APPENDIX

B

FOURIER SERIES

In this appendix we give Fourier coefficients for the exponential form of the Fourier series expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

Thus $x(t)$ is a periodic signal having period T, $\omega_0 = 2\pi/T$ is the fundamental angular frequency of the Fourier series for $x(t)$, and X_n is the complex amplitude of the nth harmonic in the Fourier series for $x(t)$, given by

$$X_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jn\omega_0 t}\,dt$$

To express a Fourier series in amplitude-phase form as

$$x(t) = \sum_{n=0}^{\infty} A_n \cos(n\omega_0 t + \theta_n)$$

use the relations

$$A_0 = |X_0| \qquad \theta_0 = \angle X_0$$

$$A_n = 2|X_n| \qquad \theta_n = \angle X_n \qquad n \neq 0$$

To express a Fourier series in quadrature form as

$$x(t) = \sum_{n=0}^{\infty} (I_n \cos n\omega_0 t + Q_n \sin n\omega_0 t)$$

use the relations

$$I_0 = X_0 \qquad Q_0 = 0$$

$$I_n = 2 \operatorname{Re} X_n \qquad Q_n = -2 \operatorname{Im} X_n \qquad n > 0$$

Rectangular pulse train See Fig. B-1:

$$X_n = \frac{a\tau}{T}\,\text{sa}\,\frac{n\pi\tau}{T}$$

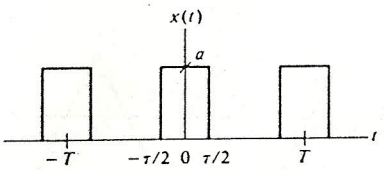


Figure B-1 Rectangular pulse train.

Triangular pulse train See Fig. B-2:

$$X_n = \frac{a\tau}{2T}\,\text{sa}^2\,\frac{n\pi\tau}{2T}$$

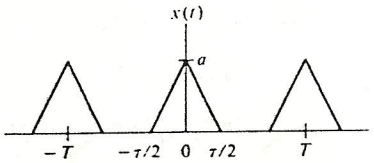


Figure B-2 Triangular pulse train.

Sawtooth pulse train See Fig. B-3:

$$X_0 = \frac{a\tau}{2T} \qquad X_n = \frac{a}{j2\pi n}e^{-jn\pi\tau/T}\left(\text{sa}\,\frac{n\pi\tau}{T} - e^{-jn\pi\tau/T}\right) \qquad n \neq 0$$

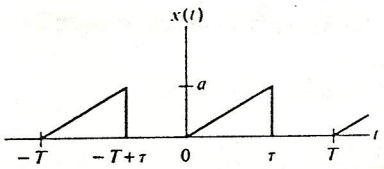


Figure B-3 Sawtooth pulse train.

Fractional sinusoid See Fig. B-4:

$$X_n = \frac{b\tau}{2T}\left[\text{sa}\,\frac{(n+1)\pi\tau}{T} + \text{sa}\,\frac{(n-1)\pi\tau}{T}\right] - \frac{(b-a)\tau}{T}\,\text{sa}\,\frac{n\pi\tau}{T}$$

where

$$b = \frac{a}{1 - \cos(\pi\tau/T)}$$

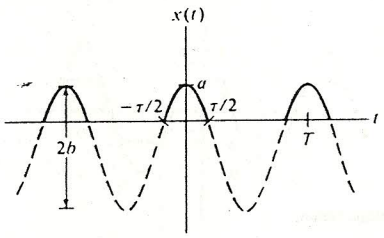


Figure B-4 Fractional sinusoid.

Impulse train See Fig. B-5:

$$X_n = \frac{a}{T}$$

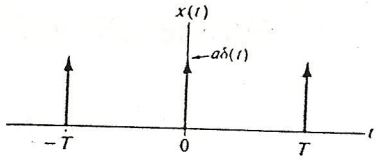


Figure B-5 Impulse train.

