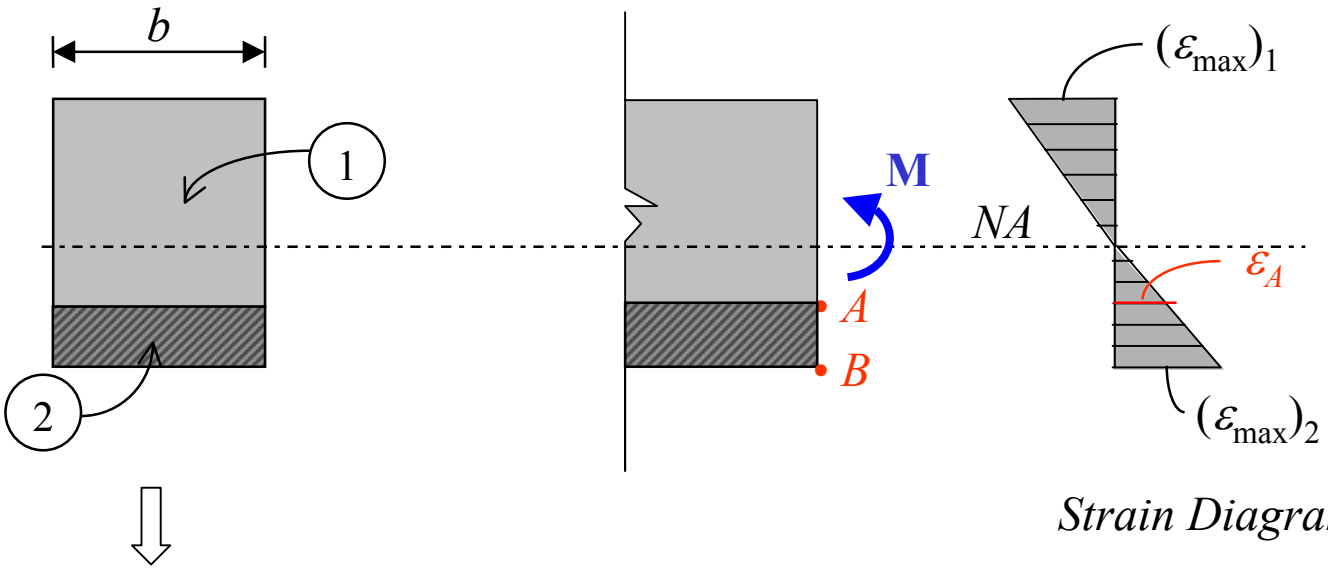
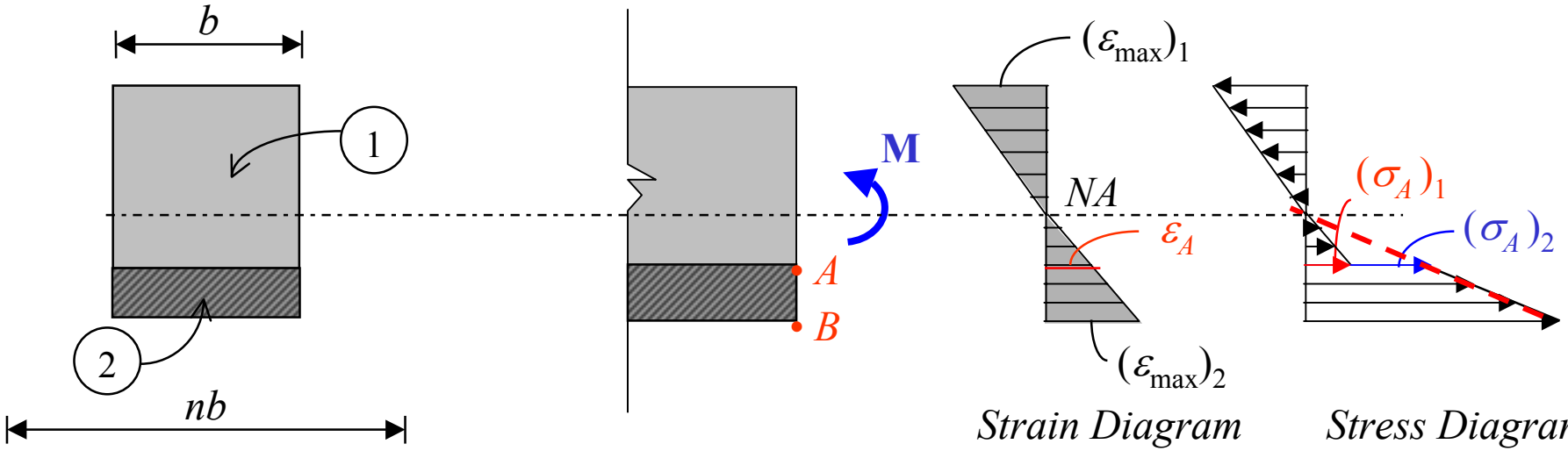
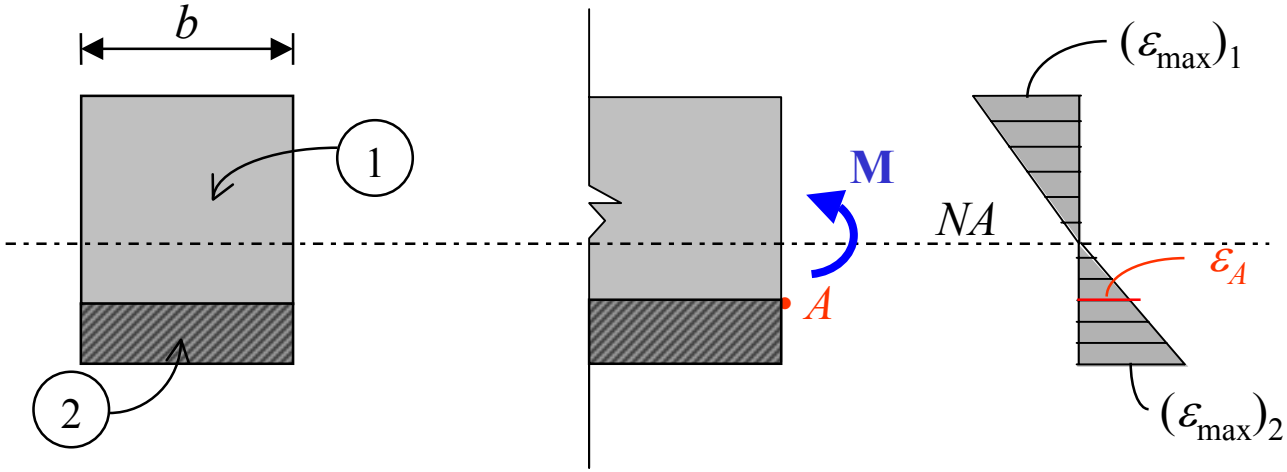
# Composite Beams




