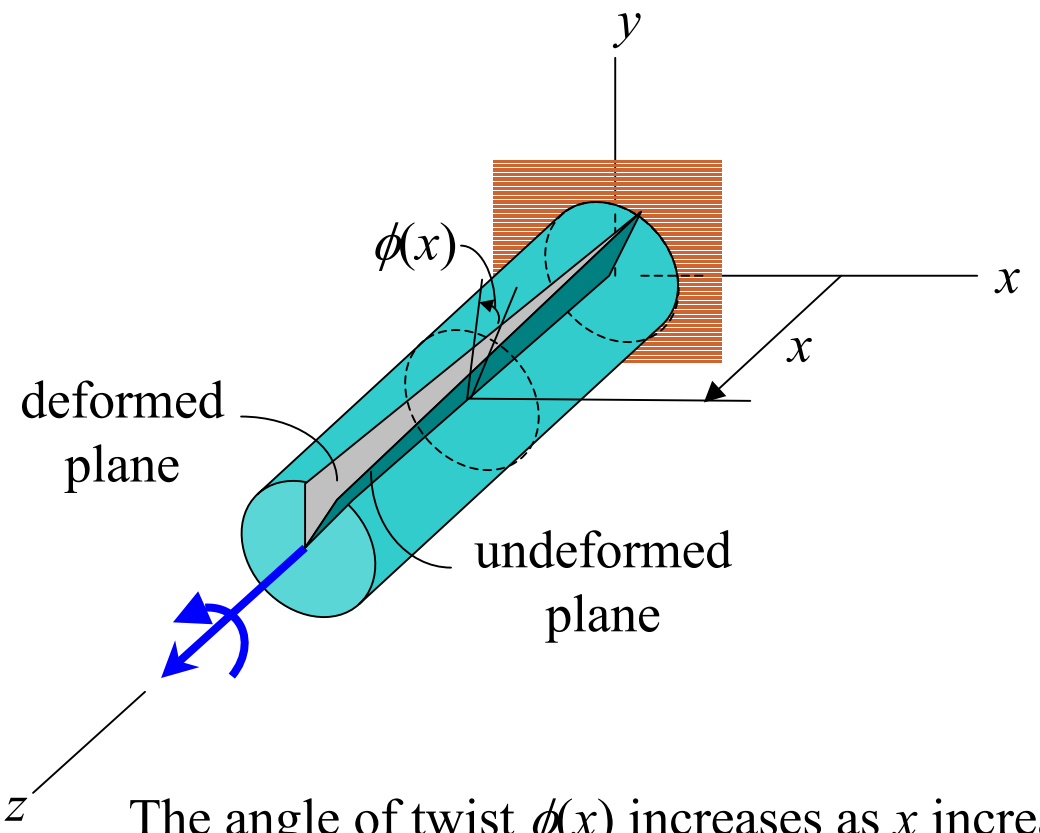
TORSION

- Torsion in a Circular Shaft
- The Torsion Formula
  - Stress
  - Strain
- Torque Diagram
- Angle of Twist
- Power Transmission
- Statically Indeterminate Torque-Loaded Members

Torsion in a Circular Shaft



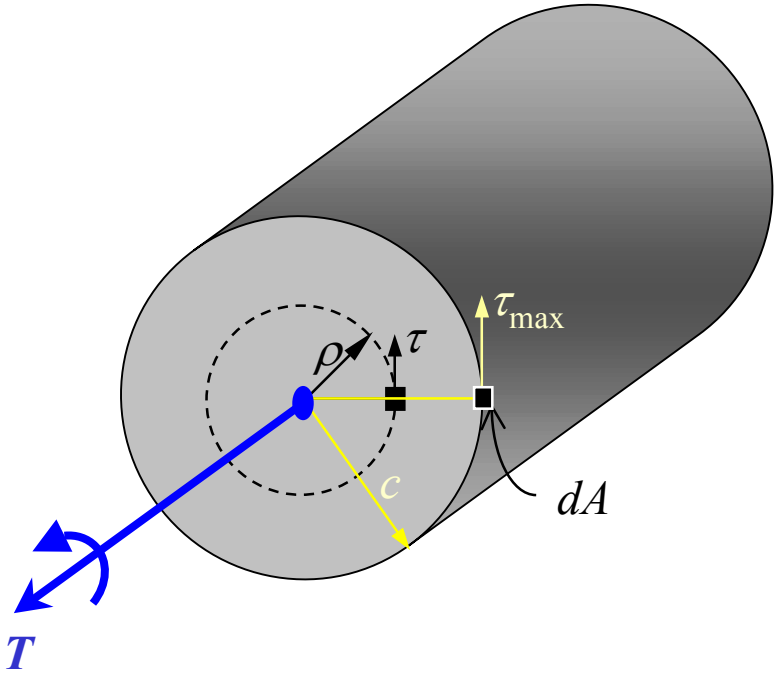
The angle of twist $\phi(x)$ increases as x increases.

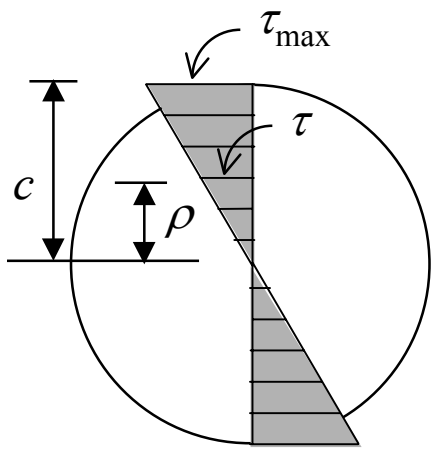
The Torsion formula

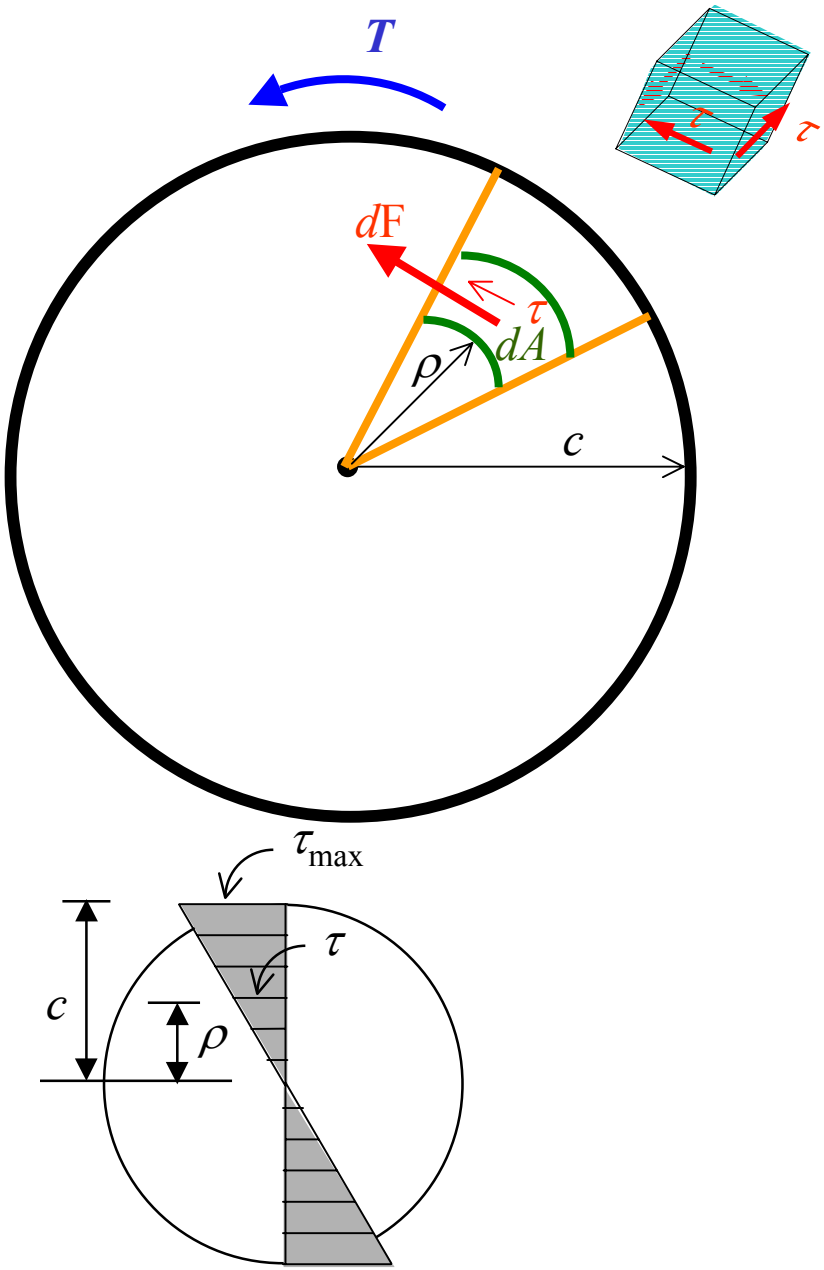
$$\frac{\tau}{\tau_{max}} = \frac{\rho}{c}$$

$$\tau = \rho \frac{\tau_{max}}{c}$$

$$\frac{\tau_{max}}{c} = constant$$







$$dT = \rho(dF) = \rho(\tau dA)$$

$$dT = \rho[\rho(\frac{\tau_{\max}}{c})dA]$$

constant

$$T = \int(\frac{\tau_{\max}}{c})\rho^2 dA$$

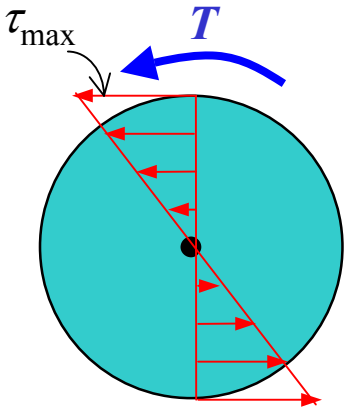
J

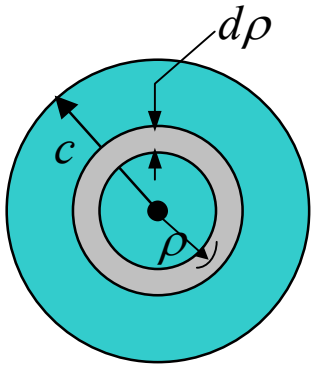
$$T = (\frac{\tau_{\max}}{c})\int \rho^2 dA$$

$$\tau_{\max} = (\frac{Tc}{J})$$

$$\tau = \frac{T\rho}{J}$$

• **Solid Shaft**

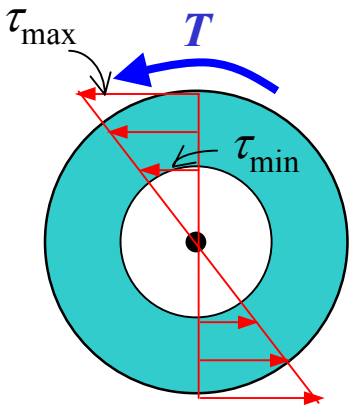


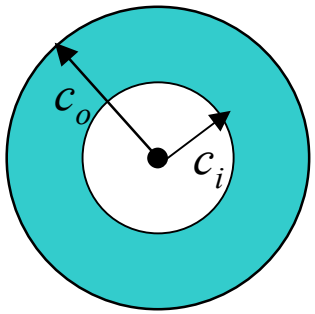


$$J = \int_A \rho^2 dA = \int_0^c \rho^2 (2\pi\rho d\rho) = 2\pi \int_0^c \rho^3 d\rho = 2\pi(\frac{1}{4})\rho^4 \Big|_0^c$$

