The transfer function from θ_1 to θ_2 is N_1/N_2. Thus the block diagram of the motor driving a load through a gear train is as shown in Figure 3.8(a). We remark that because of the loading, it is not possible to draw a block diagram as shown in Figure 3.8(b).

3.4 TRANSDUCERS

Transducers are devices that convert signals from one form to another—for example, from a mechanical shaft position, temperature, or pressure to an electrical voltage. They are also called *sensing devices*. There are all types of transducers such as thermocouples, strain gauges, pressure gauges, and others. We discuss in the following only potentiometers and tachometers.

Potentiometers

The potentiometer is a device that can be used to convert a linear or angular displacement into a voltage. Figure 2.14 shows a wire-wound potentiometer with its characteristic. The potentiometer converts the angular displacement $\theta(t)$ into an electric voltage $v(t)$ described by

$$v(t) = k\theta(t) \tag{3.30}$$

where k is a constant and depends on the applied voltage and the type of potentiometer used. The application of the Laplace transform to (3.30) yields

$$V(s) = k\Theta(s) \tag{3.31}$$

Thus the transfer function of the potentiometer is a constant. Figure 3.9 shows three commercially available potentiometers.

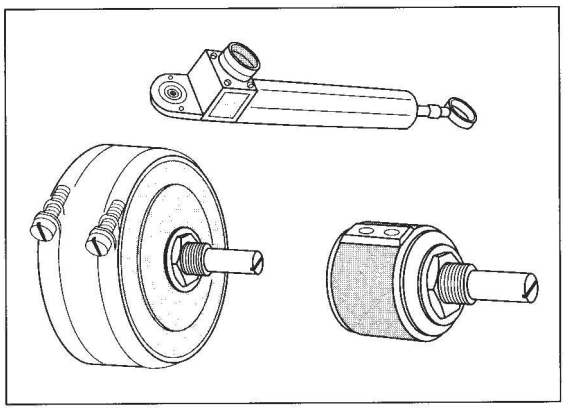


Figure 3.9 Potentiometers.

Tachometers

The tachometer is a device that can convert a velocity into a voltage. It is actually a generator with its rotor connected to the shaft whose velocity is to be measured. Therefore a tachometer is also called a tachogenerator. The output $v(t)$ of the tachometer is proportional to the shaft's angular velocity; that is,

$$v(t) = k\frac{d\theta(t)}{dt} \tag{3.32}$$

where $\theta(t)$ is the angular displacement and k is the sensitivity of the tachometer, in volts per radian per second. The application of the Laplace transform to (3.32) yields

$$G(s) = \frac{V(s)}{\Theta(s)} = ks \tag{3.33}$$

Thus the transfer function from $\theta(t)$ to $v(t)$ of the tachometer is ks.

As discussed in Section 2.4.1, improper transfer functions will amplify high-frequency noise and are not used in practice. The transfer function of tachometers is improper, therefore its employment must be justified. A tachometer is usually attached to a shaft—for example, the shaft of a motor as shown in Figure 3.10(a). Although the transfer function of the tachometer is improper, the transfer function from u to y_1 is

