cara Subtitusi

Dengan cara subtitusi ; misal u = x^2 + 1, maka

du/dx =2x atau dx=du/2x

Subtitusi du/2x untuk dx akan menghasilkan

$$\int 2x(x^2+1)dx = \int 2x.u\frac{du}{2x} = \int u.du = \frac{u^2}{2} + c_1$$

$$= \frac{1}{2}(x^4+2x^2+1) + c_1 = \frac{1}{2}x^4 + x^2 + c$$

Latihan

1. $\int 6x^2(x^3+2)^9\,dx =$

2. $\int 8e^{2x+3}dx =$

Integration by parts

- Consider two continuous functions *u=f(x)* and *v=g(x)*, then,

$$\frac{d}{dx}[f(x).g(x)] = f(x).g'(x) + f'(x).g(x)$$

Let us assume that $u = f(x)$ and $v = g(x)$

Then,

$$d(uv) = u.dv + du.v$$

Let us integrate both sides

$$uv = \int u.dv + \int v.du$$

Rearranging:

$$\int u.dv = uv - \int v.du$$

Examples

Using integration by parts, find $\int 3dx$

$u = 3, dv = dx$

$du = 0, v = x$

$\int u.dv = uv - \int v.du$

$\int 3.dx = 3x - \int x.0 = 3x + c$

Examples

Using integration by parts, find $\int (2x+3)(2x)$

$u = (2x+3), dv = 2x$

$du = 2, v = \int 2x = x^2$

$\int u.dv = uv - \int v.du$

$\int (2x+3).(2x) = (2x+3)(x^2) - \int x^2.2$

$= (2x^3 + 3x^2) - 2\int x^2$

$= 2x^3 + 3x^2 - \frac{2}{3}x^3 = \frac{4}{3}x^3 + 3x^2 + c$

Examples

Verification $\int (2x+3)(2x) = \int (4x^2 + 6x)dx$

$$= \int 4x^2 dx + \int 6x\ dx = \frac{4}{3}x^3 + \frac{6}{2}x^2 + c = \frac{4}{3}x^3 + 3x^2 + c$$

Integral Tertentu (*Definite Integral*)

- Integral dari suatu fungsi yang nilai-nilai variabel bebasnya memiliki batas tertentu
- Integral tertentu sering digunakan untuk menghitung luas area yang terletak diantara kurva y=f(x) dan sumbu x dalam suatu rentangan wilayah yang dibatasi oleh x=a dan x=b
- Proses pencarian luas suatu area yang batas-batas atau limit dari area tersebut sudah ditentukan
- Dalam integral tak tentu kita temukan bahwa :

$$\int f(x)dx = F(x) + c$$

Integral Tertentu (*Definite Integral*)

- Untuk mengetahui hasil integrasi teresbut untuk suatu rentangan wilayah tertentu, katakanlah antara x = a dan x = b, dimana a<b, maka persamaan diatas menjadi :
- [F(b)+c] – [F(a)+c] = F(b) – F(a)
- F(b) – F(a) : hasil integral tertentu dari f(x) antara a dan b.
- Secara lengkap persamaan pertama tadi dapat dituliskan menjadi :

$$\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

Integral Tertentu (*Definite Integral*)

- Notasi $\int_a^b f(x)dx$ dibaca integral f(x) untuk rentangan wilayah x dari a ke b. selanjutnya, mengingat a<b, a dinamakan batas bawah integrasi (lower limit integration) sedangkan b disebut batas atas integrasi (upper limit integration)

Teorema Kalkulus Fundamental (Fundamental Theorem of Calculus)

- Jika suatu fungsi f(x) adalah kontinu dalam suatu interval, maka fungsi tersebut mempunyai integral (antiderivatif) dalam intervalnya dan lebih lanjut jika F(x) adalah integral dari f(x) maka untuk 2 titik a dan b dalam interval kita dapatkan :

$$\int_a^b f(x)dx = F(b) - F(a)$$

Kaidah atau Aturan Integrasi tertentu

$$1.\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

$$\int_2^5 x^4 dx = \left[\frac{1}{5}x^5\right]_2^5 = \frac{1}{5}(5)^5 - \frac{1}{5}(2)^5 = 625 - 6{,}4 = 618{,}6$$

$$2.\int_a^a f(x)dx = 0$$

$$\int_2^2 X^4 = \left[\frac{1}{5}x^5\right]_2^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(2)^5 = 0$$

$$3.\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_2^5 x^4 dx = -\int_5^2 x^4 = -\left[\frac{1}{5}x^5\right]_5^2 = -\left[\frac{1}{5}(2)^5 - \frac{1}{5}(5)^5\right] = -(6{,}4 - 625) = 618{,}6$$

Kaidah atau Aturan Integrasi tertentu

$$4.\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_2^5 5x^4\,dx = 5\int_4^5 x^4 = 5\left[\frac{1}{5}x^5\right]_2^5 = 5\left[\frac{1}{5}(5)^5 - \frac{1}{5}(2)^5\right] = 5(618{,}6) = 3093$$

$$5.\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)\,dx + \int_a^b g(x)dx$$

$$\int_2^5 (X^4+5X^4) = \int_2^5 (x^4)dx + \int_2^5 5X^4\,dx = \left[\frac{1}{5}x^5\right]_2^5 + \left[x^5\right]_2^5 = \left[\left(\frac{1}{5}(5)^5\right) - \left(\frac{1}{5}(2)^5\right)\right] + \left[(5)^5 - (2)^5\right] = 3711{,}6$$

$$6.\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

$$\int_2^3 x^4\,dx + \int_3^5 x^4\,dx = \left[\frac{1}{5}x^5\right]_2^3 + \left[\frac{1}{5}x^5\right]_3^5 =$$