$$J = \frac{\pi}{2} c^4$$

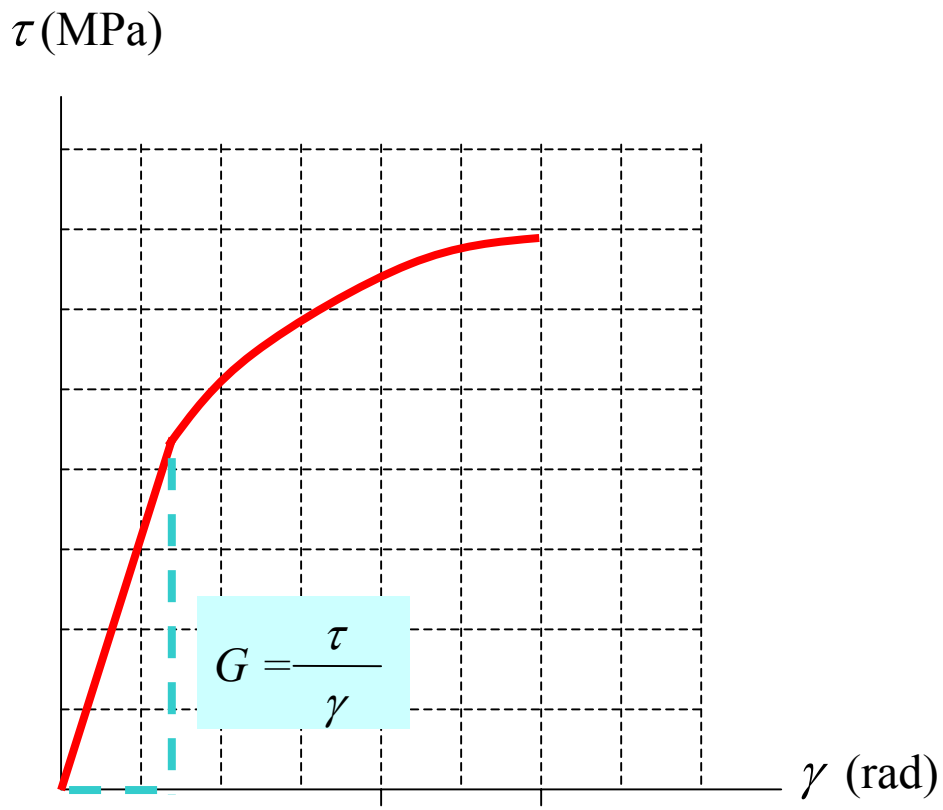
• **Tubular Shaft**



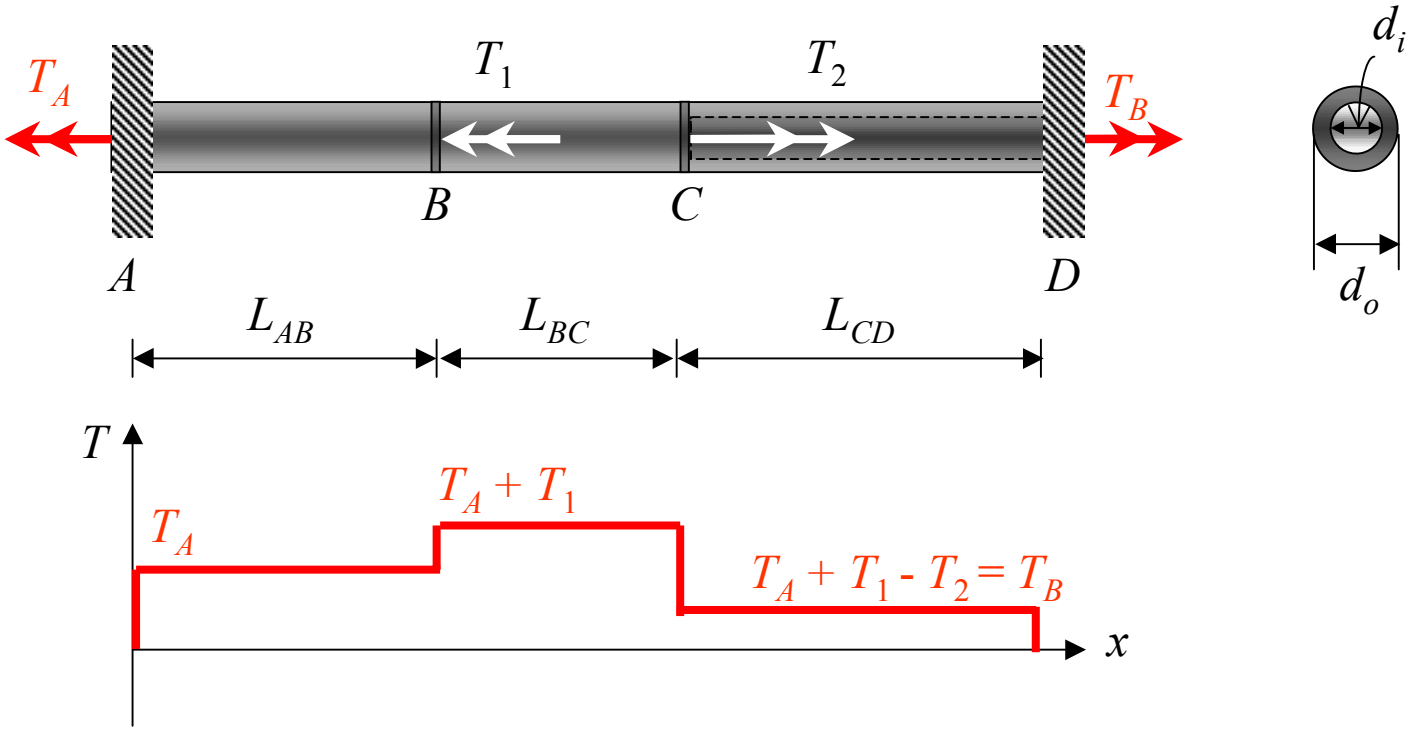


$$J = \frac{\pi}{2}(c_o^{\ 4} - c_i^{\ 4})$$

- **Stress strain relationship**



- **Equilibrium and compatibility condition**



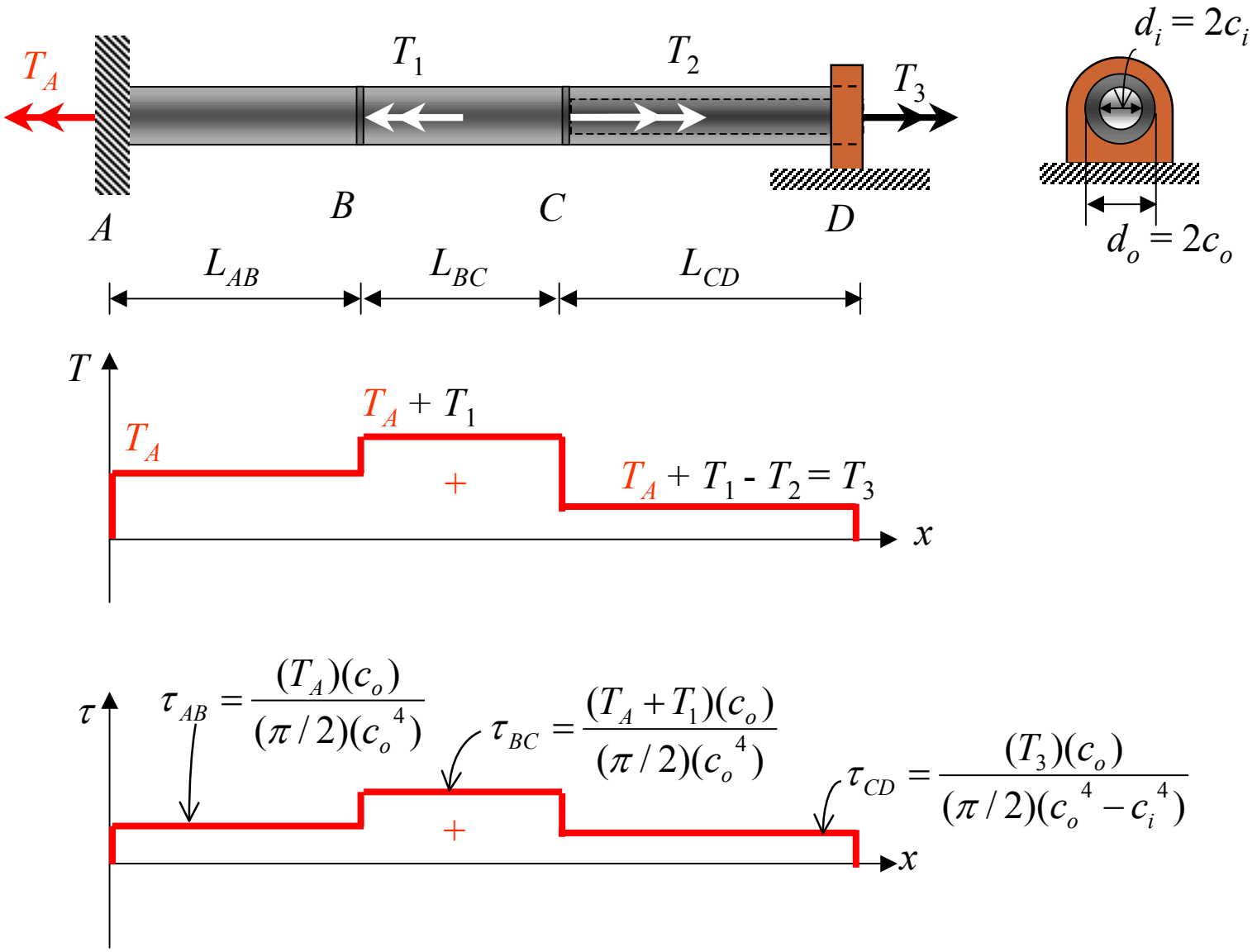
Equilibrium equation:

$$T_A + T_1 - T_2 - T_B = 0$$

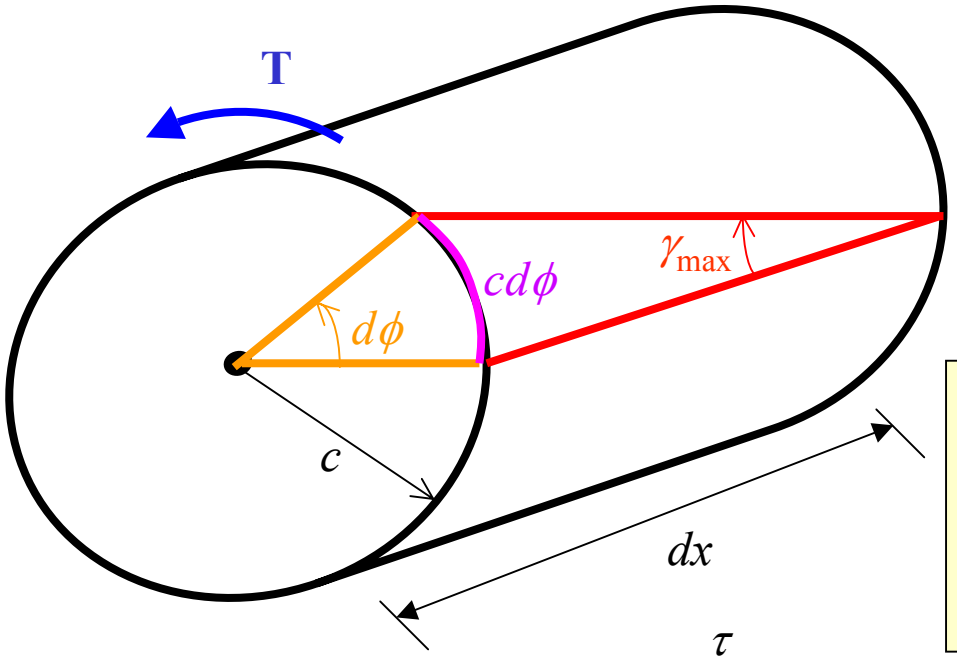
Compatibility equation:

$$\phi_{D/A} = \phi_{B/A} + \phi_{C/B} + \phi_{D/C} = 0$$

- **Torsion and shear stress diagram**



Angle of twist



$$\tau_{max} = \left(\frac{Tc}{J}\right)$$

$$\tau = \frac{T\rho}{J}$$

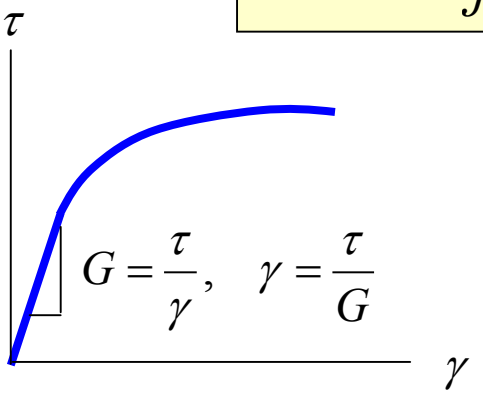
$$cd\phi = \gamma_{max} dx$$

$$d\phi = \frac{\gamma_{max} dx}{c}$$

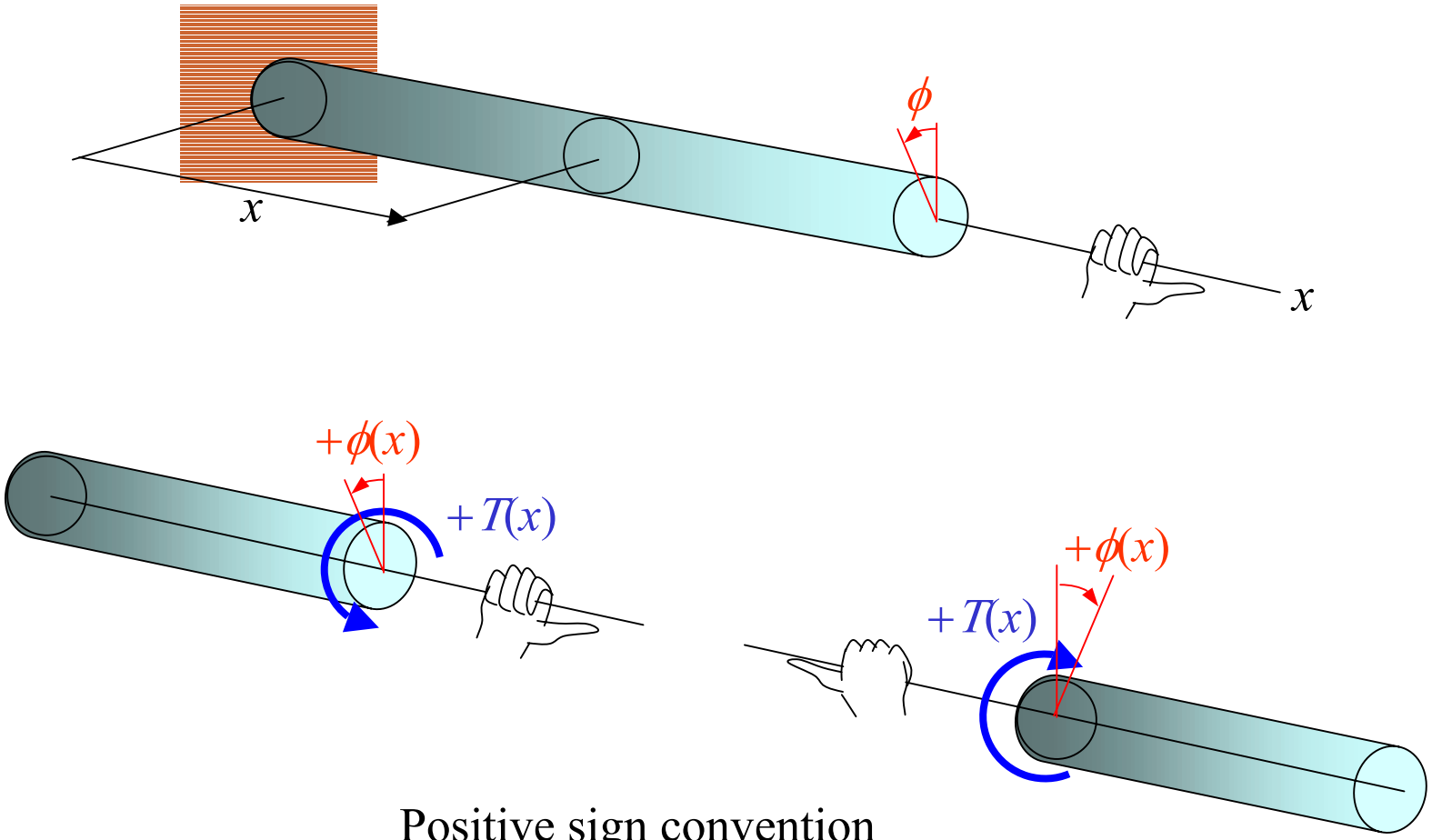
$$d\phi = \frac{\tau_{max} dx}{Gc} = \frac{Tcdx}{JGc}$$