$$\frac{k_m}{s(\tau_m s + 1)} \cdot ks = \frac{kk_m}{\tau_m s + 1}$$

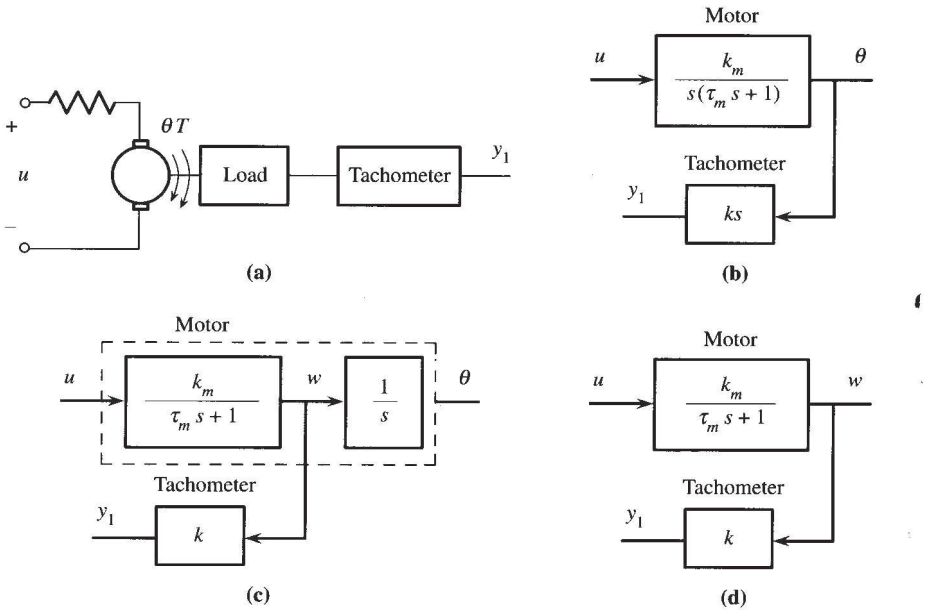


Figure 3.10 Use of tachometer.

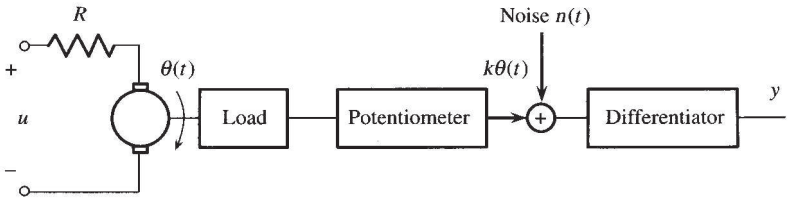


Figure 3.11 Unacceptable way of generating velocity signal.

as shown in Figure 3.10(b). It is strictly proper. Thus electrical noise entered at the armature circuit will not be amplified. The transfer function from motor torque $T(t)$ to y_1 is, from Figure 3.2,

$$\frac{1}{s(Js + f)} \cdot ks = \frac{k}{Js + f}$$

which is again strictly proper. Thus, mechanical noise, such as torque generated by gusts, is smoothed by the moment of inertia of the motor. In conclusion, tachometers will not amplify electrical and mechanical high-frequency noises and are widely used in practice. See also Problem 3.17. Note that the block diagram in Figure 3.10(b) can also be plotted as shown in Figure 3.10(c). This arrangement is useful in computer computation and operational amplifier circuit realization, as will be discussed in Chapter 5. We mention that the arrangement shown in Figure 3.11 cannot be used to measure the motor angular velocity. Although the potentiometer generates signal $k\theta(t)$, it also generates high-frequency noise $n(t)$ due to brush jumps, wire irregularities, and variations of contact resistance. The noise is greatly amplified by the differentiator and overwhelms the desired signal $kd\theta/dt$. Thus the arrangement cannot be used in practice.

The transfer function of a tachometer is ks only if its input is displacement. If its input is velocity $w(t) = d\theta(t)/dt$, then its transfer function is simply k. In velocity control systems, the transfer function of motors is $k_m/(\tau_m s + 1)$ as shown in Figure 3.4(b). In this case, the block diagram of a motor and a tachometer is as shown in Figure 3.10(d). Therefore, it is important to specify what are the input and output of each block.

Error Detectors

Every error detector has two input terminals and one output terminal. The output signal is proportional to the difference of the two input signals. An error detector can be built by connecting two potentiometers as shown in Figure 3.12(a). The two potentiometers may be located far apart. For example, one may be located inside a room and the other, attached to an antenna on the rooftop. The input signals θ_r and θ_o are mechanical positions, either linear or rotational; the output $v(t)$ is a voltage signal. They are related by