*Strain Diagram*




*Strain Diagram* *Stress Diagram*



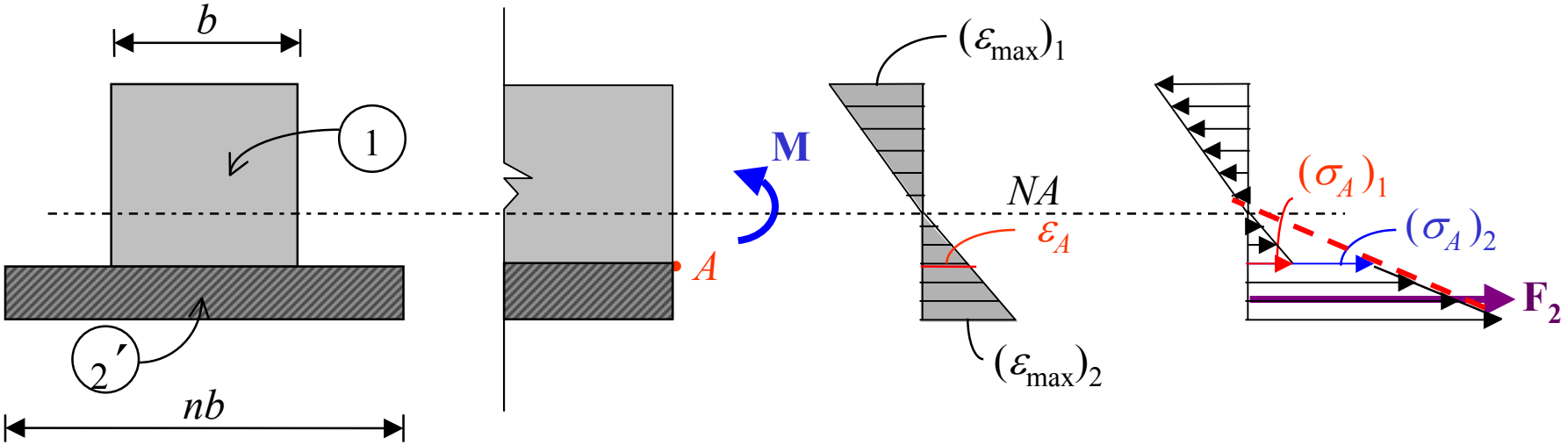

- *At the interface*

$$\varepsilon_A = (\varepsilon_A)_1 = (\varepsilon_A)_2$$

$$\frac{(\sigma_A)_1}{E_1} = \frac{(\sigma_A)_2}{E_2}$$

$$\frac{E_2}{E_1} = \frac{(\sigma_A)_2}{(\sigma_A)_1} = n$$

$$\boxed{\sigma_2 = n\sigma_1}$$




- *Force*

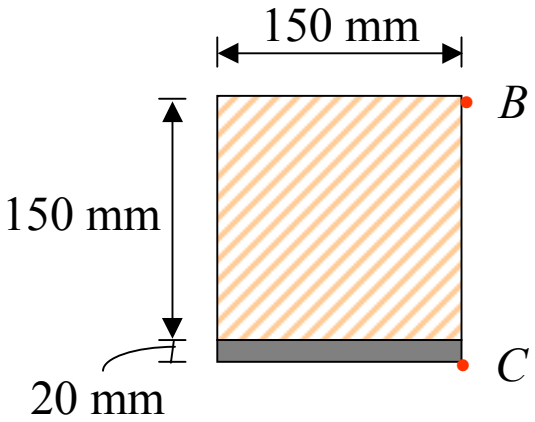
$$F_2 = \int \sigma_2 dA$$

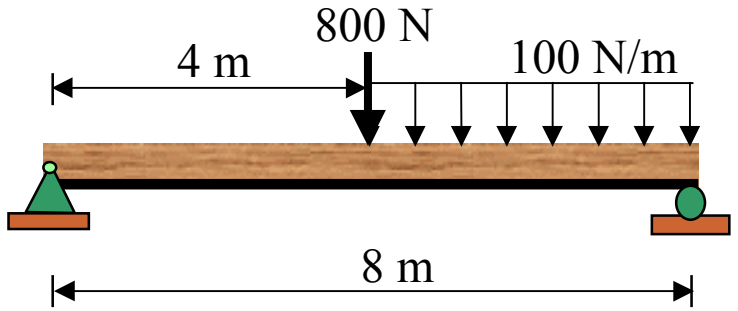
$$= \int (n\sigma_1) dA$$

$$\boxed{F_2 = \int \sigma_1 (n dA)}$$