$$d\phi = \frac{Tdx}{JG}$$

$$\phi = \int \frac{Tdx}{JG} \quad \dashrightarrow \quad \phi = \frac{TL}{JG}$$

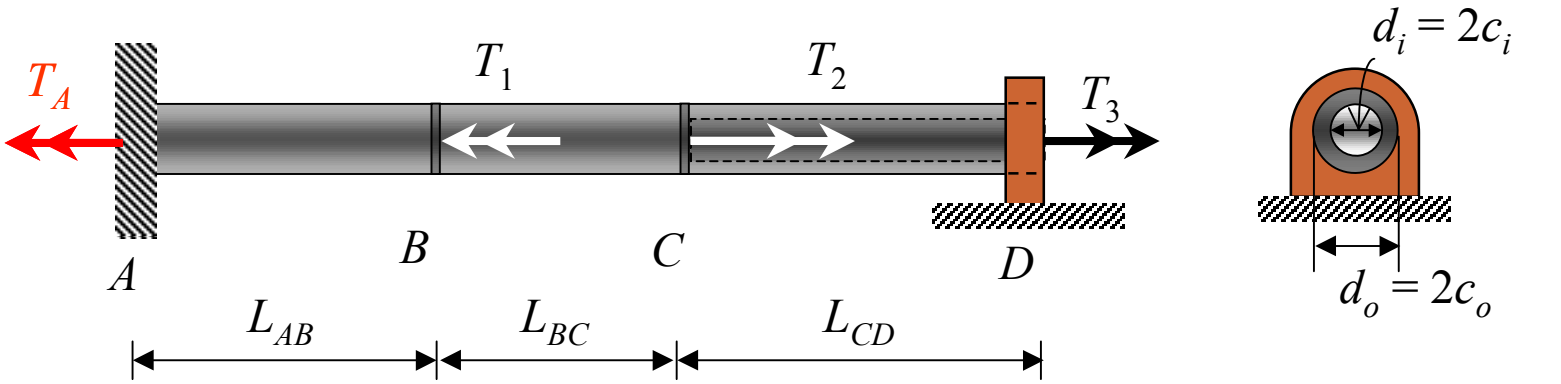


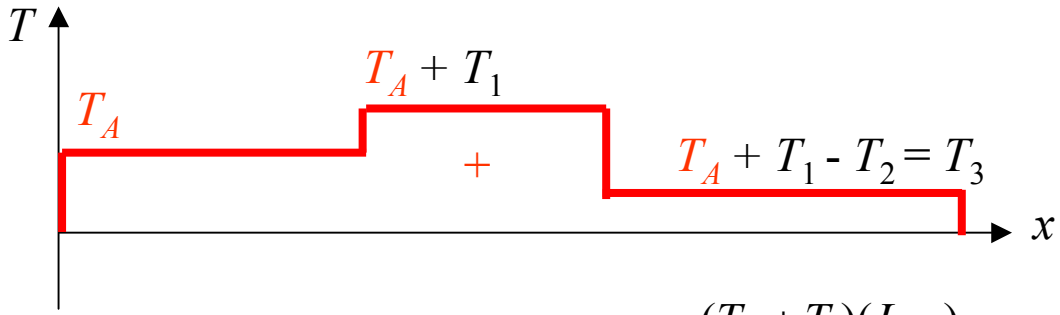
• **Sign Convention**



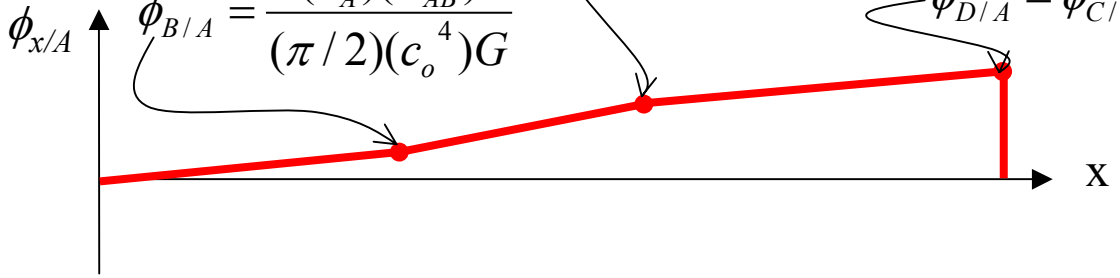
Positive sign convention
for T and ϕ

• **Torsion and angle of twist diagram**



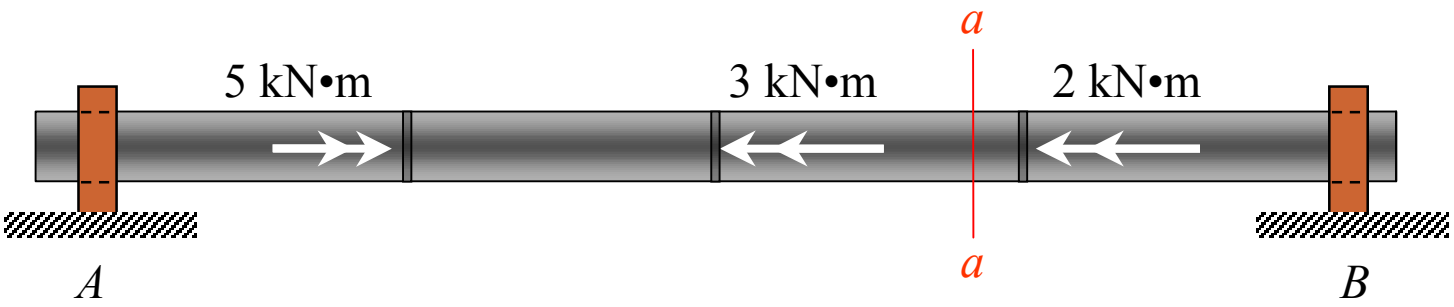


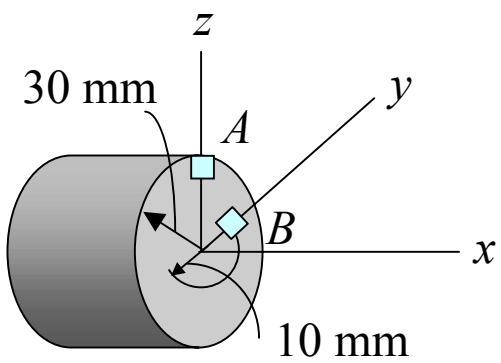
$$\phi_{C/A} = \phi_{B/A} + \frac{(T_A + T_1)(L_{BC})}{(\pi/2)(c_o^{\,4})G}$$



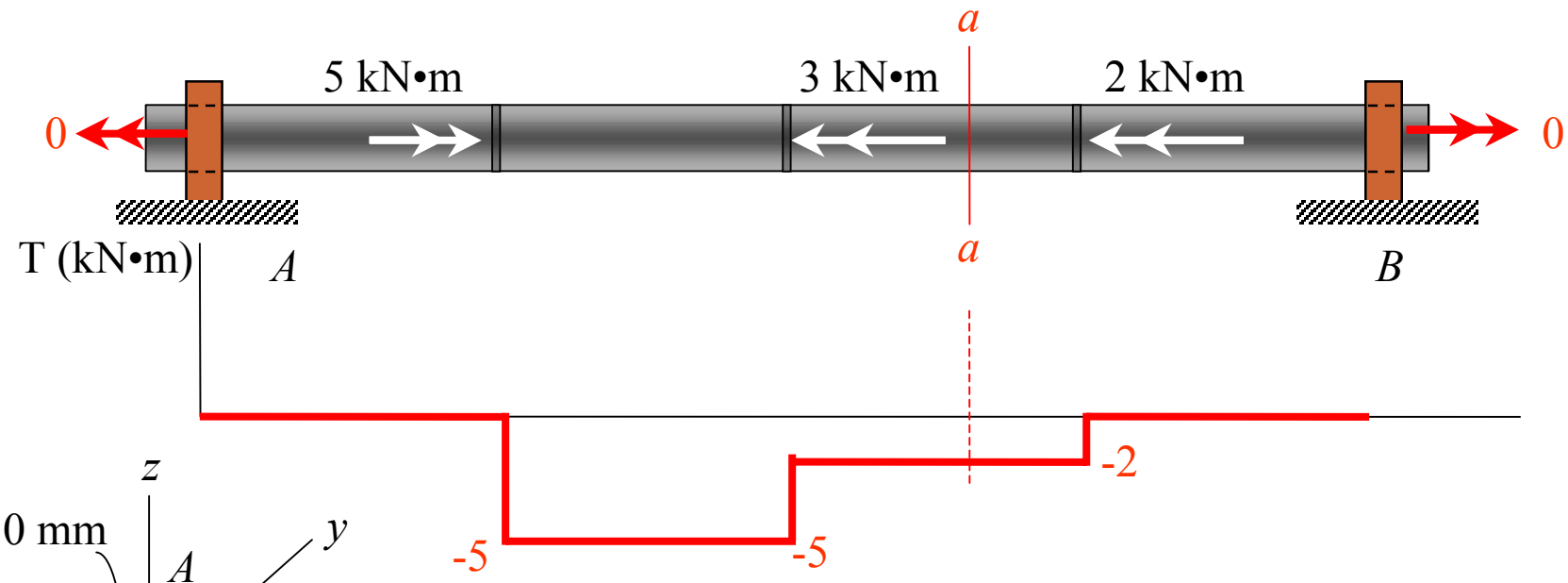
Example 1

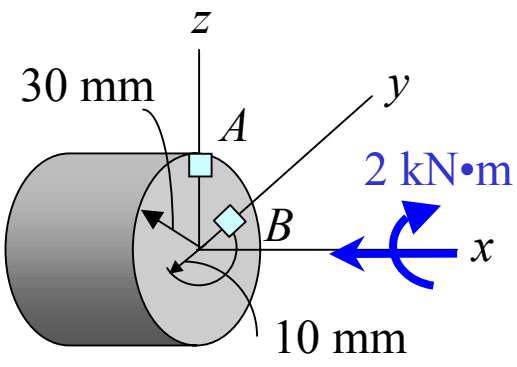
The shaft shown is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points *A* and *B*, located at section a-a of the shaft.



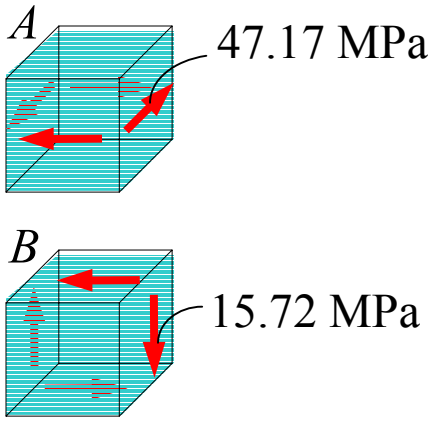


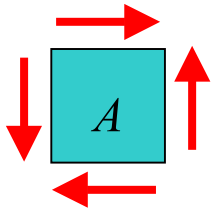
Section a-a





Section a-a





Top view *A*

- **Section Property**

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.03\ \text{m})^4 = 1.272\,(10^{-6})\ \text{m}^4$$

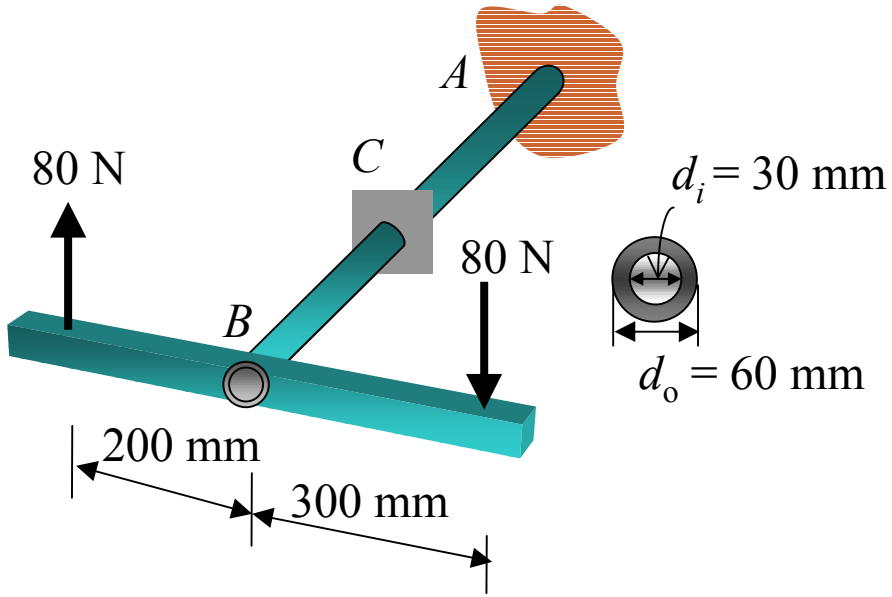
- **Shear Stress**

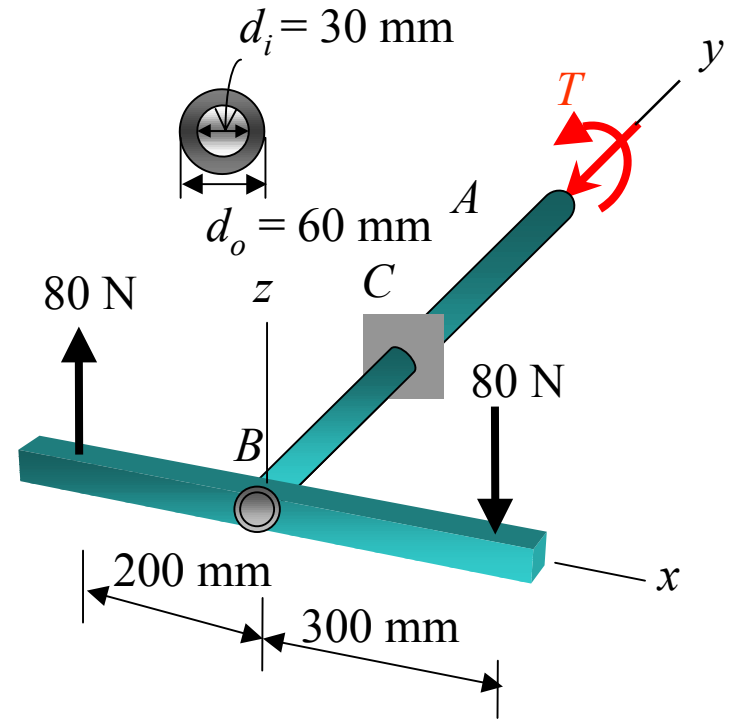
$$\tau_A = \frac{Tc}{J} = \frac{(2\times 10^3\ \text{N}\bullet\text{m})(0.03\ \text{m})}{1.272\,(10^{-6})\ \text{m}^4} = 47.17\ \text{MPa}$$

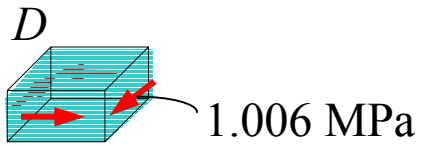
$$\tau_B = \frac{T\rho}{J} = \frac{(2\times 10^3\ \text{N}\bullet\text{m})(0.01\ \text{m})}{1.272\,(10^{-6})\ \text{m}^4} = 15.72\ \text{MPa}$$

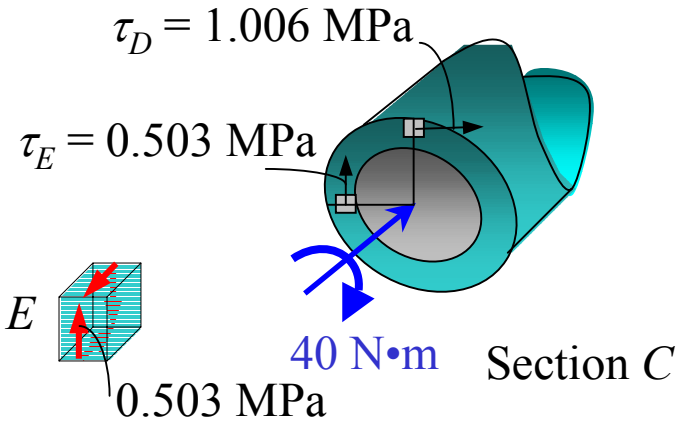
Example 2

The pipe shown in has an inner diameter of 30 mm and an outer diameter of 60 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.









- **Internal Torque**

$$\sum M_x = 0; \qquad 80\ \text{N}(0.5\ \text{m}) - T = 0$$

$$T = 40\ \text{N•m}$$

- **Section Property**

$$J = \frac{\pi}{2}[(0.03\ \text{m})^4 - (0.015\ \text{m})^4] = 1.193(10^{-6})\ \text{m}^4$$

- **Shear Stress**

- For any point lying on the outside surface of the pipe, $\rho = c_o = 0.03$ m

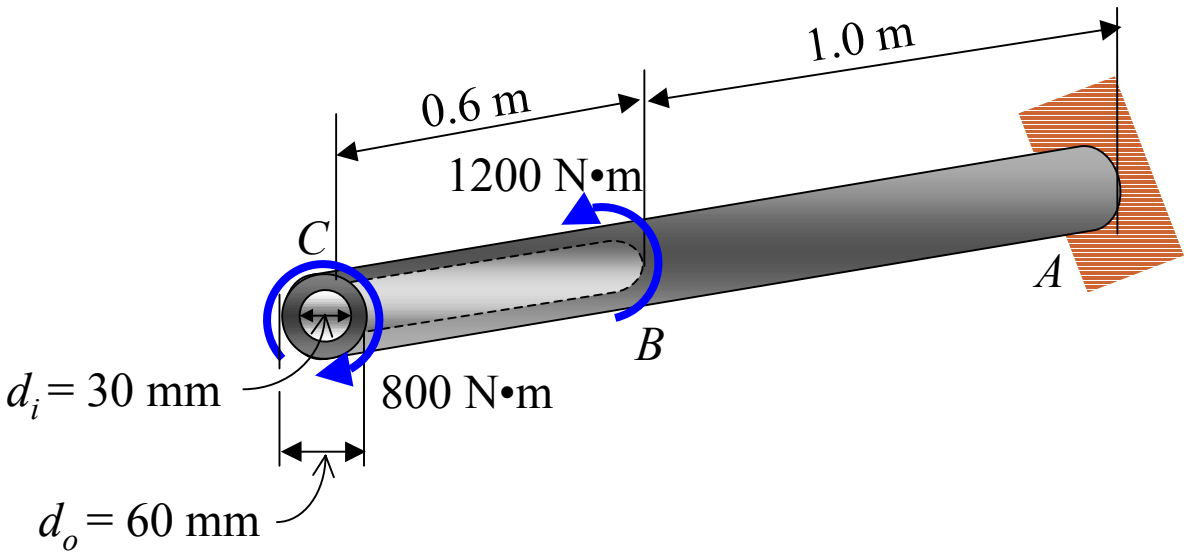
$$\tau_o = \frac{Tc_o}{J} = \frac{40\ \text{N} \bullet \text{m}(0.03\ \text{m})}{1.193(10^{-6})\ \text{m}^4} = 1.006\ \text{MPa}$$