$$v(t) = k[\theta_r(t) - \theta_o(t)] \tag{3.34}$$

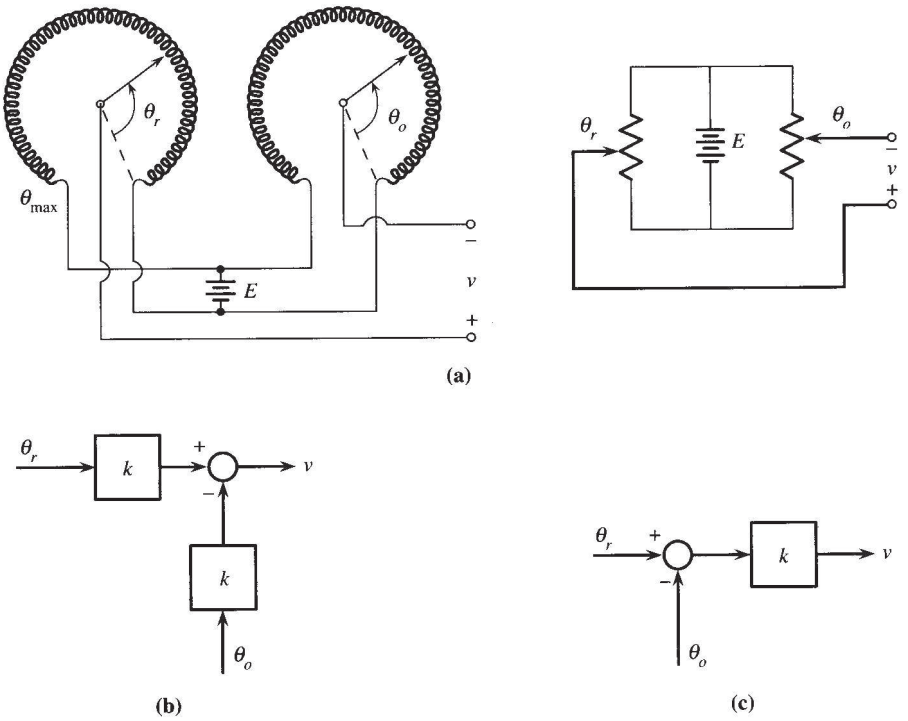


Figure 3.12 Pair of potentiometers and their schematic representations.

or

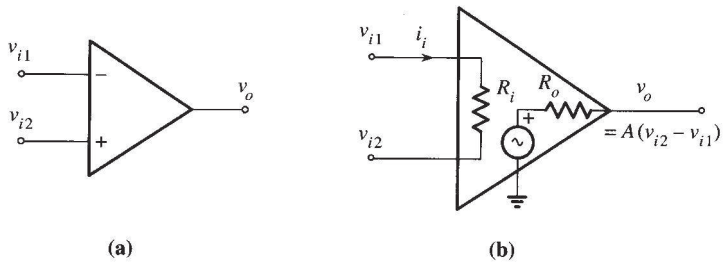
$$V(s) = k[\Theta_r(s) - \Theta_o(s)]$$

where k is a constant. The pair of potentiometers can be represented schematically as shown in Figure 3.12(b) or (c). The circle where the two signals enter is often called the *summing point.*

3.5 OPERATIONAL AMPLIFIERS (OP-AMPS)

The operational amplifier is one of the most important circuit elements. It is built in integrated-circuit form. It is small, inexpensive, and versatile. It can be used to build buffers, amplifiers, error detectors, and compensating networks, and is therefore widely used in control systems.

The operational amplifier is usually represented as shown in Figure 3.13(a) and is modeled as shown in Figure 3.13(b). It has two input terminals. The one with a "$-$" sign is called the inverting terminal and the one with a "$+$" sign the non-inverting terminal. The output voltage v_o equals $A(v_{i2} - v_{i1})$, and A is called the *open-loop gain.* The resistor R_i in Figure 3.13(b) is called the *input resistance* and



(a) (b)

Figure 3.13 Operational amplifier.

R_o, the *output resistance*. R_i is generally very large, greater than $10^4\ \Omega$, and R_o is very small, less than 50 Ω. The open-loop gain A is very large, usually over 10^5, in low frequencies. Signals in op-amps are limited by supply voltages, commonly $\pm$ 15 V. Because of this limitation, if A is very large or infinity, then we have

$$v_{i1} = v_{i2} \tag{3.35}$$

This equation implies that the two input terminals are virtually short-circuited. Because R_i is very large, we have

$$i_i = 0 \tag{3.36}$$

This equation implies that the two input terminals are virtually open-circuited. Thus the two input terminals have the two conflicting properties: open-circuit and short-circuit. Using these two properties, operational amplifier circuits can easily be analyzed.

Consider the operational amplifier circuit shown in Figure 3.14(a). Because of the direct connection of the output terminal and inverting terminal, we have $v_{i1} = v_o$. Thus we have, using the short-circuit property, $v_o = v_{i2}$. It means that the output voltage is identical to the input voltage v_{i2}. One may wonder why we do not connect v_o directly to v_{i2}, rather than through an op-amp. There is an important reason for doing this. The input resistance of op-amps is very large and the output resistance is very small, so op-amps can isolate the circuits before and after them, and thus eliminate the loading problem. Therefore, the circuit is called a *voltage follower*, *buffer*, or *isolating amplifier* and is widely used in practice.