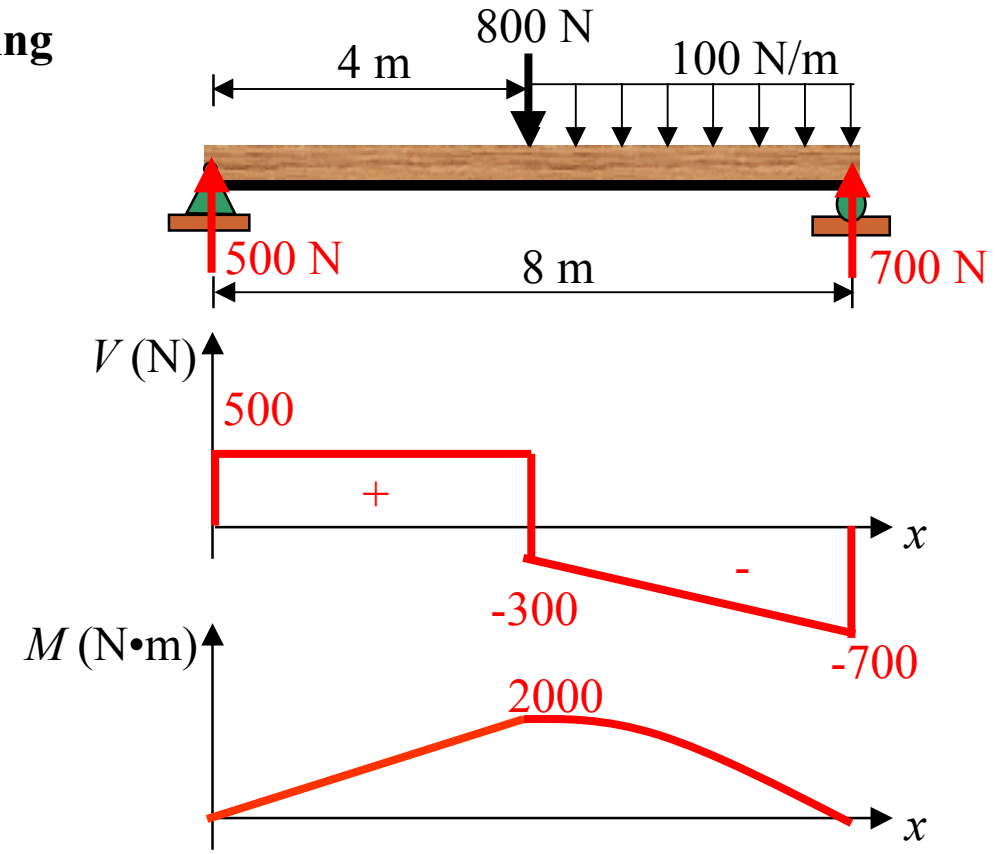
## Example 8

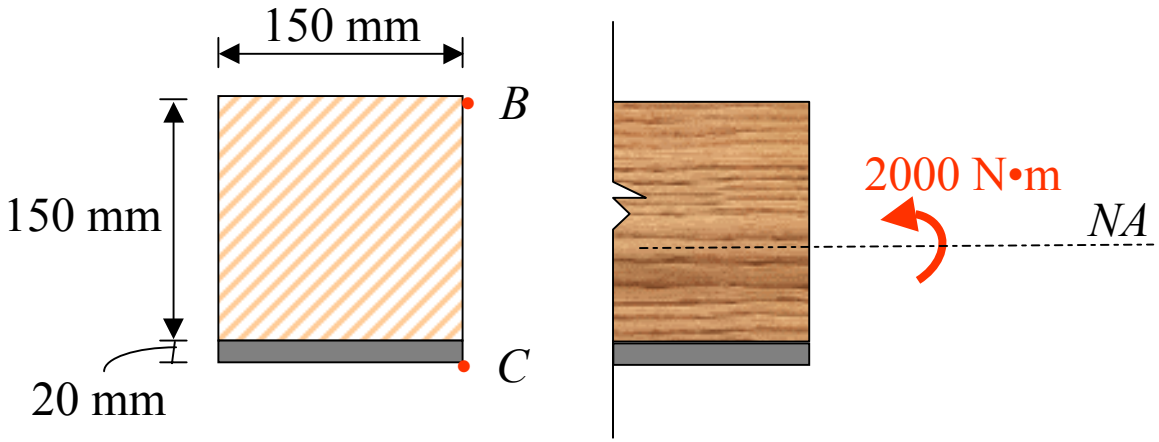
A composite beam is made of wood (150mmx150mm) and reinforced with a steel strap (150 mm x 20 mm) located on its bottom side, determine the normal stress at points *B* and *C*. Take $E_{st}$ = 200 GPa and $E_w$ = 12 GPa.



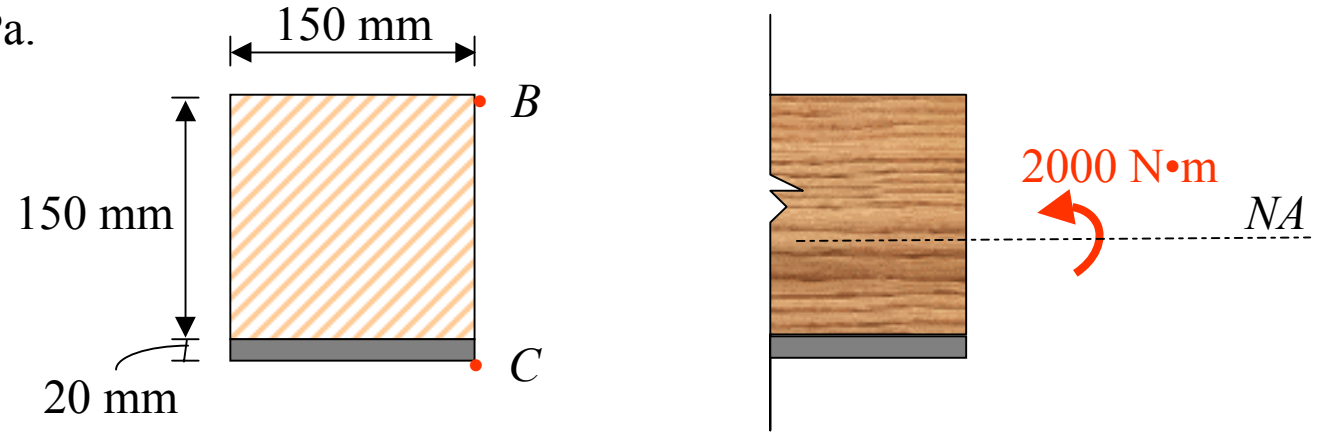
- **Internal loading**



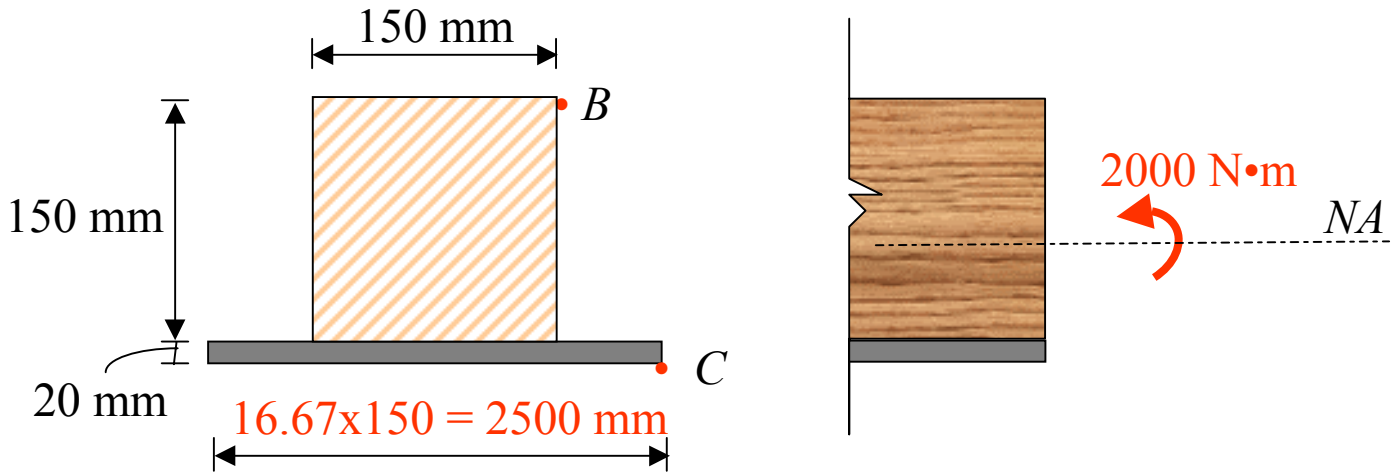



- **Modulus ratio n**

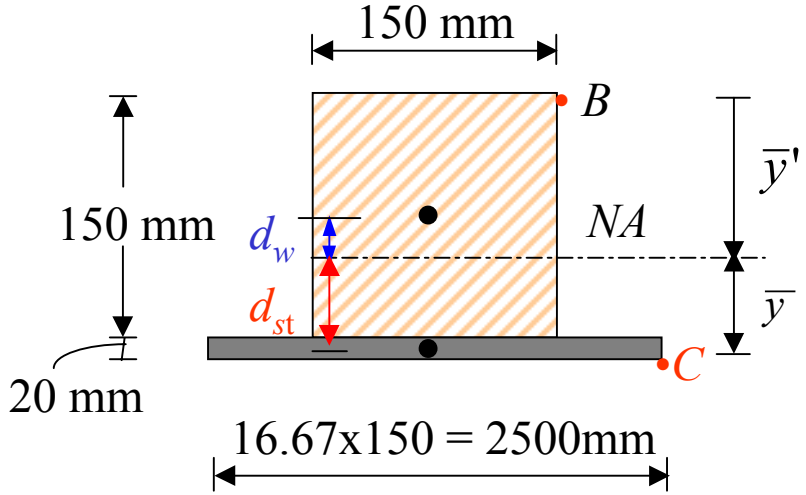
$E_{st}$ = 200 GPa
$E_w$ = 12 GPa.