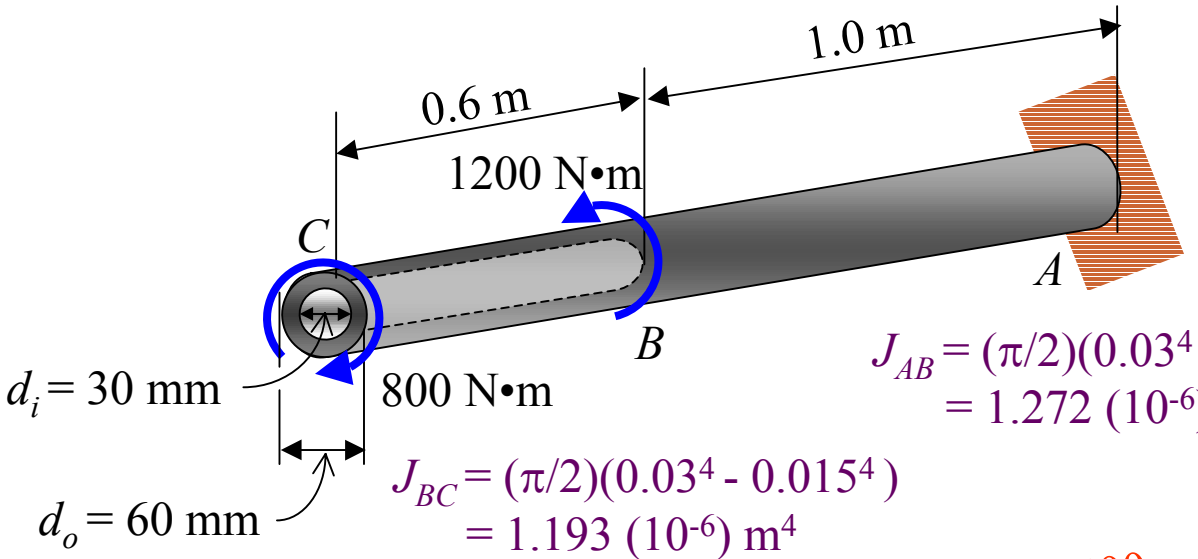
- For any point located on the inside surface, $\rho = c_i = 0.015$ m

$$\tau_i = \frac{Tc_i}{J} = \frac{40\ \text{N} \bullet \text{m}(0.015\ \text{m})}{1.193(10^{-6})\ \text{m}^4} = 0.503\ \text{MPa}$$

Example 3

A solid steel shaft shown in the figure (G = 80 GPa), having an inner diameter of 30 mm and outer diameter of 60 mm. Draw the quantitative torsion, shear stress and the angle of twist diagram of the shaft.

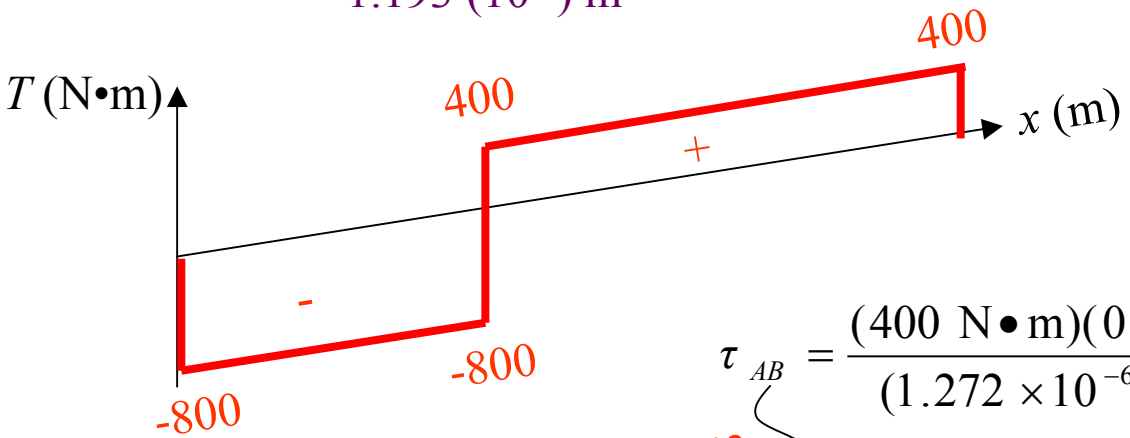




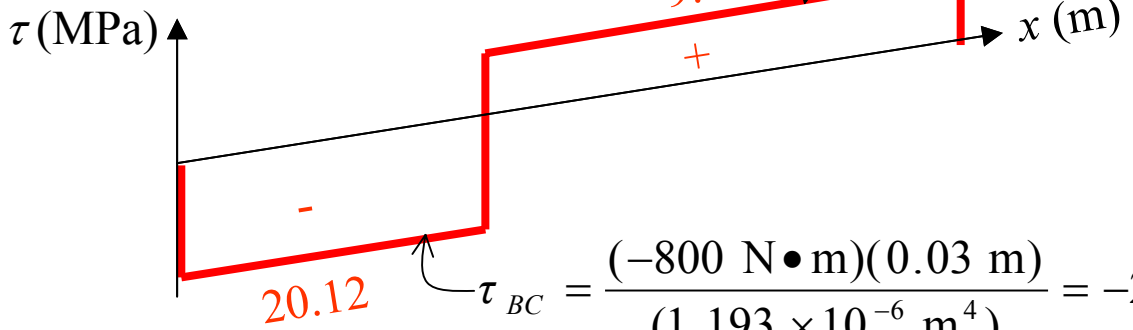
$J = (\pi/2)(c_o^4 - c_i^4)$

$J_{AB} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ \text{m}^4$

$J_{BC} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ \text{m}^4$

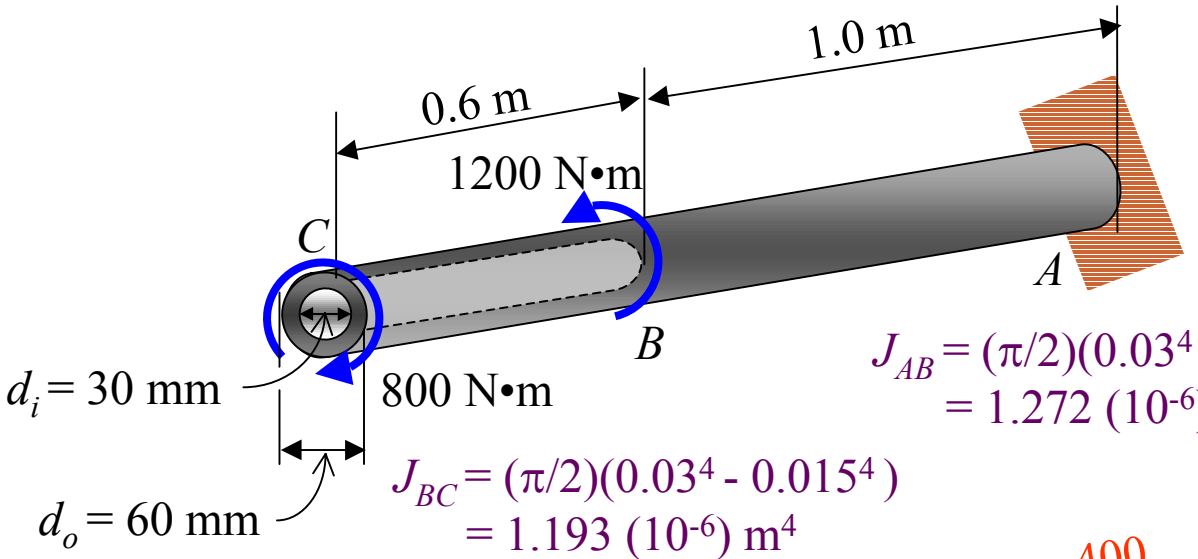


$$\tau_{AB} = \frac{(400\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(1.272\times10^{-6}\ \text{m}^4)} = 9.43\ \text{MPa}$$



$$\tau = \frac{Tc}{J}$$

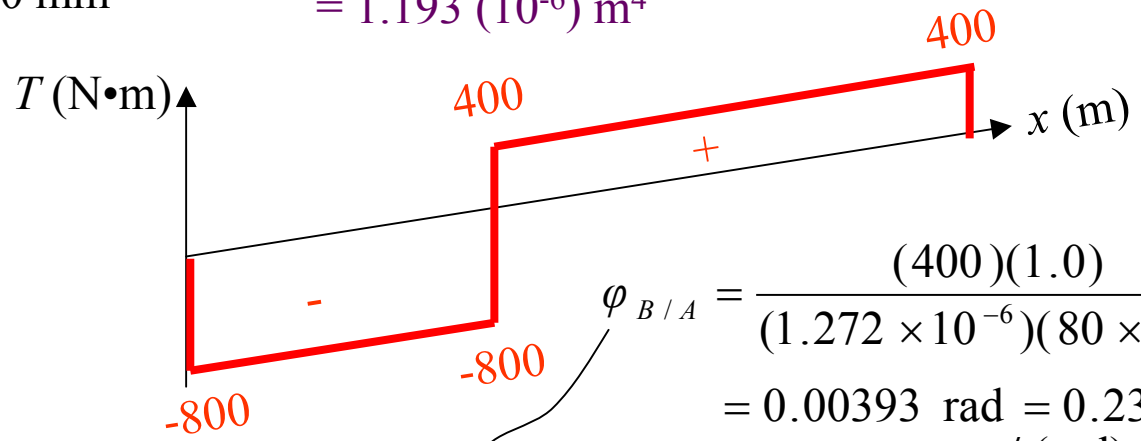
$$\tau_{BC} = \frac{(-800\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(1.193\times10^{-6}\ \text{m}^4)} = -20.12\ \text{MPa}$$



$J = (\pi/2)(c_o^4 - c_i^4)$

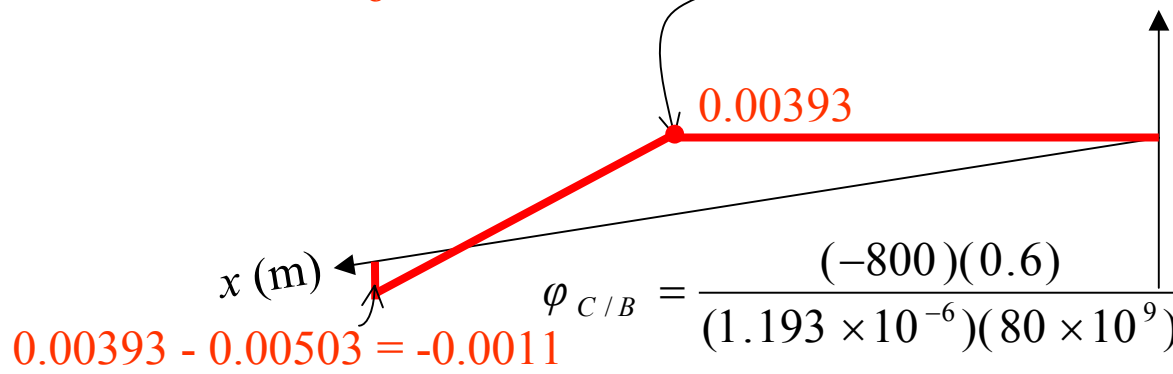
$J_{AB} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ m^4$

$J_{BC} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ m^4$



$$\varphi_{B/A} = \frac{(400)(1.0)}{(1.272\times10^{-6})(80\times10^{9})}$$

$= 0.00393\ \text{rad} = 0.23^o$

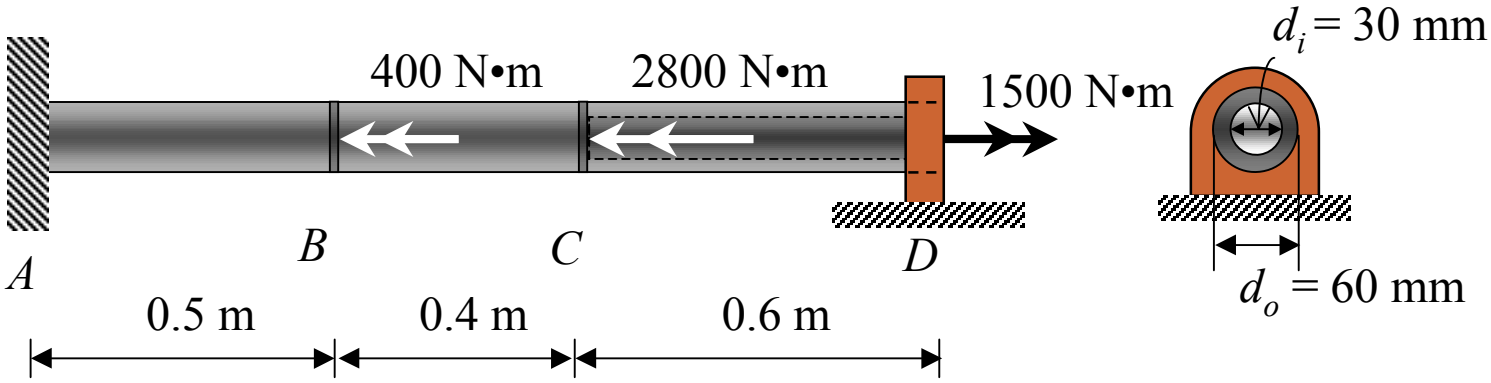


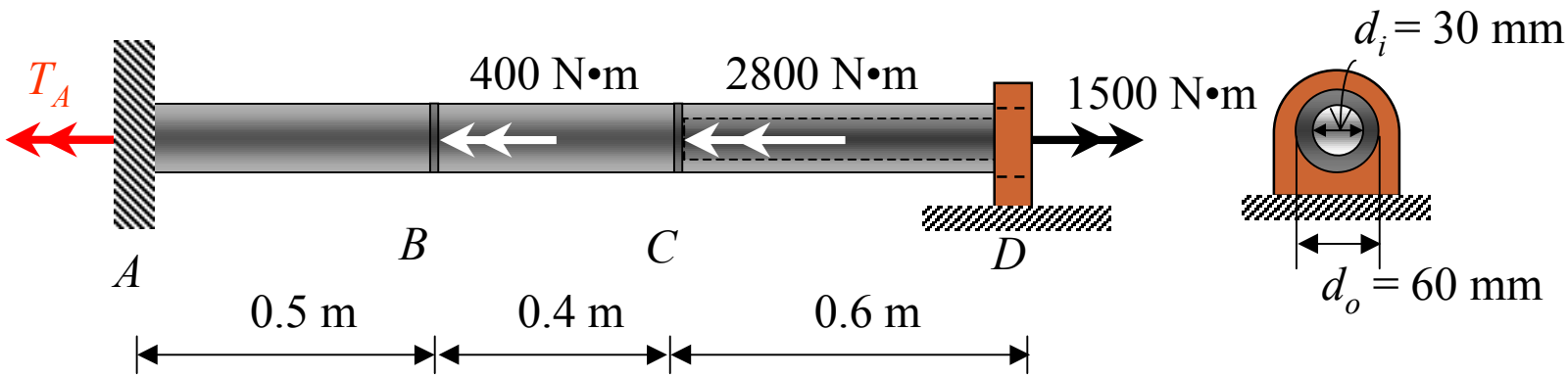
$$\phi = \frac{TL}{JG}$$

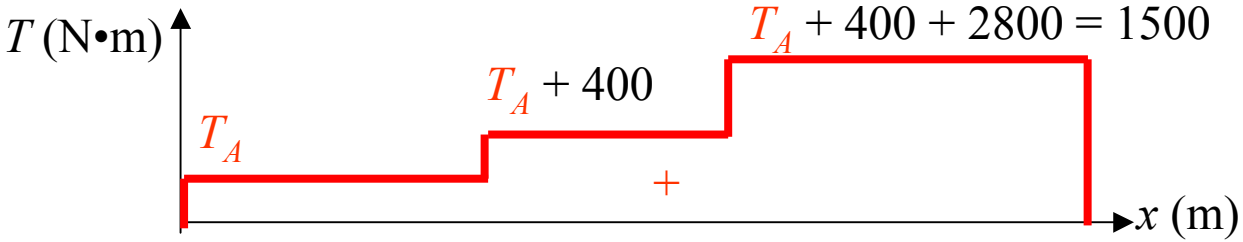
$$\varphi_{C/B} = \frac{(-800)(0.6)}{(1.193\times10^{-6})(80\times10^{9})} = -0.00503\ \text{rad} = -0.29^o$$

Example 4

A solid steel shaft shown in the figure(G = 80 GPa), having an inner diameter of 30 mm and outer diameter of 60 mm. Draw the quantitative torque, shear stress and the angle of twist diagrams of the shaft.