Half-wave-rectified sinusoid See Fig. B-6:

$$X_n = \frac{a}{4}\left[\text{sa}\,\frac{(n+1)\pi}{2} + \text{sa}\,\frac{(n-1)\pi}{2}\right]$$

Note that $X_n = 0$ for $n = \pm 3, \pm 5, \pm 7, \ldots$.

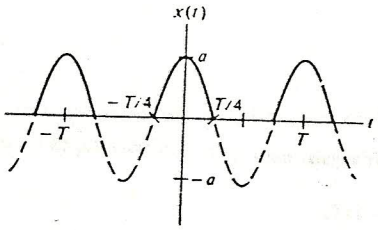


Figure B-6 Half-wave rectified sinusoid.

Full-wave-rectified sinusoid See Fig. B-7:

$$X_n = \frac{a[1 + (-1)^n]}{4}\left[\text{sa}\,\frac{(n+1)\pi}{2} + \text{sa}\,\frac{(n-1)\pi}{2}\right]$$

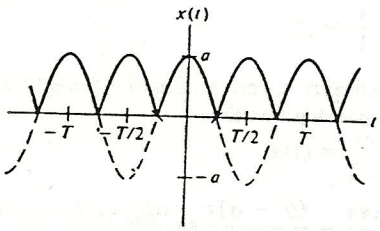


Figure B-7 Full-wave rectified sinusoid.

Square wave See Fig. B-8:

$$X_0 = 0 \qquad X_n = \frac{a}{2}\,\text{sa}\,\frac{n\pi}{2} \qquad n \neq 0$$

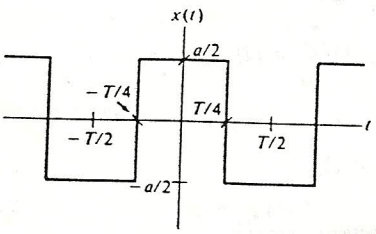


Figure B-8 Square wave.

Triangular wave See Fig. B-9:

$$X_0 = 0 \qquad X_n = \frac{a}{2}\,\mathrm{sa}^2\frac{n\pi}{2} \qquad n \neq 0$$

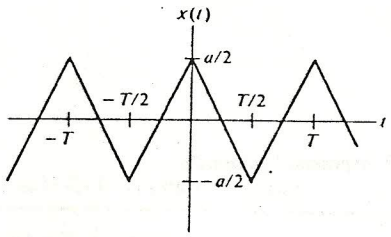


Figure B-9 Triangular wave.

Sawtooth wave See Fig. B-10:

$$X_0 = 0 \qquad X_n = \frac{ja(-1)^n}{2n\pi} \qquad n \neq 0$$

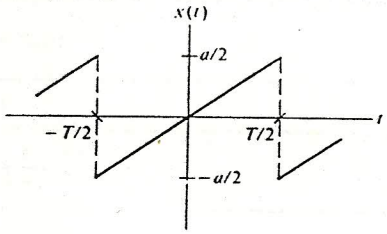


Figure B-10 Sawtooth wave.

Clipped sinusoid See Fig. B-11:

$$X_0 = 0 \qquad X_{\pm 1} = \frac{b}{2} - \frac{b\tau}{T}\left(\mathrm{sa}\,\frac{2\pi\tau}{T} + 1\right) + \frac{2a\tau}{T}\,\mathrm{sa}\,\frac{\pi\tau}{T}$$

$$X_n = [(-1)^n - 1]\left\{\frac{b\tau}{2T}\left[\mathrm{sa}\,\frac{(n+1)\pi\tau}{T} + \mathrm{sa}\,\frac{(n-1)\pi\tau}{T}\right] - \frac{a\tau}{T}\,\mathrm{sa}\,\frac{n\pi\tau}{T}\right\}$$

$$n = \pm 2, \pm 3, \ldots$$

where a = peak amplitude after clipping
b = peak amplitude before clipping
τ = duration of clipping = $(T/\pi)\cos^{-1}(a/b)$

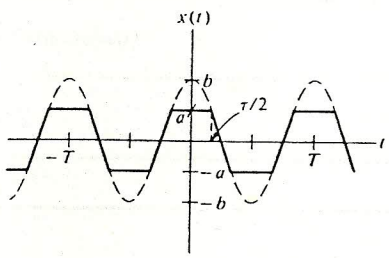


Figure B-11 Clipped sinusoid.