$$n = \frac{E_{st}}{E_w} = \frac{200}{12} = 16.67$$



- **Section properties**




$$\bar{y}' = \frac{\sum \bar{y}A}{\sum A} = \frac{(150\times150)(75)+(2500\times20)(160)}{(150\times150)+(2500\times20)} = 133.6\,\text{mm}$$

$$\bar{y} = 100 - 133.6 = 36.4\,\text{mm}$$

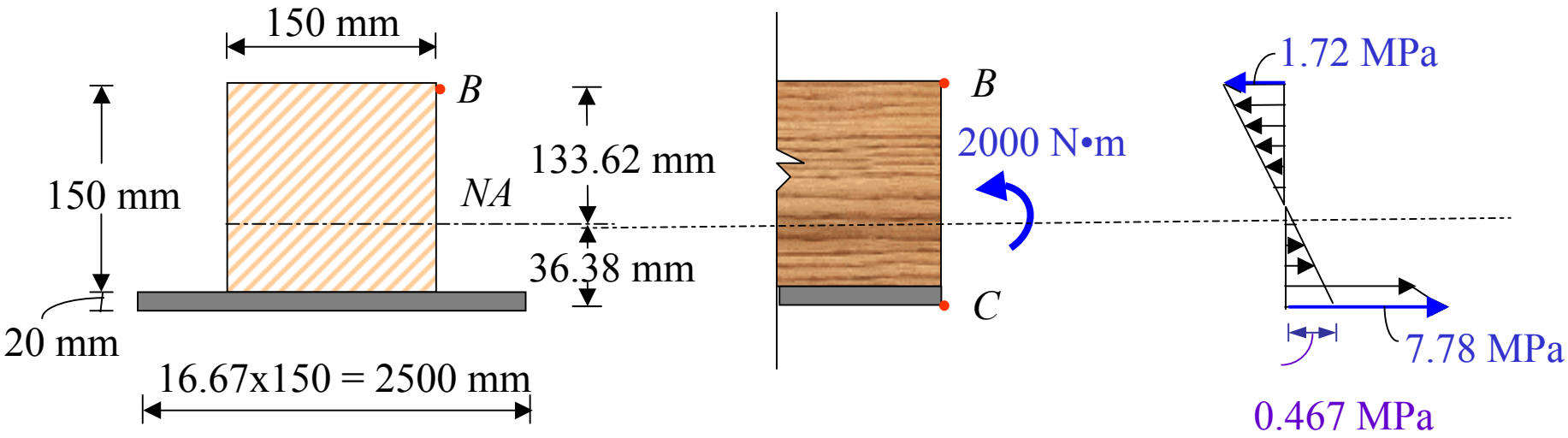
$$I = (\bar{I} + Ad^2)_w + (\bar{I} + Ad^2)_{st}$$

$$= [\frac{1}{12}(0.15\text{ m})(0.15\text{ m})^3 + (0.15\text{ m}\times0.15\text{ m})(0.1336\text{ m} - 0.075\text{ m})^2]$$

$$+ [\frac{1}{12}(2.5\text{ m})(0.02\text{ m})^3 + (2.5\text{ m}\times0.02\text{ m})(0.0364\text{ m} - 0.01\text{ m})^2]$$

$$= 155.9\times10^{-6}\text{ m}^4$$

- **Normal Stress**




$$I = 155.9 \text{ x} 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I}$$

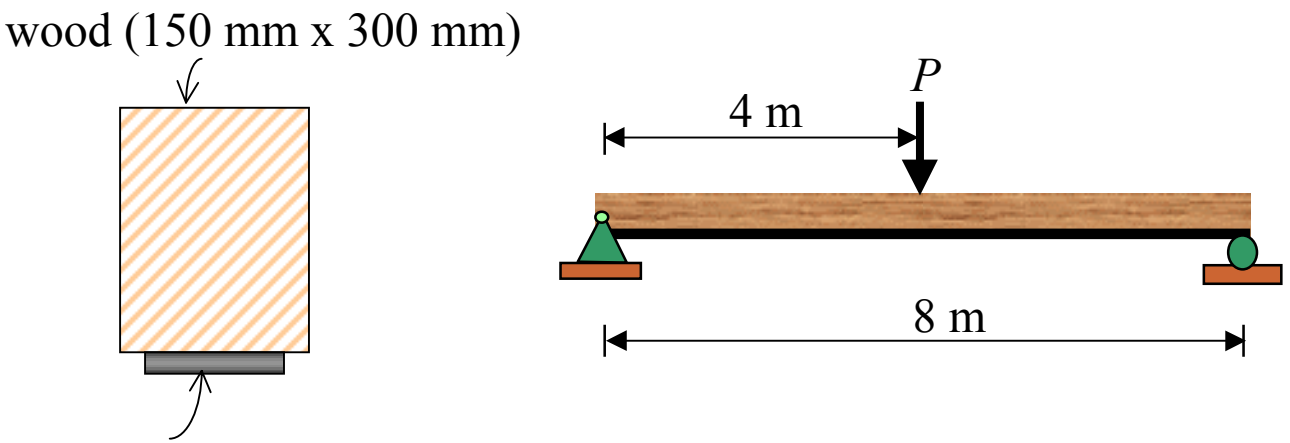
$$\sigma_B = -\frac{2\,000 \text{ N} \bullet \text{m}(0.134 \text{ m})}{155.9 \times 10^{-6} \text{m}^4} = -1.72 \text{ MPa} \quad (C) \Leftarrow$$

$$\sigma_{C'} = \frac{2\,000 \text{ N} \bullet \text{m}(0.0364 \text{ m})}{155.9 \times 10^{-6} \text{ m}^4} = 0.467 \text{ MPa} \quad (T)$$