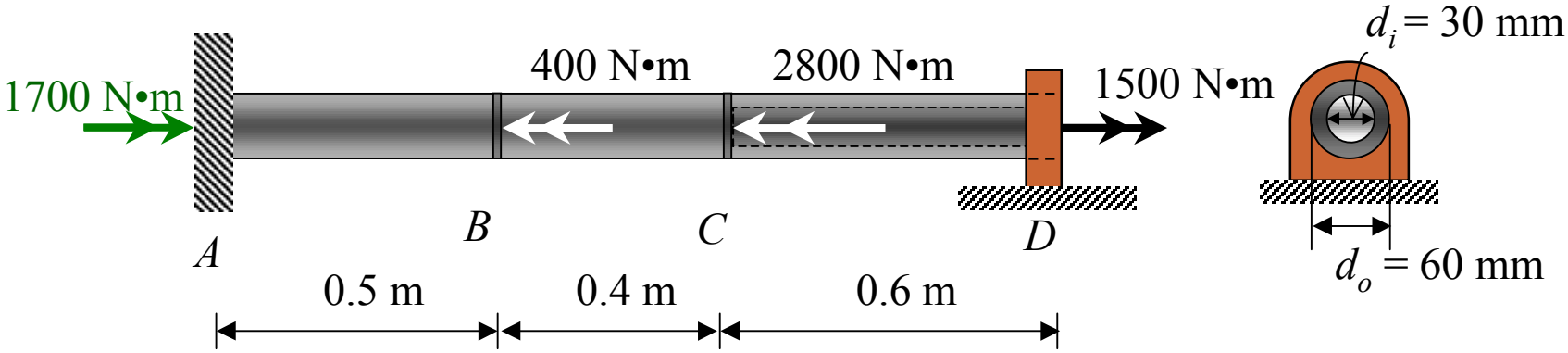




Equilibrium equation:

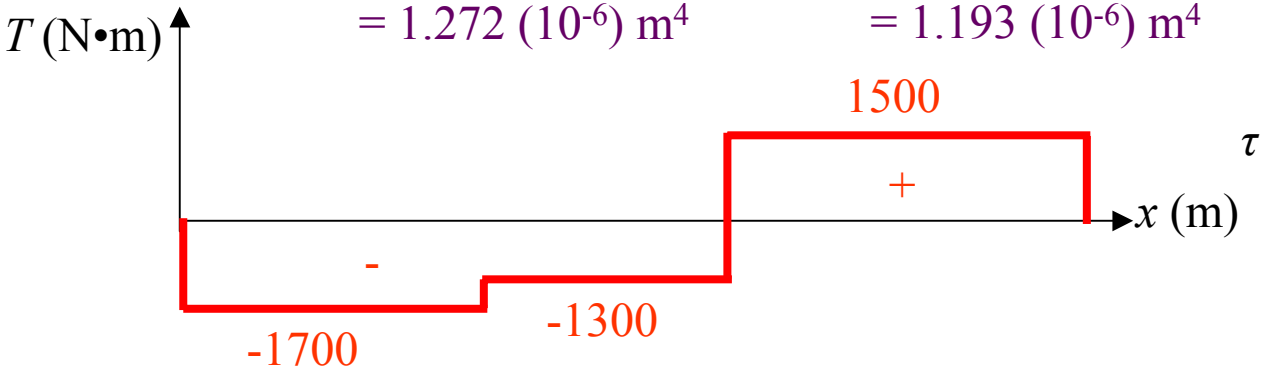
$$T_A + 400 + 2800 - 1500 = 0$$

$$T_A = -1700 \text{ N•m}$$

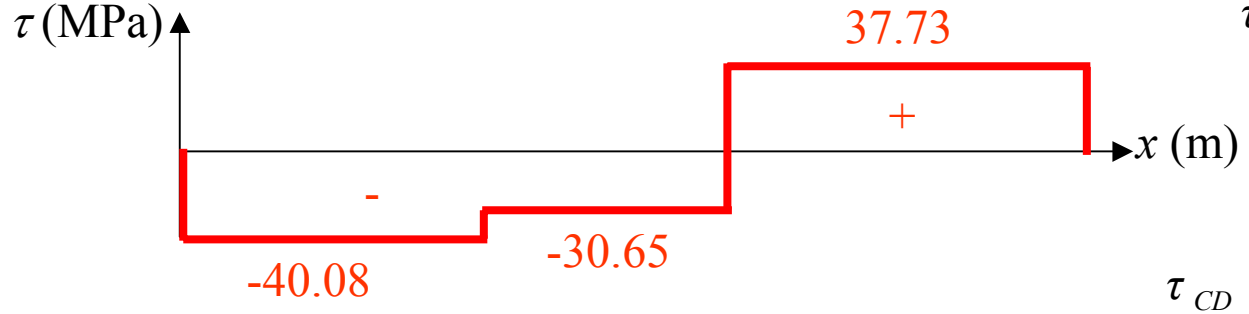


$J_{AC} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ m^4$

$J_{CD} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ m^4$

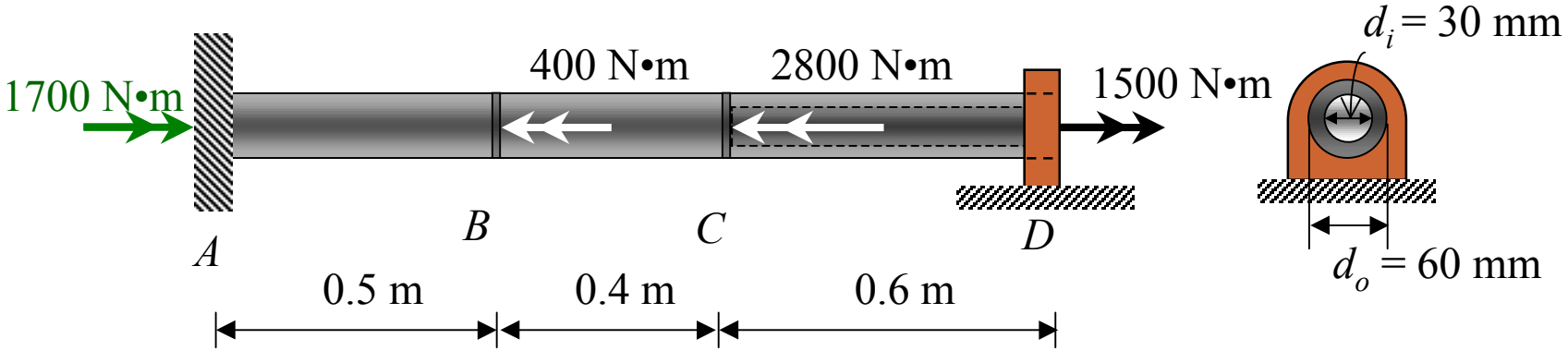


$$\tau_{AB} = \frac{(-1700\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4)\ \text{m}^4} = -40.08\ \text{MPa}$$



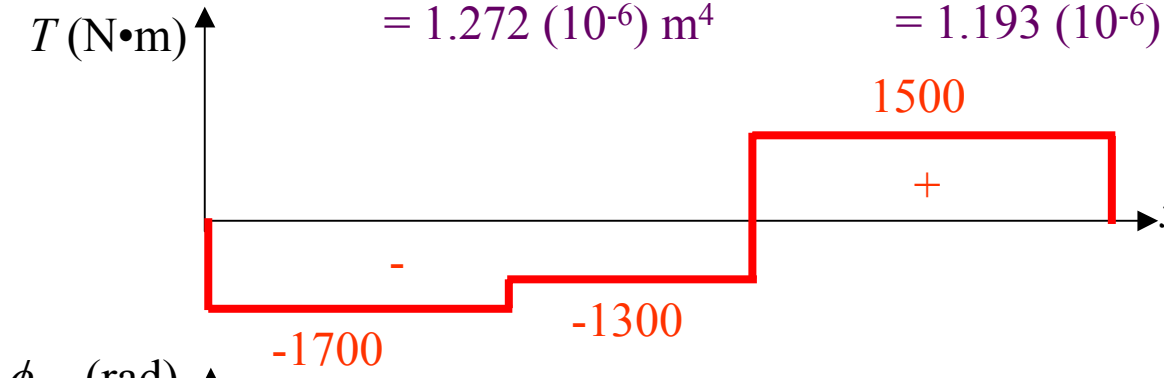
$$\tau_{BC} = \frac{(-1300\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4)\ \text{m}^4} = -30.65\ \text{MPa}$$

$$\tau_{CD} = \frac{(1500\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4 - 0.015^4)\ \text{m}^4} = 37.73\ \text{MPa}$$

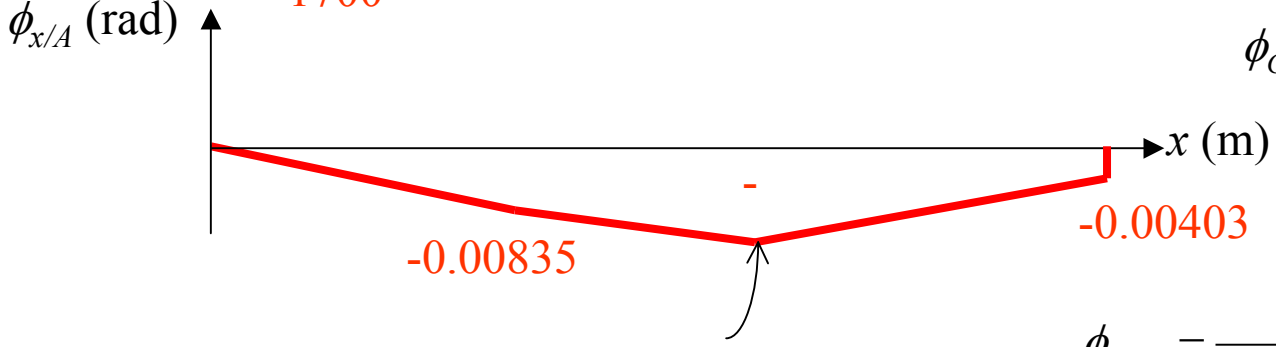


$J_{AC} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ m^4$

$J_{CD} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ m^4$



$$\varphi_{B/A} = \frac{(-1700)(0.5)}{(\pi/2)(0.03^4)(80\times10^9)} = -0.00835 \text{ rad}$$

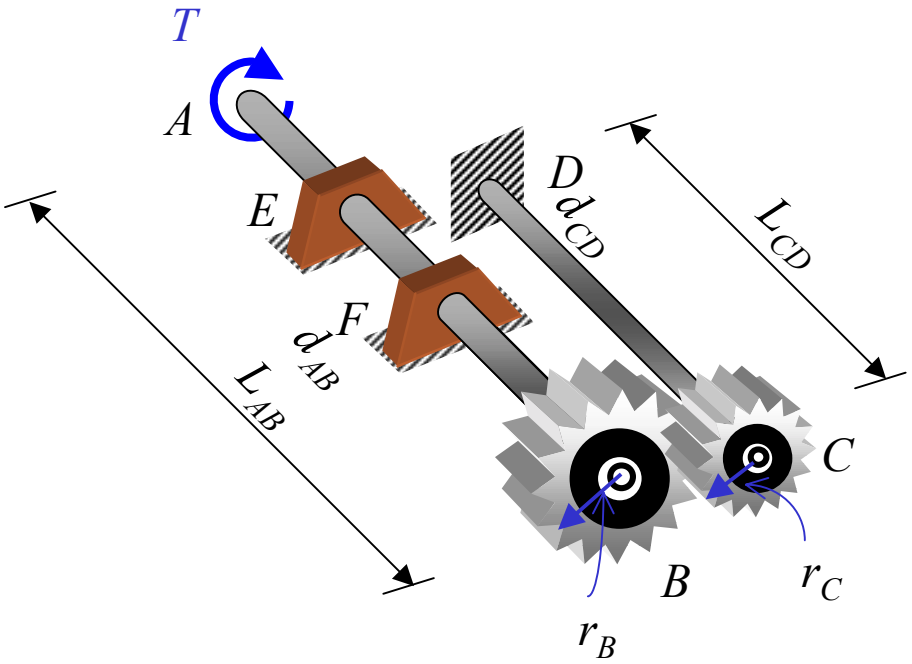


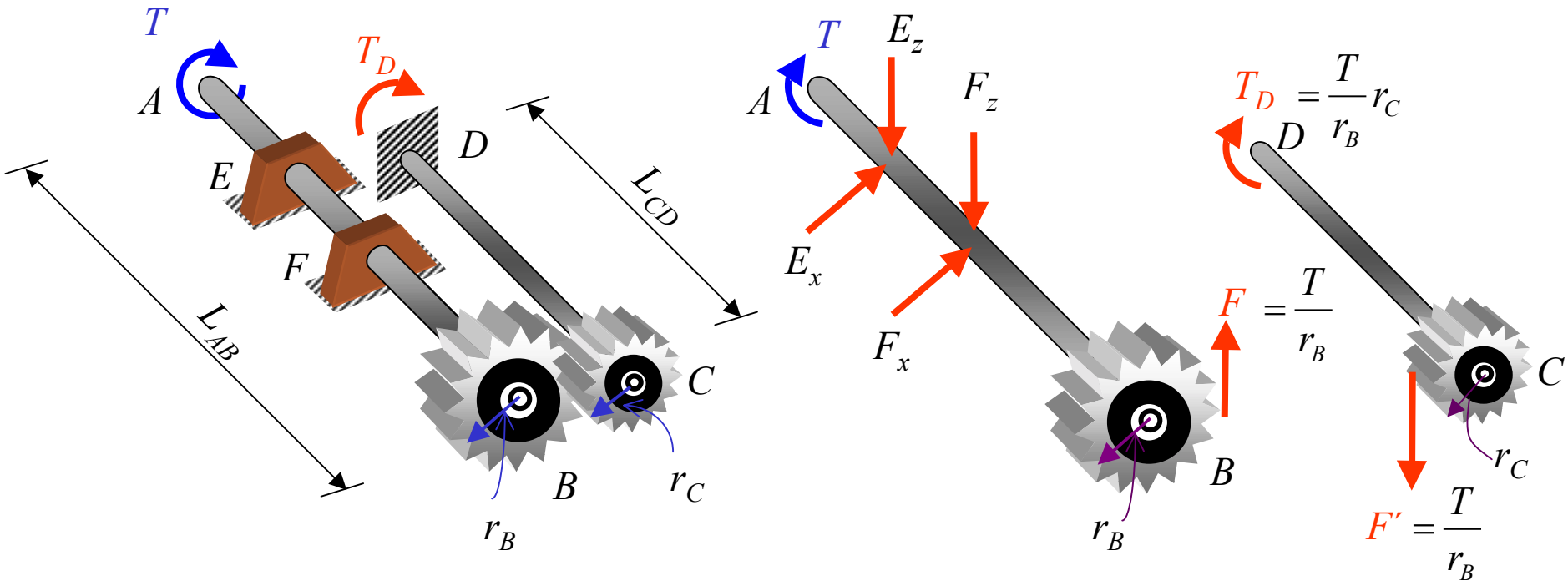
$$\phi_{C/B} = \frac{(-1300)(0.4)}{(\pi/2)(0.03^4)(80\times10^9)} = -0.00511 \text{ rad}$$