Consider the circuit shown in Figure 3.14(b) where Z_1 and Z_f are two impedances. The open-circuit property $i_i = 0$ implies $i_1 = -i_o$. Thus we have[4]

$$\frac{V_1 - V_{i_1}}{Z_1} = -\frac{V_o - V_{i_1}}{Z_f} \tag{3.37}$$

[4]Because impedances are defined in the Laplace transform domain (see Section 2.4), all variables must be in the same domain. We use V_i to denote the Laplace transform of v_i.

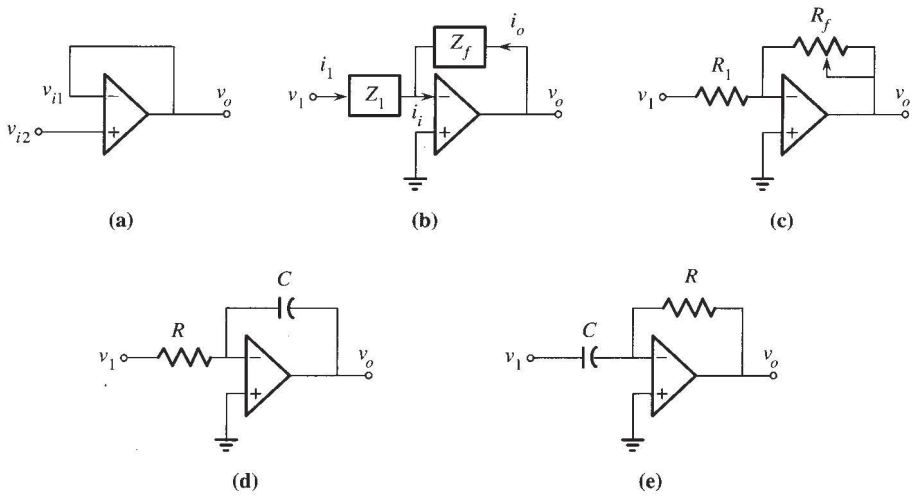


Figure 3.14 Op-amp circuits.

Because the noninverting terminal is grounded and because of the short-circuit property, we have $v_{i_1} = v_{i_2} = 0$. Thus (3.37) becomes

$$\frac{V_o}{V_1} = -\frac{Z_f}{Z_1} \tag{3.38}$$

If $Z_f = R_f$ and $Z_1 = R_1$, then the transfer function from v_1 to v_o is $-R_f/R_1$ and the circuit can be used as an amplifier with fixed gain $-R_f/R_1$. If R_f is replaced by a potentiometer or an adjustable resistor as shown in Figure 3.14(c), then the gain of the amplifier can be easily adjusted.

Exercise 3.5.1

Show that the transfer function of the circuit in Figure 3.14(d) equals $-1/RCs$. Thus, the circuit can act as an integrator. Show that the transfer function of the circuit in Figure 3.14(e) equals $-RCs$. Thus, the circuit can act as a pure differentiator. Integrators and differentiators can be easily built by using operational amplifier circuits. However, differentiators so built may not be stable and cannot be used in practice. See Reference [18]. Integrators so built are stable and are widely used in practice.

Consider the op-amp circuit shown in Figure 3.15. The noninverting terminal is grounded, so $v_{i1} = v_{i2} = 0$. Because of $i_i = 0$, we have

$$i_f = -(i_1 + i_2 + i_3)$$

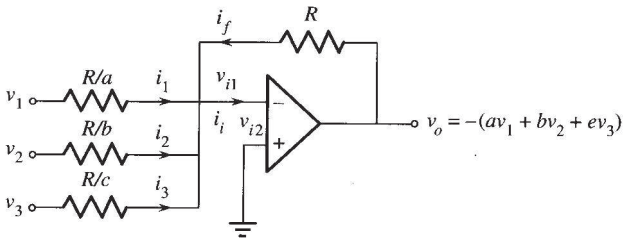


Figure 3.15 Op-amp circuit.

which together with $v_{i1} = 0$ implies

$$\frac{v_o}{R} = -\left(\frac{v_1}{R/a} + \frac{v_2}{R/b} + \frac{v_3}{R/c}\right)$$