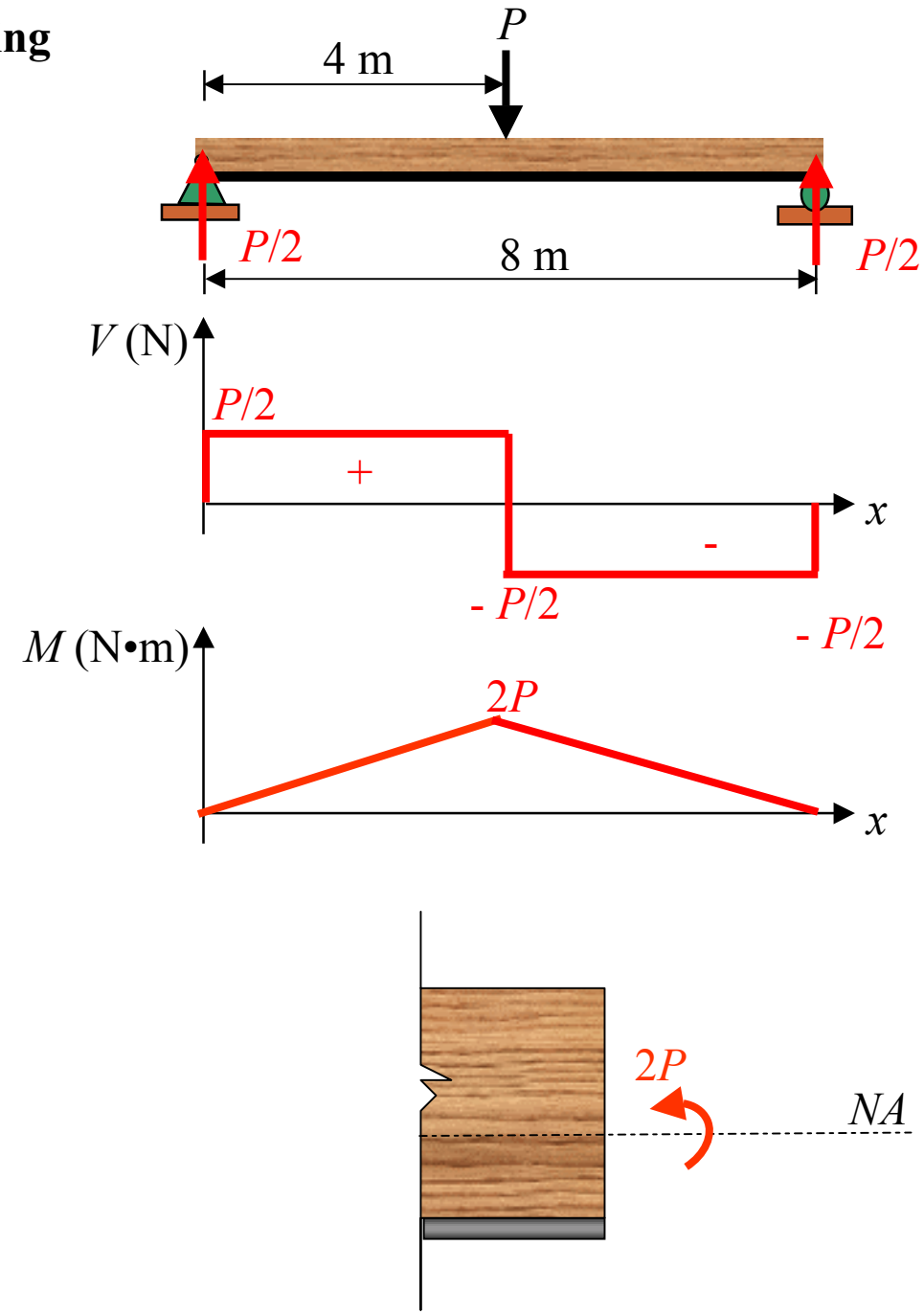
$$\sigma_C = n(\sigma_B) = 16.67 \frac{(2\,000 \text{ N} \bullet \text{m})(0.0364 \text{ m})}{155.9 \times 10^{-6} \text{ m}^4} = 7.78 \text{ MPa} \quad (T) \Leftarrow$$

## Example 9

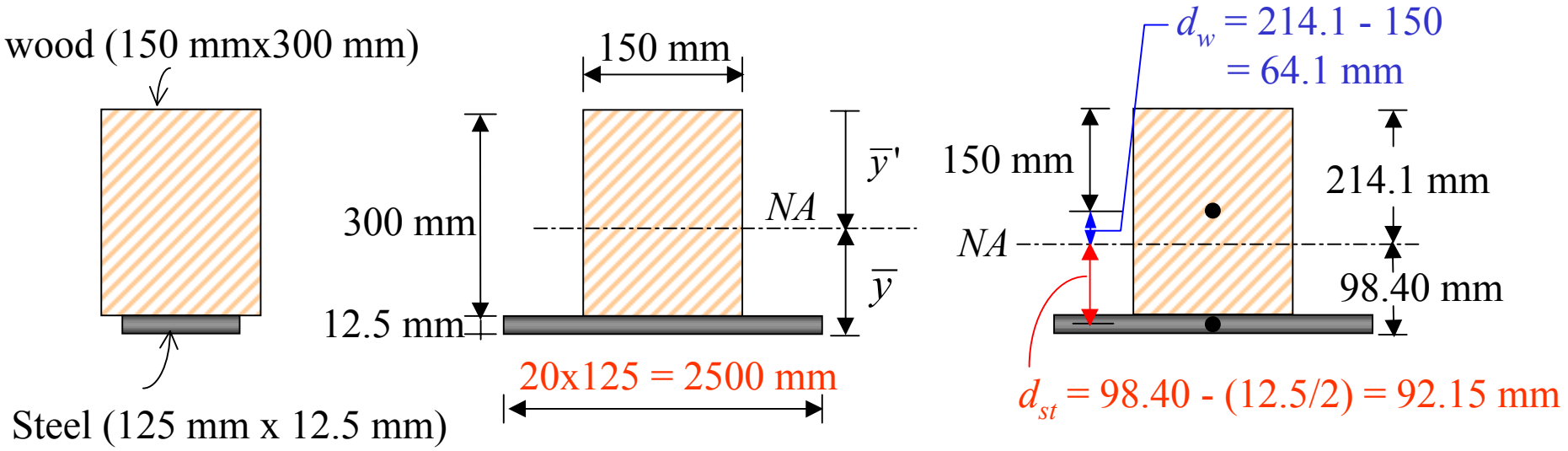
A composite beam is made of wood (150 mm x 300 mm) and reinforced with a steel strap (125 mm x 12.5 mm). If the allowable normal stress for steel is $(\sigma_{\text{allow}})_{st}$ = 175 MPa, and for the wood $(\sigma_{\text{allow}})_{w}$ = 12 MPa , determine the maximum load $P$ the beam can support. Take $E_{st}$ = 200 GPa and $E_{w}$ = 10 GPa.



- **Internal loading**



- **Section Property**




$$n = 200/10 = 20$$

$$\bar{y}' = \frac{\sum \bar{y}A}{\sum A} = \frac{(150 \times 300)(150) + (2500 \times 12.5)(306.3)}{(150 \times 300) + (2500 \times 12.5)} = 214.1 \text{ mm}$$