-0.00835 -0.00511 = -0.0135

$$\phi_{D/C} = \frac{(1500)(0.6)}{(\pi/2)(0.03^4 - 0.015^4)(80\times10^9)} = 0.00943 \text{ rad}$$

The meshed gears





- ***Internal torque***

$$+\circlearrowright \Sigma M_{AB} = 0: \quad T - F(r_B) = 0$$

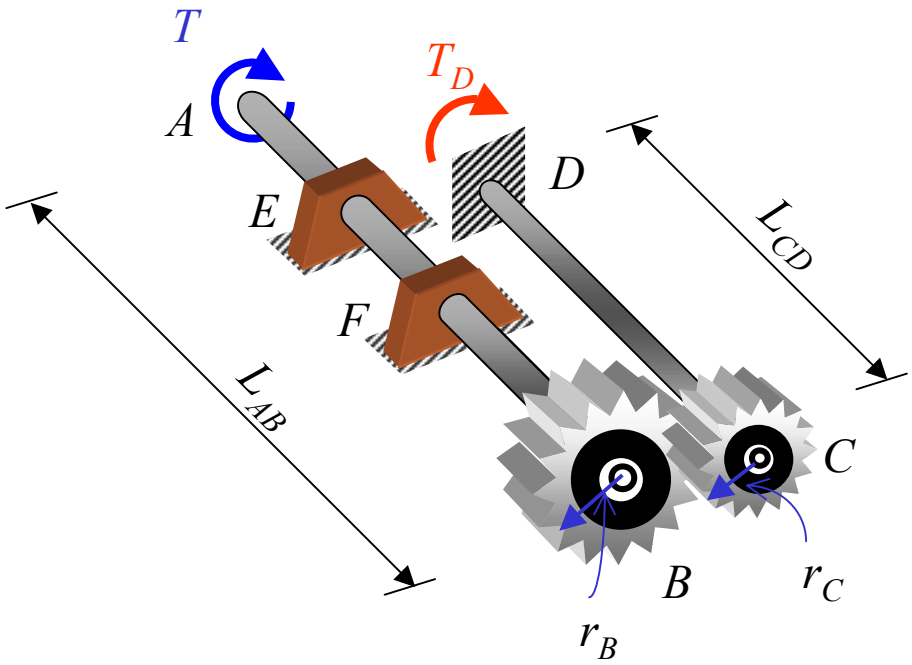
$$F = \frac{T}{r_B}$$

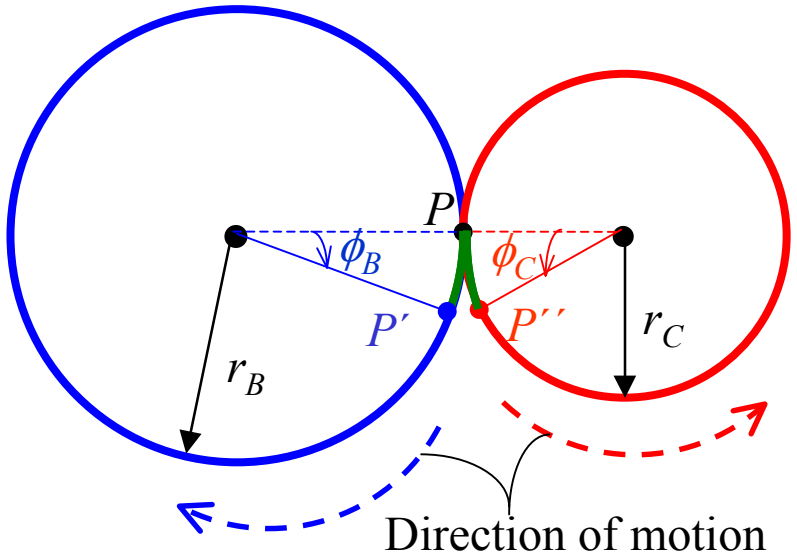
$$F' = F = \frac{T}{r_B}$$

$$+\circlearrowright \Sigma M_{CD} = 0:$$

$$T_D - \frac{T}{r_B} r_C = 0$$

$$\boxed{T_D = \frac{r_C}{r_B} T}$$



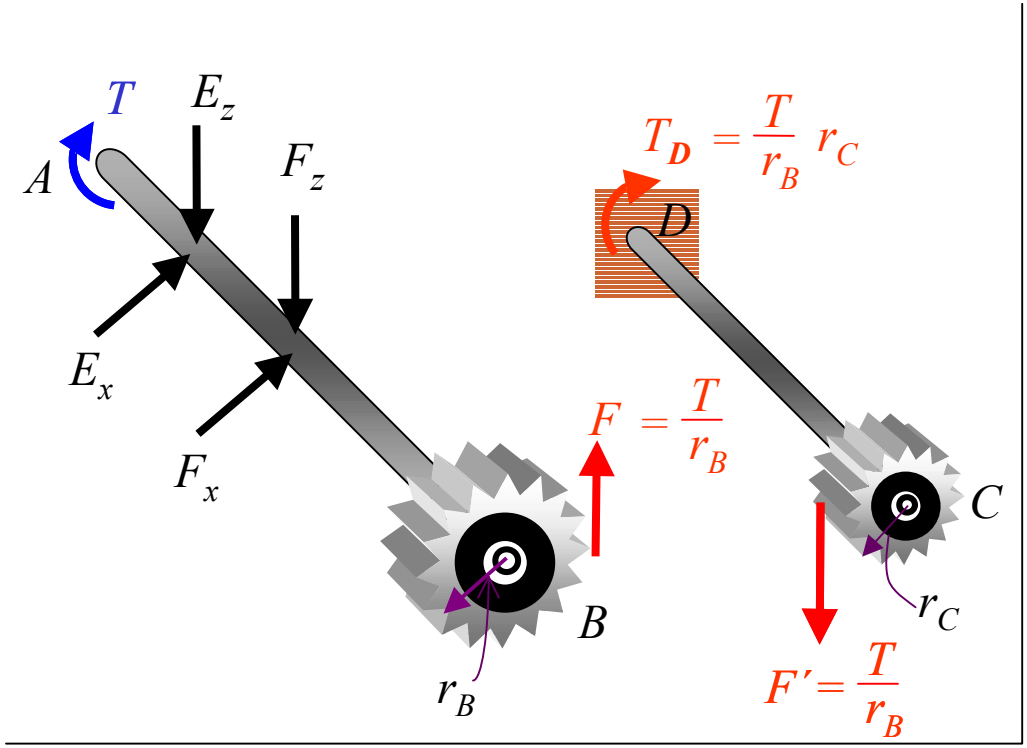


- ***Angle of twist***

$$\text{arc } PP' = \text{arc } PP''$$

$$\phi_B r_B = \phi_C r_C$$

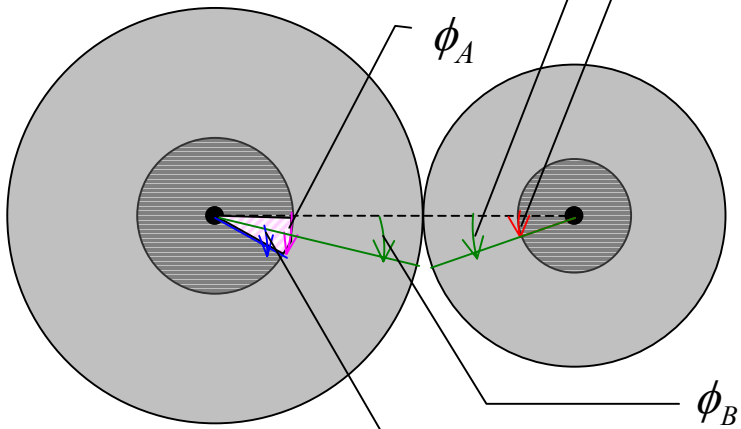
$$\boxed{\phi_B = \frac{r_C}{r_B}\phi_C}$$



Start at $D \rightarrow C \rightarrow B \rightarrow A$

$$\phi_{C/D} = \frac{(\frac{r_C}{r_B} T) L_{CD}}{J_{CD} G_{CD}}$$

$$\phi_C = \phi_{C/D}$$



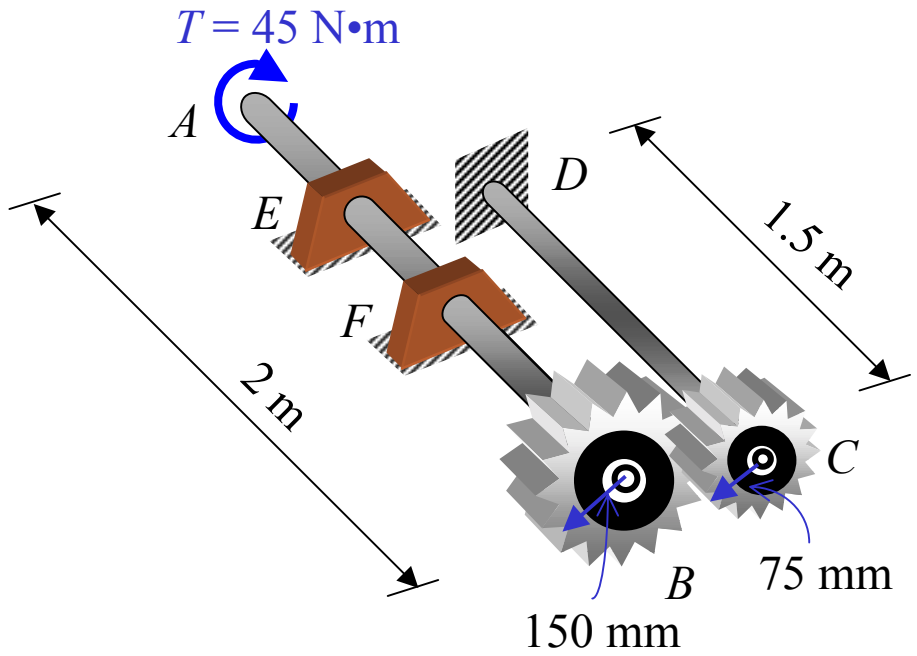
$$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{r_C}{r_B} [\frac{(\frac{r_C}{r_B} T) L_{CD}}{J_{CD} G_{CD}}]$$

$$\phi_{A/B} = \frac{T L_{AB}}{J_{AB} G_{AB}}$$

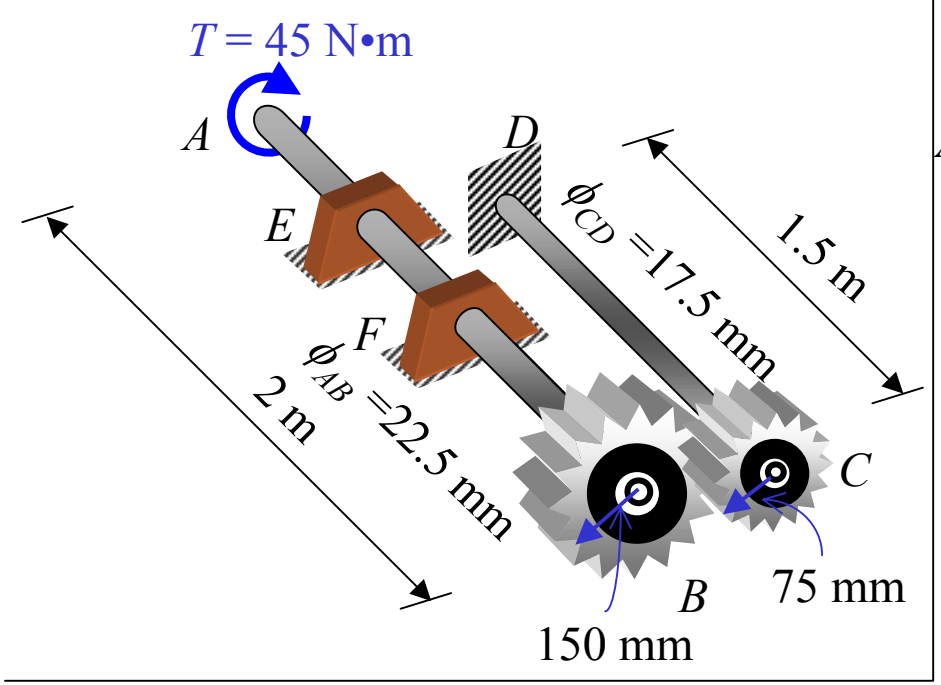
$$\phi_A = \phi_{A/B} + \phi_B = \frac{T L_{AB}}{J_{AB} G_{AB}} + [\frac{(\frac{r_C}{r_B})(\frac{r_C}{r_B} T) L_{CD}}{J_{CD} G_{CD}}]$$

Example 5a

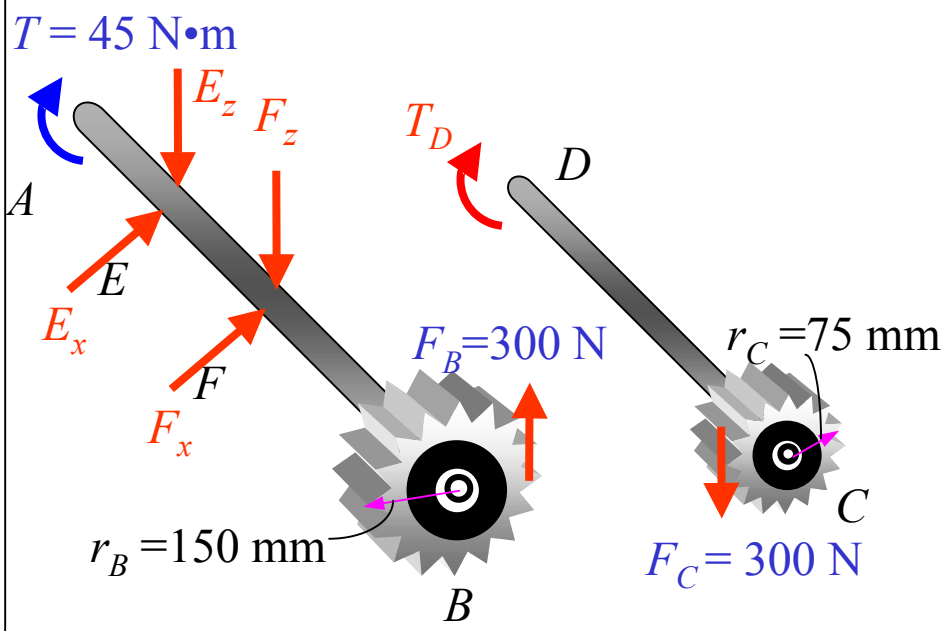
The two solid steel shafts are coupled together using the meshed gears. Determine the shearing stress in two solid and the angle of twist of end *A* of shaft *AB* when the torque $T = 45$ N•m is applied. The upper shaft has a diameter of 17.5 mm and the lower shaft has a diameter of 22.5 mm. Take $G = 80$ GPa. Shaft *AB* is free to rotate within bearings *E* and *F*, whereas shaft *DC* is fixed at *D*.



$G = 80$ MPa.