Thus we have

$$v_o = -(av_1 + bv_2 + cv_3) \tag{3.39}$$

If $a = 1$ and $b = c = 0$, then (3.39) reduces to $v_o = -v_1$. It is called an *inverting amplifier* as shown in Figure 3.16(a). The op-amp circuit shown in Figure 3.16(b) can serve as an error detector. The output of the first op-amp is

$$e = -(r - v_w)$$

where v_w is the voltage generated by a tachometer. Note the polarity of the tachometer output. The output of the second op-amp, an inverting amplifier, is $u = -e$, thus we have $u = r - v_w$. This is an error detector. In conclusion, op-amp circuits are versatile and widely used. Because of their high input resistances and low output resistances, their connection will not cause the loading problem.

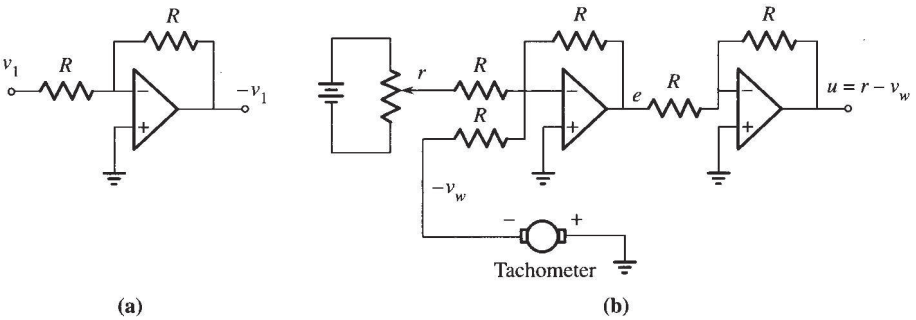


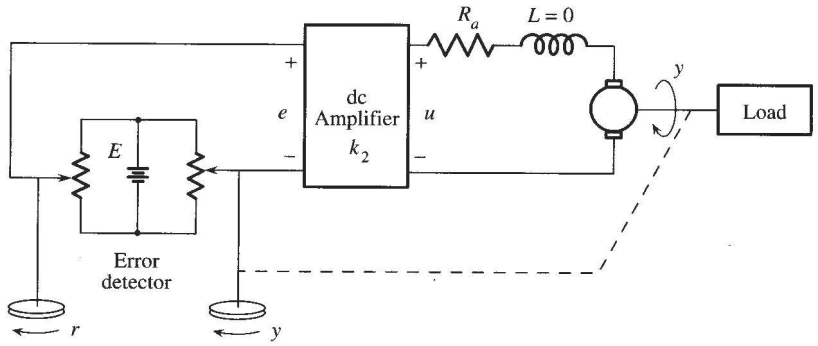
Figure 3.16 (a) Inverting amplifier. (b) Error detector.

3.6 BLOCK DIAGRAMS OF CONTROL SYSTEMS

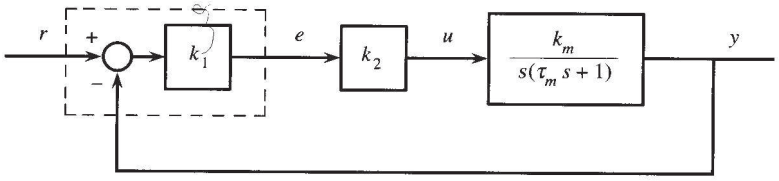
In this section, we show how block diagrams are developed for control systems.

Example 3.6.1

Consider the control system shown in Figure 3.17(a). The load could be a telescope or an antenna and is driven by an armature-controlled dc motor. The system is designed so that the actual angular position of the load will follow the reference signal. The error e between the reference signal r and the controlled signal y is detected by a pair of potentiometers with sensitivity k_1. The dotted line denotes mechanical coupling, therefore their signals are identical. The error e is amplified by a dc amplifier with gain k_2 and then drives the motor. The block diagram of this system is shown in Figure 3.17(b). The diagram is self-explanatory.



(a)



(b)

Figure 3.17 Position control system.

Example 3.6.2

In a steel or paper mill, the products are moved by rollers, as shown in Figure 3.18(a). In order to maintain a prescribed tension, the roller speeds are kept constant and equal to each other. This can be achieved by using the control system shown in Figure 3.18(b). Each roller is driven by an armature-controlled dc motor. The desired