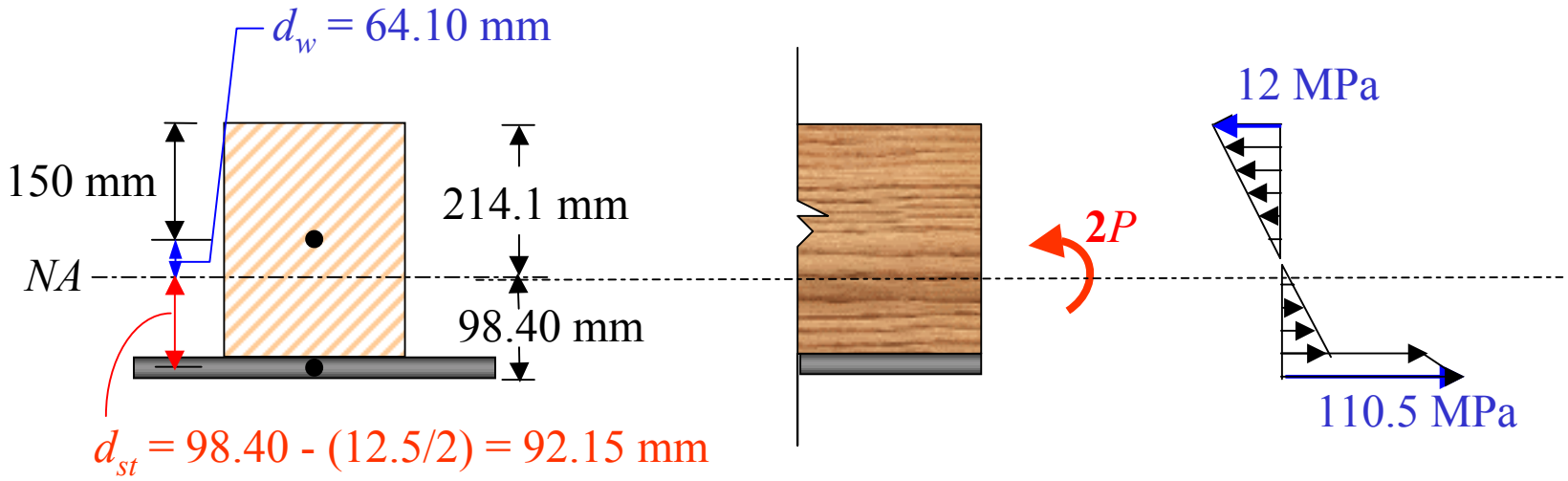
$$\bar{y} = 312.5 - 214.1 = 98.40 \text{ mm}$$

$$I = (\bar{I} + Ad^2)_w + (\bar{I} + Ad^2)_{st}$$

$$= [(1/12)(150)(300^3) + (150\text{x}300)(64.10^2)] + [(1/12)(2500)(12.5^3) + (2500\text{x}12.5)(92.15^2)]$$

$$= 788.2\text{x}10^6 \text{ mm}^4$$

- **Allowable normal stress**




$$\sigma = \frac{Mc}{I}$$

$$(\sigma_{\text{allow}})_w = 12 \times 10^6 = \frac{(2P)(0.2141)}{(788.2 \times 10^{-6})}$$

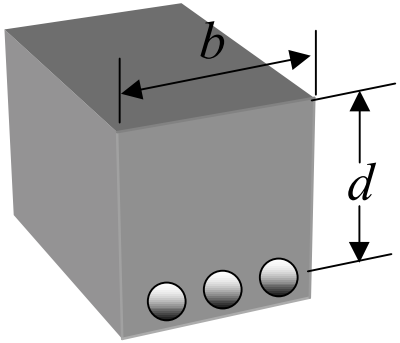
$$P = 22.1\,\text{kN}$$

$$(\sigma_{\text{allow}})_{st} = 175 \times 10^6 = \frac{(2P)(0.0984)}{(788.2 \times 10^{-6})}(20)$$

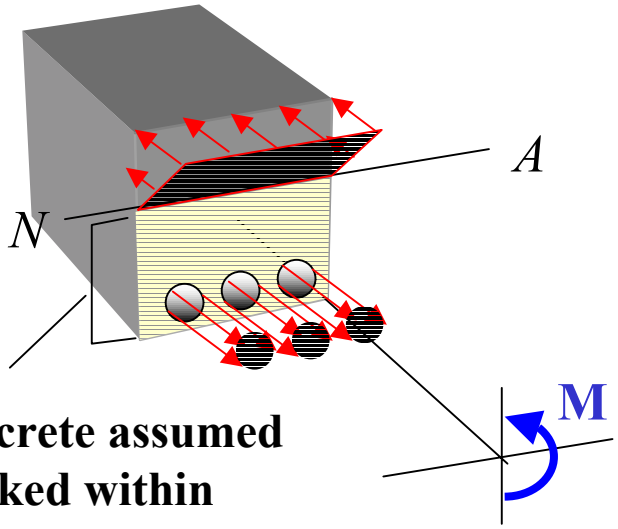
$$P = 35.0\,\text{kN}$$

The maximum load $P$ the beam can support is $P = 22.1$ kN ⇦

# Reinforce-Concrete Beams

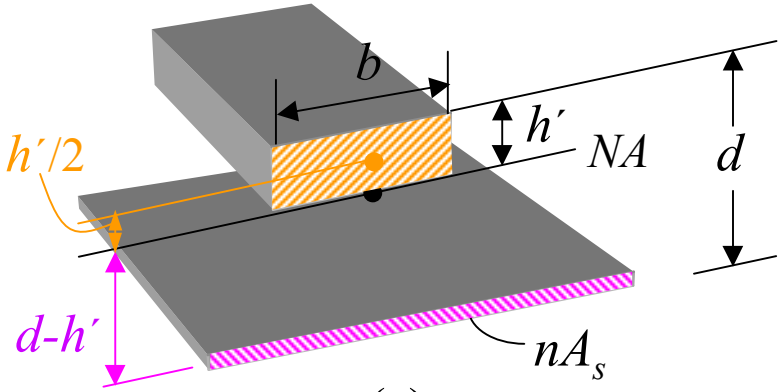



(a)




**Concrete assumed cracked within this region.**

(b)