Exponential pulse train See Fig. B-12:

$$X_n = \frac{a(1 - e^{-T/\tau})}{T/\tau + j2\pi n}$$

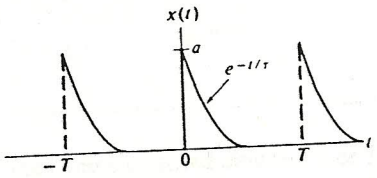


Figure B-12 Exponential pulse train.

APPENDIX

C

FOURIER TRANSFORMATION

Table C-1 Transform pairs

$e(t) = \mathcal{F}^{-1}\{E(j\omega)\}$	$E(j\omega) = \mathcal{F}\{e(t)\}$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega}$ (see Sec. 4-3H)
$e^{-t/\tau}u(t)$	$\frac{\tau}{1 + j\omega\tau}$
1	$2\pi\delta(\omega)$
$r\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sa}\frac{\omega\tau}{2}e^{-j\omega\tau/2}$
$\frac{t}{\tau}e^{-t/\tau}u(t)$	$\frac{\tau}{(1 + j\omega\tau)^2}$
$\cos(\omega_0 t + \theta)$	$\pi e^{j\theta}\delta(\omega - \omega_0) + \pi e^{-j\theta}\delta(\omega + \omega_0)$
$(1 - e^{-t/\tau})u(t)$	$\frac{1}{j\omega(1 + j\omega\tau)}$
$(e^{-t/\tau_1} - e^{-t/\tau_2})u(t)$	$\frac{\tau_1 - \tau_2}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$
$[1 + \beta^{-1}e^{-\zeta\omega_0 t}\sin(\beta\omega_0 t - \theta)]u(t)$ $\beta = \sqrt{1 - \zeta^2}$ $\theta = \operatorname{Tan}^{-1}(\beta, -\zeta)$	$\frac{1}{j\omega[(j\omega/\omega_0)^2 + 2\zeta j\omega/\omega_0 + 1]}$
$e^{-t/\tau}(\sin\omega_0 t)u(t)$	$\frac{\omega_0\tau^2}{(1 + j\omega\tau)^2 + (\omega_0\tau)^2}$
$e^{-\lvert t\rvert/\tau}$	$\frac{2\tau}{1 + (\omega\tau)^2}$
$e^{-t^2/2\tau^2}$	$\tau\sqrt{2\pi}e^{-(\omega\tau)^2/2}$

Table C-2 Operational properties

$e(t) = \mathcal{F}^{-1}\{E(j\omega)\}$	$E(j\omega) = \mathcal{F}\{e(t)\}$
$\int_{-\infty}^{\infty} e_1(\lambda)e_2(t-\lambda)\,d\lambda$	$E_1(j\omega)E_2(j\omega)$
$e(t)e^{j\omega_c t}$	$E(j\omega - j\omega_c)$
$e(t-t_0)$	$E(j\omega)e^{-j\omega t_0}$
$e(\eta t)$	$\frac{1}{\lvert\eta\rvert}E\left(\frac{j\omega}{\eta}\right)$
$\frac{d^n e(t)}{dt^n}$	$(j\omega)^n E(j\omega)$
$\int_{-\infty}^{t} e(\lambda)\,d\lambda$	$\frac{1}{j\omega}E(j\omega)$
$\int_{-\infty}^{\infty} \lvert e(t)\rvert^2\,dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert E(j\omega)\rvert^2\,d\omega$
$e_1(t)e_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} E_1(j\lambda)E_2(j\omega - j\lambda)\,d\lambda$
$e(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{T}\sum_{n=-\infty}^{\infty}E\left(j\omega - j\frac{2n\pi}{T}\right)$