- **Internal Torque**



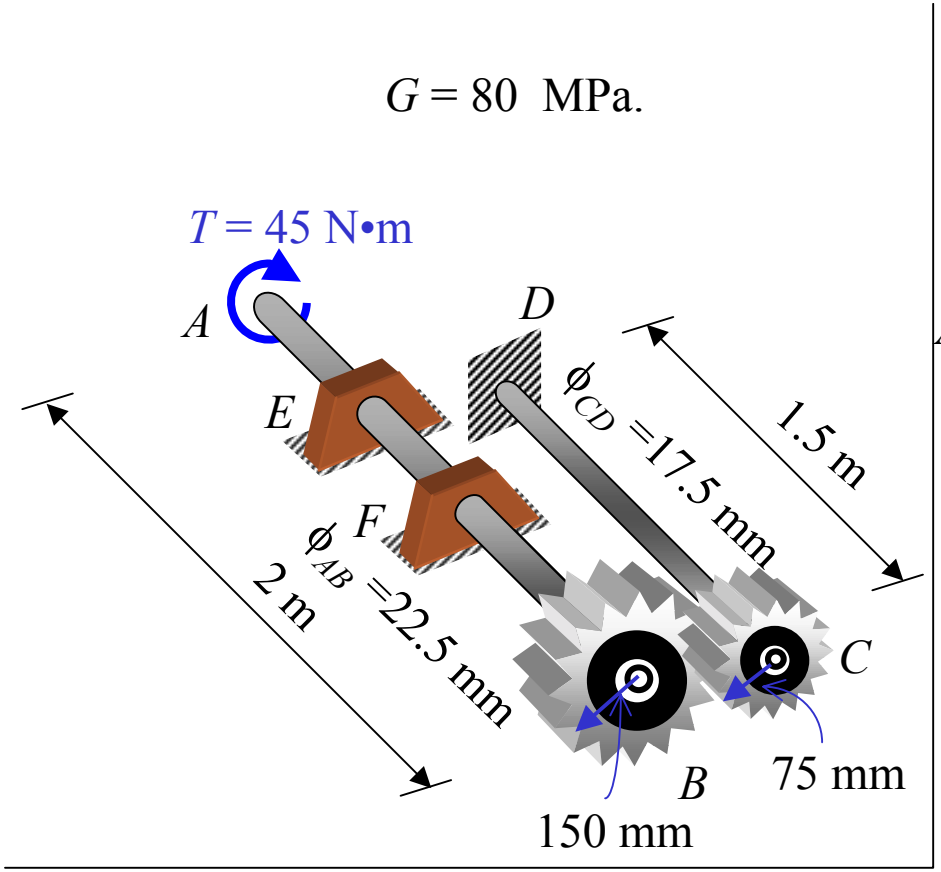
$$T_D = \frac{r_C}{r_B} T_A = \frac{75 \text{ mm}}{150 \text{ mm}} (45 \text{ N} \bullet \text{m})$$

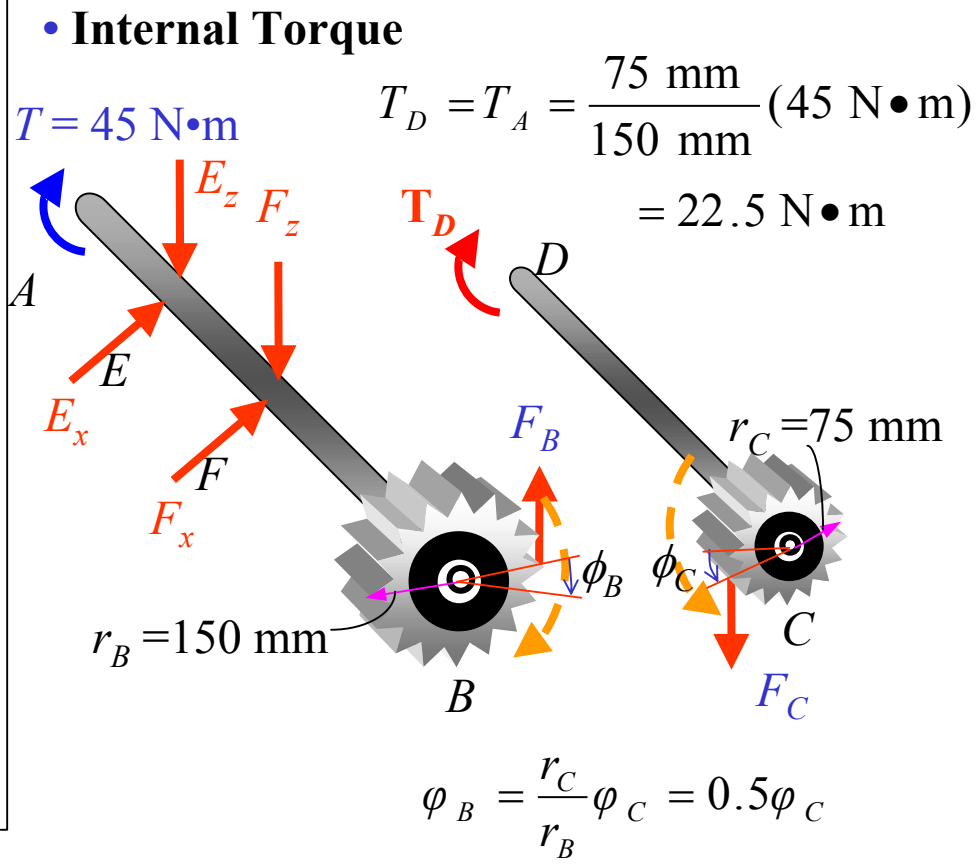
$$= 22.5 \text{ N} \bullet \text{m}$$

- **Shearing Stress**

$$\tau_{AB} = \frac{T_{AB} c_{AB}}{J_{AB}} = \frac{(45 \text{ N} \bullet \text{m})(0.01125 \text{ m})}{(\pi / 2)(0.01125 \text{ m})^4} = 20.12 \text{ MPa} \Leftarrow$$

$$\tau_{CD} = \frac{T_{CD} c_{CD}}{J_{CD}} = \frac{(22.5 \text{ N} \bullet \text{m})(0.00875 \text{ m})}{(\pi / 2)(0.00875 \text{ m})^4} = 21.38 \text{ MPa} \Leftarrow$$





• **Angle of twist of end *A***

$$\phi_A = \phi_{A/B} + \phi_B = \phi_{A/B} + 0.5\phi_C$$

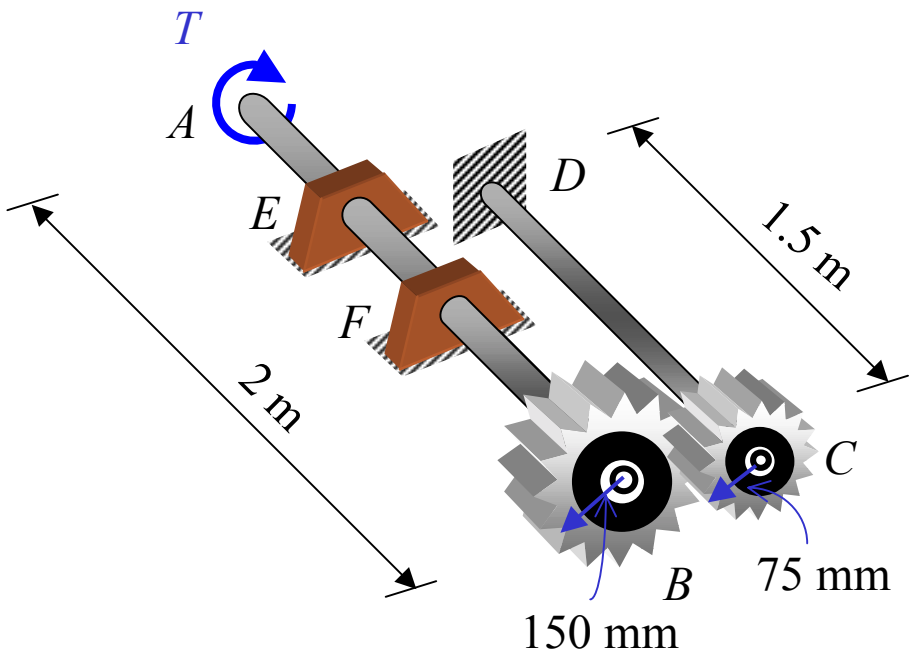
$$\phi_A = \frac{T_{AB}L_{AB}}{J_{AB}G} + \left(\frac{r_C}{r_B}\right)\left[\frac{T_{CD}L_{CD}}{J_{CD}G}\right] = \frac{T\,L_{AB}}{J_{AB}G} + \left(\frac{r_C}{r_B}\right)\left[\left(\frac{r_C}{r_B}\right)\frac{T\,L_{CD}}{J_{CD}G}\right]$$

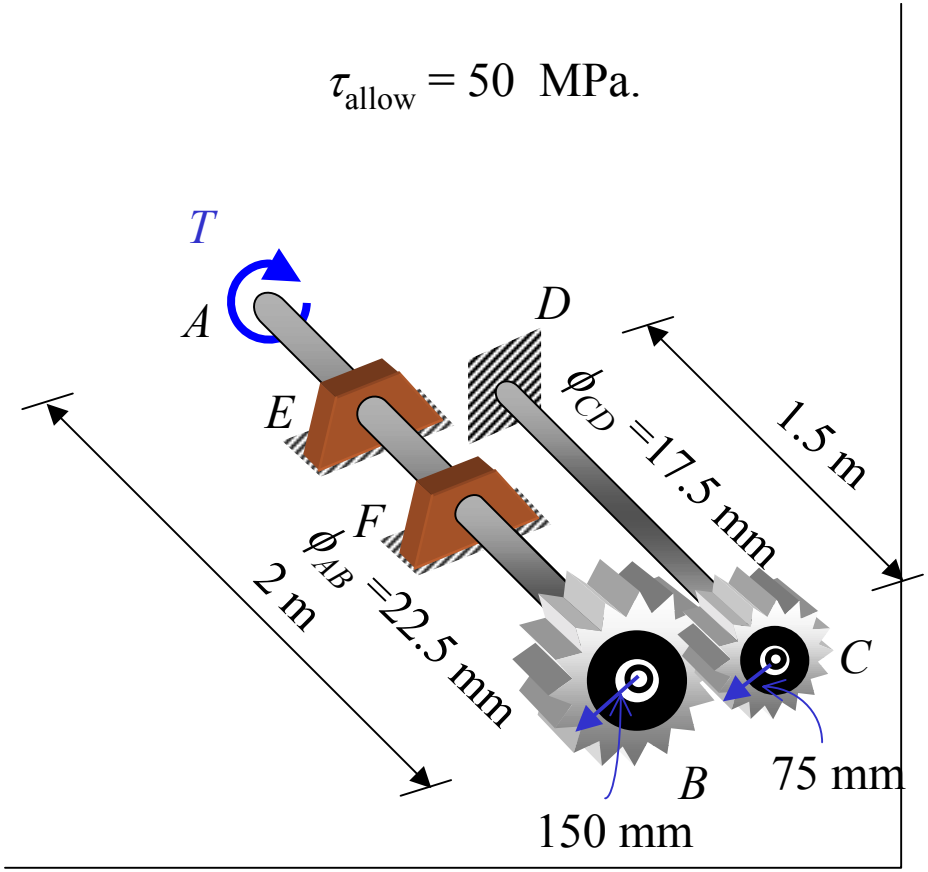
$$\varphi_A = \frac{(45\ \text{N}\bullet\text{m})(2\ \text{m})}{(\pi/2)(0.01125\ \text{m})^4(80\times10^9\ \text{Pa})} + 0.5\left[\frac{(.5)(45\ \text{N}\bullet\text{m})(1.5\ \text{m})}{(\pi/2)(0.00875\ \text{m})^4(80\times10^9\ \text{Pa})}\right]$$

$$\phi_A = 0.0447 + 0.0229 = 0.0676\ \text{rad} = 0.0676(180/\pi) = 3.87^\circ$$

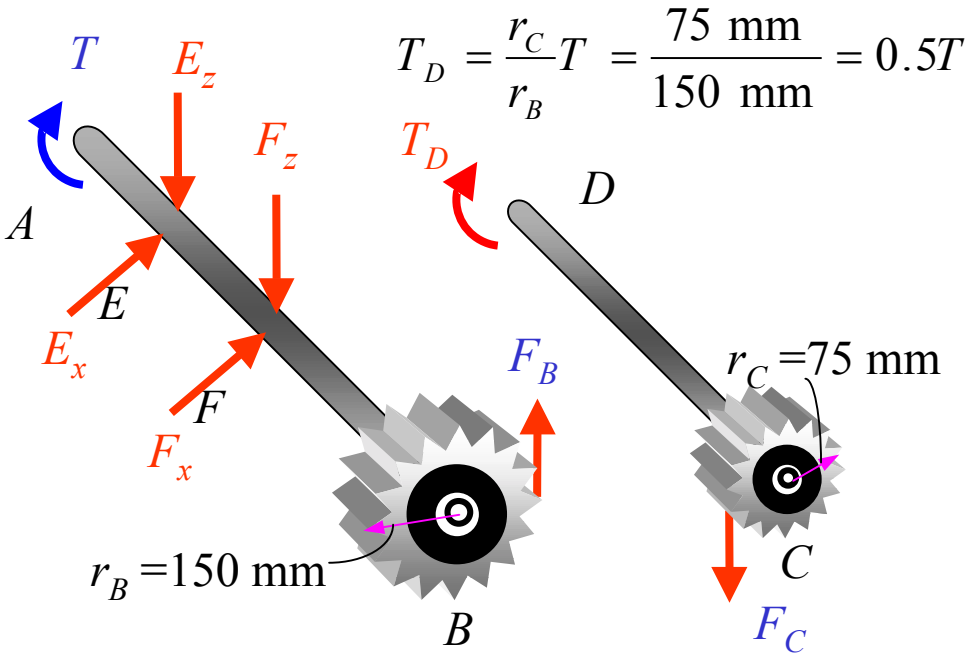
Example 5b

The two solid steel shafts are coupled together using the meshed gears. Shaft *AB* is free to rotate within bearings *E* and *F*, whereas shaft *DC* is fixed at *D*. The upper shaft has a diameter of 17.5 mm and the lower shaft has a diameter of 22.5 mm. Determine the maximum torque the shaft can have if it is restricted not to twist more than 10^{o} and the allowable shear stress is $\tau_{allow} = 50$ MPa. Take $G = 80$ GPa.