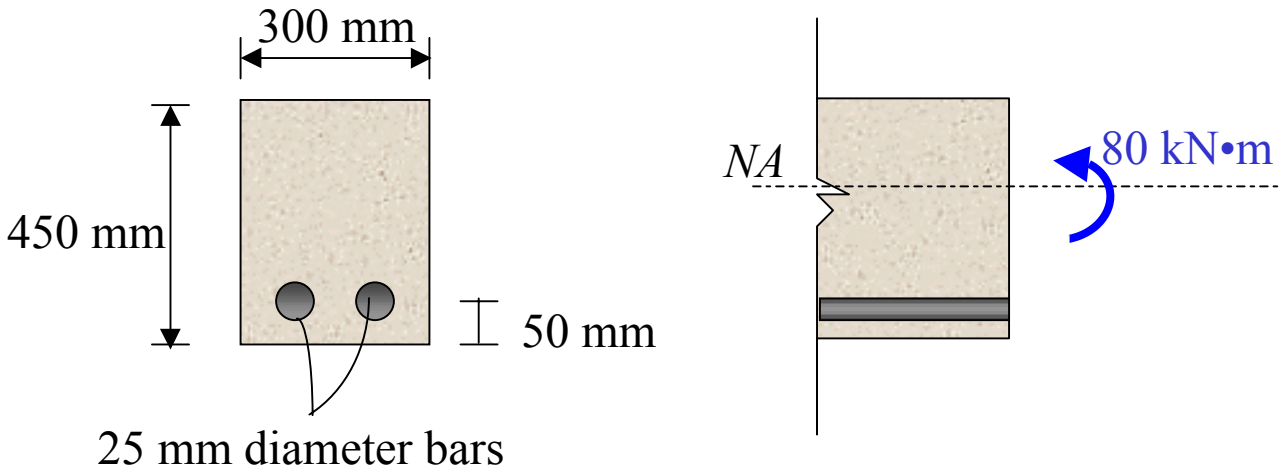


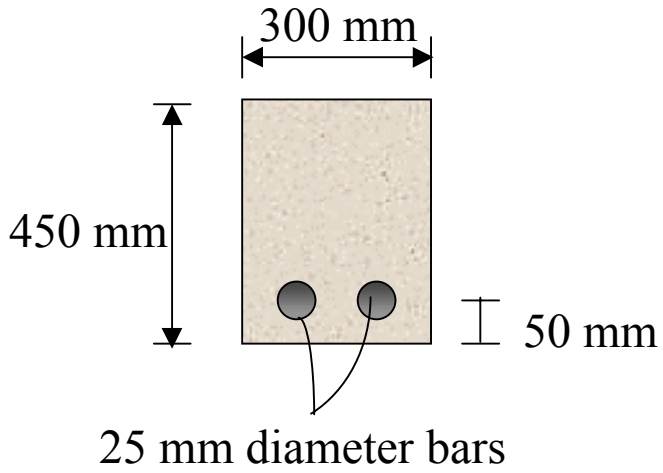

(c)

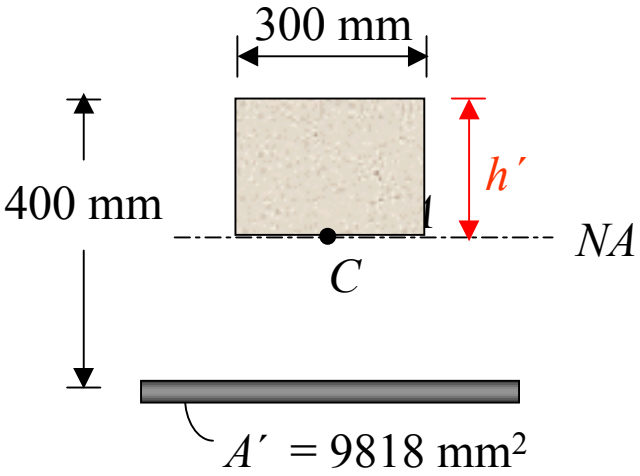
$$\sum (\bar{y}A)_{NA}: \quad bh'(\frac{h'}{2}) - nA_s(d-h') = 0$$

## Example 10a

The reinforced concrete beam has the cross-section area shown . If it is subjected to a bending moment of $M = 80$ kN•m, determine the normal stress in each of the steel reinforcing rods and the maximum normal stress in the concrete. Take $E_{st} = 200$ GPa and $E_c = 20$ GPa.






- ***Section Property***

$$n = E_{st}/E_c = 200/20 = 10$$

$$A_{st} = 2(\pi/4)(25^2) = 981.8 \text{ mm}^2$$

$$A_{st}' = nA_{st} = 9818 \text{ mm}^2$$

Take moment area about the neutral axis:

$$(\Sigma \bar{y}A)_{NA} = 0$$

$$(\bar{y}A)_c - (\bar{y}A)_{st} = 0$$

$$(300xh')(h'/2) - (9818)(400-h') = 0$$

$$150h'^2 + 9818h' - (9818)(400) = 0$$

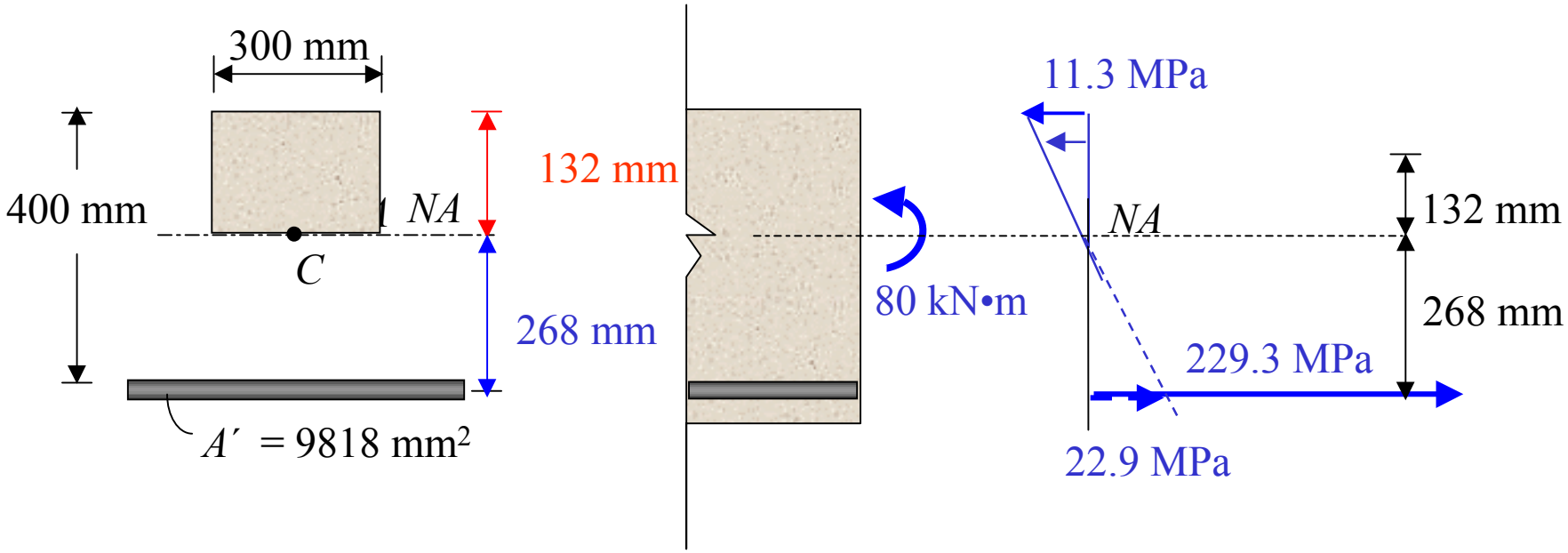
$$h' = 132 \text{ mm}$$

$$I = (\bar{I} + Ad^2)_c + (\bar{I}^{\approx 0} + Ad^2)_{st}$$

$$= [(1/12)(300)(132^3) + (300x132)(132/2)^2]$$

$$+ [(9818)(400-132)^2]$$

$$= 935.1x10^6 \text{ mm}^4$$




- ***Normal stress;***

$$\sigma = \frac{Mc}{I}$$

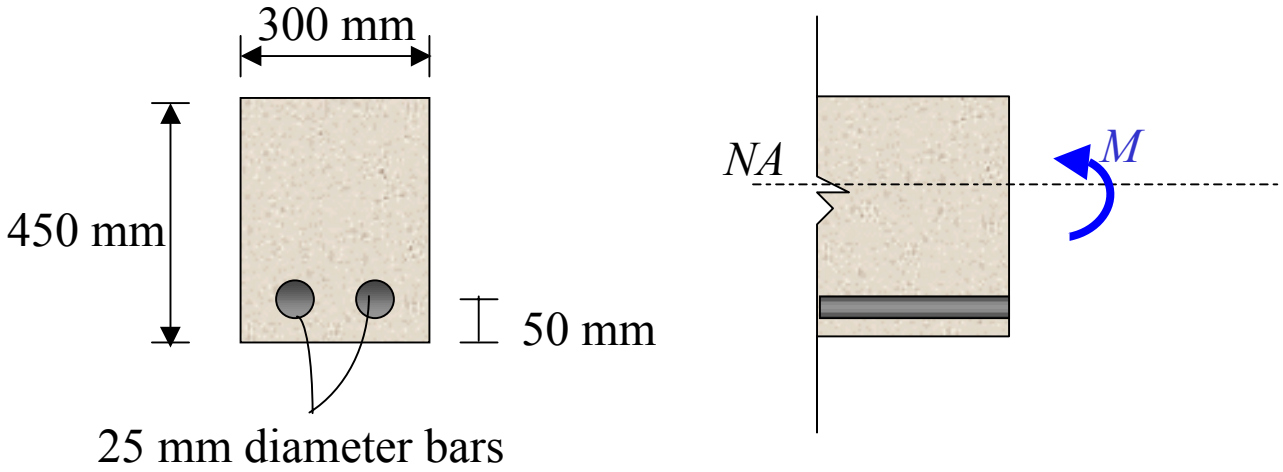
$$(\sigma_{\max})_{conc} = \frac{(80\times10^{3})(0.132)}{(935.1\times10^{-6})} = 11.3 \text{ MPa} \Leftarrow$$

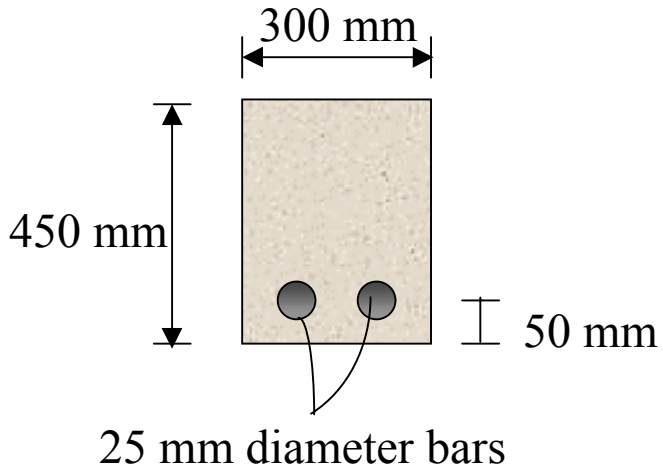
$$(\sigma')_{conc} = \frac{(80\times10^{3})(0.268)}{(935.1\times10^{-6})} = 22.9 \text{ MPa}$$

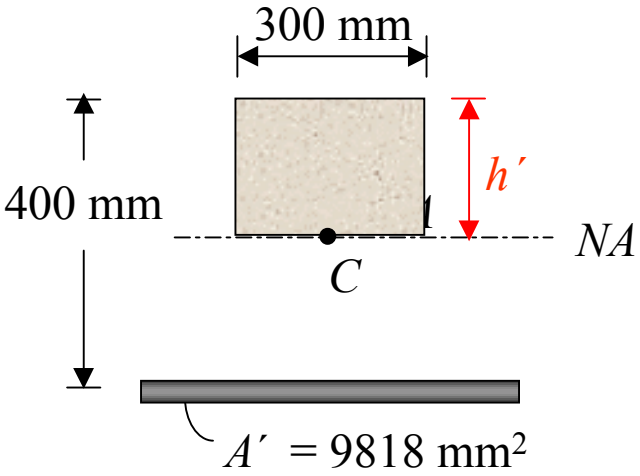
$$(\sigma_{\max})_{st} = \frac{(80\times10^{3})(0.268)}{(935.1\times10^{-6})}(10) = 229 \text{ MPa} \Leftarrow$$

## Example 10b

The reinforced concrete beam has the cross-section area shown . If the allowable normal stress for concrete is $(\sigma_{\text{allow}})_c = 20$ MPa, and for the steel is $(\sigma_{\text{allow}})_{st} = 300$ MPa , determine the maximum moment $M$ the beam can support.Take $E_{st} = 200$ GPa and $E_c = 20$ GPa.






- **Section Property**

$n = E_{st}/E_c = 200/20 = 10$

$A_{st} = 2(\pi/4)(25^2) = 981.8 \text{ mm}^2$

$A_{st}' = nA_{st} = 9818 \text{ mm}^2$

Take moment area about the neutral axis:

$$(\Sigma \bar{y}A)_{NA} = 0$$

$$(\bar{y}A)_c - (\bar{y}A)_{st} = 0$$

$$(300xh')(h'/2) - (9818)(400-h') = 0$$

$$150h'^2 + 9818h' - (9818)(400) = 0$$

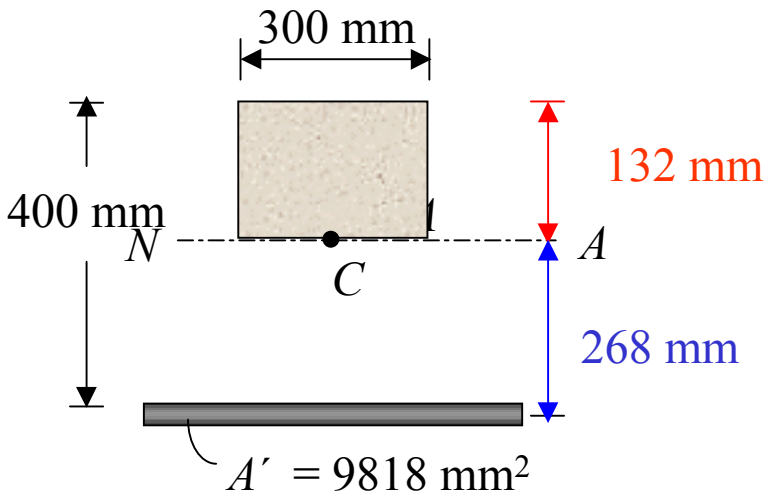
$$h' = 132 \text{ mm}$$

$$I = (\bar{I} + Ad^2)_c + (\bar{I}^{\approx 0} + Ad^2)_{st}$$

$$= [(1/12)(300)(132^3) + (300x132)(132/2)^2]$$

$$+ [(9818)(400-132)^2]$$

$$= 935.1x10^6 \text{ mm}^4$$




$I = 935.1\text{x}10^6\ \text{mm}^4$

- **Allowable Stress**

$(\sigma_{\text{allow}})_c = 20\ \text{MPa}$ and $(\sigma_{\text{allow}})_{st} = 300\ \text{MPa}$

$$\sigma = \frac{My}{I}$$

concrete : $$20 \times 10^6\ \text{Pa} = \frac{M(0.132\ \text{m})}{935.1 \times 10^{-6}\ \text{m}^4}$$

$$M = 141.7\ \text{kN} \bullet \text{m}$$

steel : $$300 \times 10^6\ \text{Pa} = \frac{M(0.268\ \text{m})}{935.1 \times 10^{-6}\ \text{m}^4} \quad (10)$$

$$M = 104.7\ \text{kN} \bullet \text{m}$$