- **Internal Torque**

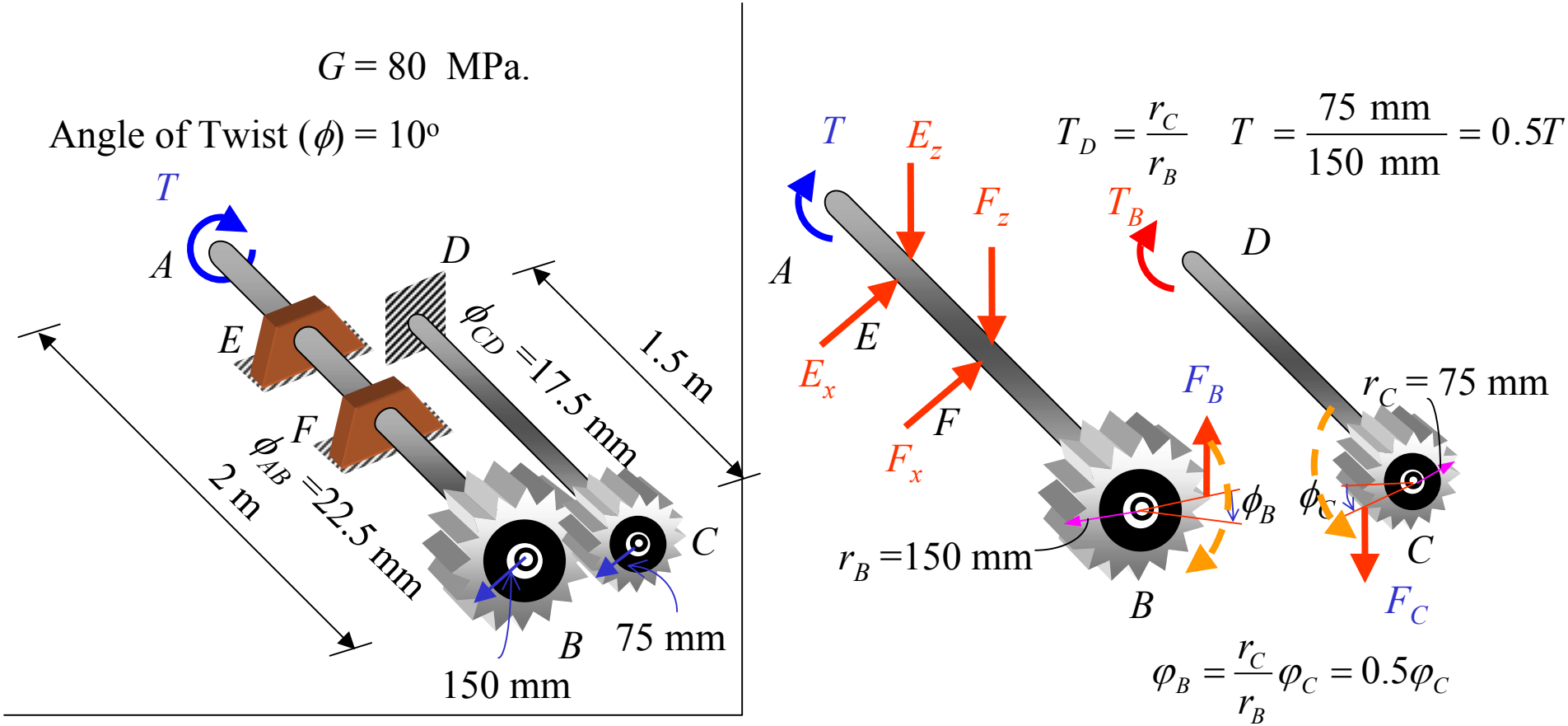


$$T_D = \frac{r_C}{r_B}T = \frac{75\text{ mm}}{150\text{ mm}} = 0.5T$$

- **Allowable Shearing Stress:** $\tau = Tc/J$

$$(\tau_{AB})_{allow} = 50\text{x}10^6\text{ Pa} = \frac{(T)(0.01125\text{ m})}{(\pi/2)(0.01125\text{ m})^4}, \qquad T_1 = 111.8\text{ N•m}$$

$$(\tau_{CD})_{allow} = 50\text{x}10^6\text{ Pa} = \frac{(0.5T)(0.00875)}{(\pi/2)(0.00875\text{ m})^4}, \qquad T_2 = 105.2\text{ N•m}$$



- **Angle of twist**

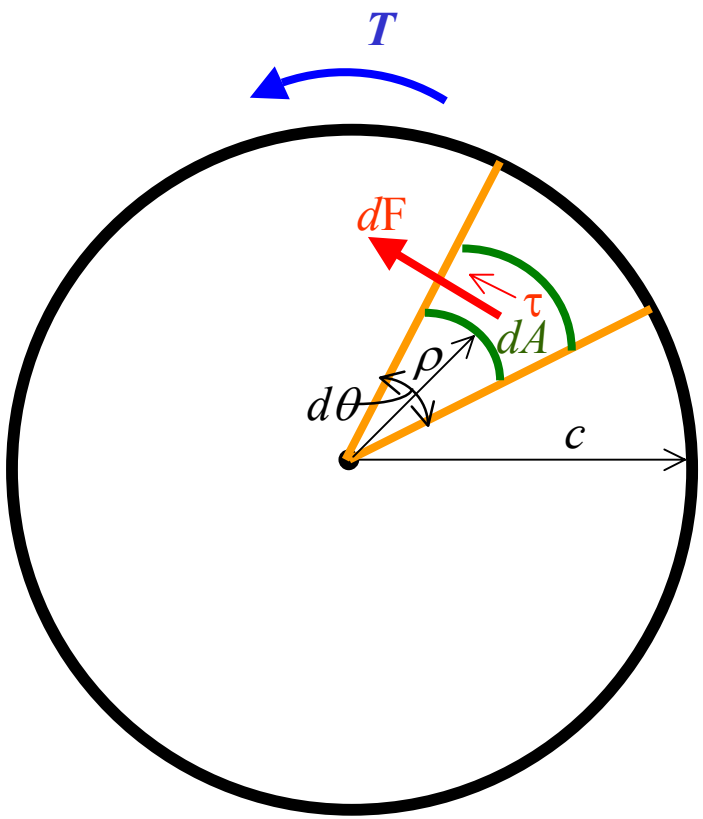
$$\phi_A = \phi_{A/B} + \phi_B = \phi_{A/B} + 0.5\phi_C$$

$$10\left(\frac{\pi}{180}\right)\text{ rad} = \frac{T_A L_{AB}}{J_{AB} G} + 0.5\left[\frac{T_D L_{CD}}{J_{CD} G}\right]$$

$$10\left(\frac{\pi}{180}\right)\text{ rad} = \frac{T\ (2\text{ m})}{(\pi/2)(0.01125\text{ m})^4 (80\times10^9\ Pa)} + 0.5\left[\frac{(0.5T\ )(1.5\text{ m})}{(\pi/2)(0.00875\text{ m})^4 (80\times10^9\ \text{Pa})}\right]$$

T_3= 116.15 N•m ⟶ The maximum torque at A is 105.2 N•m ⟸

Power transmission



Power = Work performed per unit of time

P = Power in N•m/s, or W or watts
1 W = 1 N•m/s

$dP = (dF)\ \rho\ (d\theta)/(dt)$
$=(dT)\ \omega$

$P = T\,\omega$

ω = Angular velocity, in rad/s

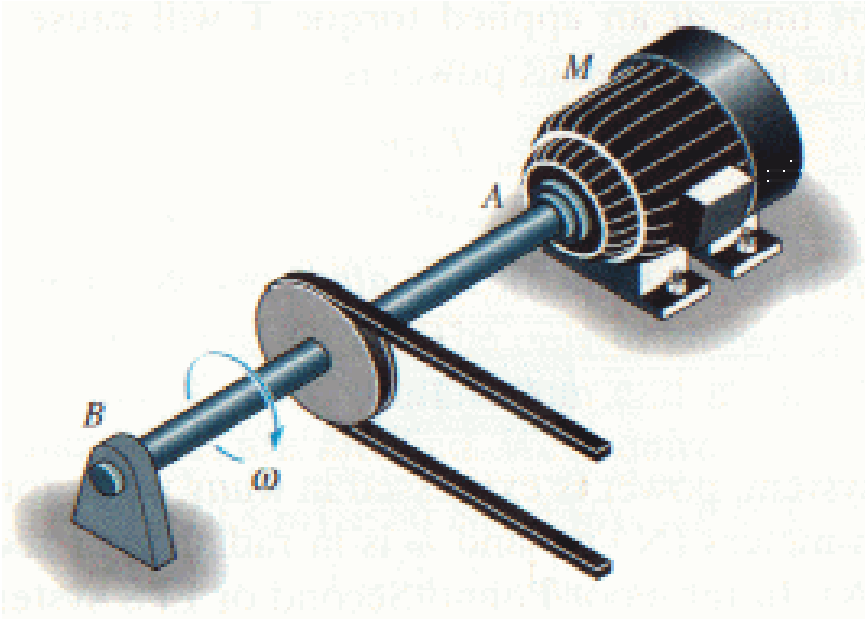
$P = 2\pi f\,T$

f = Frequency, in Hz

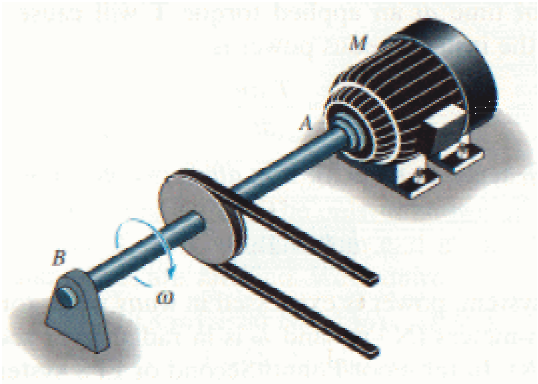
Note
1 hp = 745.7 N•m/s

Example 6a

A solid steel shaft *AB* is to be used to transmit 5 hp from the motor M to which it is attached. If the shaft rotates at ω = 175 rpm and the steel has an allowable shear stress of τ_{allow} = 100 MPa, determine the required diameter of the shaft.



Solution



5 hp

$\omega = 175$ rpm

$\tau_{allow} = 100$ MPa

$\phi = ?$

$$\Rightarrow \quad P = T\omega$$

$$(5 \text{ hp})\left(\frac{745.7 \text{ N} \bullet \text{m/s}}{1 \text{ hp}}\right) = T\left[\frac{175 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}}\right]$$

$$T = 203.4 \text{ N} \bullet \text{m}$$

$$\Rightarrow \quad \tau_{allow} = \frac{Tc}{J} = \frac{Tc}{(\pi/2)(c^4)} = \frac{2T}{\pi(c^3)}$$

$$c = \left[\frac{2T}{\pi\tau_{allow}}\right]^{1/3}$$

$$= \left[\frac{2(203.41 \text{ N} \bullet \text{m})}{\pi(100 \times 10^6 \text{ N/m}^2)}\right]^{1/3}$$

$$= 10.90 \text{ mm}$$

$$d = 2c = 2(10.90 \text{ mm}) = 21.8 \text{ mm}$$

Select a shaft having a diameter of 22 mm

Example 6b

A tubular shaft, having an inner diameter of 30 mm and an outer diameter of 42 mm, is to be used to transmit 90 kW of power. Determine the frequency of rotation of the shaft so that the shear stress will not exceed 50 MPa.

Solution

$$\tau_{\max} = \frac{Tc}{J}$$

$$50 \times 10^6 \text{ N/m}^2 = \frac{T\,(0.021 \text{ m})}{(\pi / 2)\left[(0.021 \text{ m})^4 - (0.015 \text{ m})^4\right]}$$

$$T = 538 \text{ N} \bullet \text{m}$$

$$P = 2\pi f T$$

$$90 \times 10^3 \text{ N} \bullet \text{m/s} = 2\pi f\,(538 \text{ N} \bullet \text{m})$$

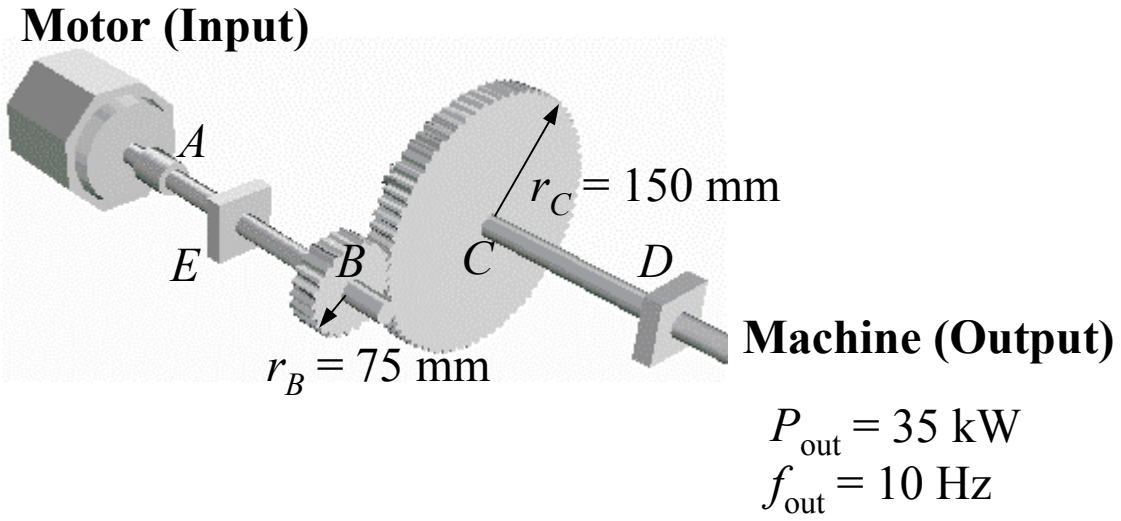
$$f = 26.6 \text{ Hz}$$

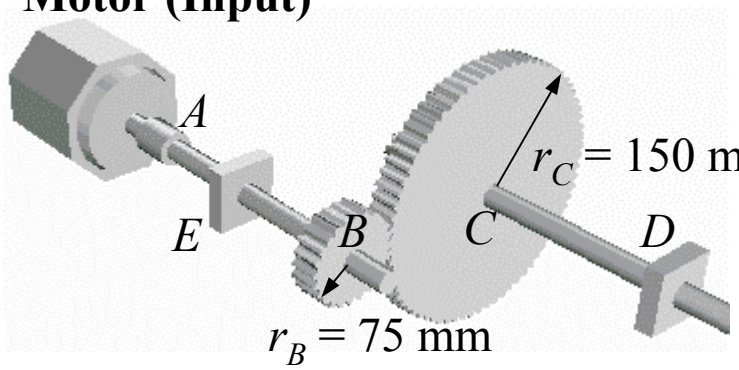
Example 7

The two solid steel shaft are coupled together using the meshed gears. Shaft *CD* connected to the machine which required power of 35 kW at frequency 10 Hz, whereas shaft *AB* connected to motor to rotate within bearing *E*.

(a) Determine the required power and frequency of motor.

(b) If the allowable stress in both shafts must be limited to 200 MPa, determine the size of the diameter required for each shaft.





$P_{\text{out}} = 35$ kW

$f_{\text{out}} = 10$ Hz

- **Internal torque**

-Output (Machine)

$$P = Tw = T(2\pi f)$$

$$35 \times 10^3 = T_D(2\pi \times 10)$$

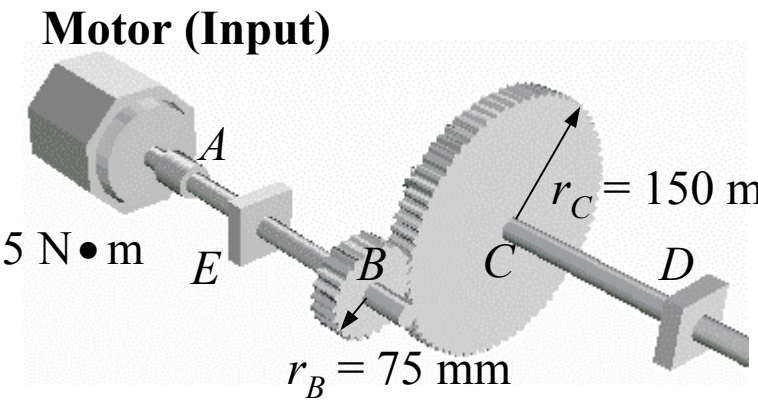
$$T_C = T_D = 557 \text{ N} \bullet \text{m}$$

-Input (Motor)

$$F_B = F_C$$

$$\frac{T_B}{r_B} = \frac{T_C}{r_C}$$

$$T_B = \frac{r_B}{r_C} T_C = \frac{75}{150}(557 \text{ N} \bullet \text{m}) = 278.5 \text{ N} \bullet \text{m}$$



$T_A = T_B = 278.5\ \text{N} \bullet \text{m}$

allowable stress in both shaft : $\tau_{allow} = 200$ MPa

$P_{\text{out}} = 35$ kW

$f_{\text{out}} = 10$ Hz

$T_C = T_D = 557\ \text{N} \bullet \text{m}$

- **(a) The power and frequency of motor**

$$f_B = \frac{r_C}{r_B} f_C$$
$$= 2f_C$$
$$= 2(10\ \text{Hz}) = 20\ \text{Hz}$$

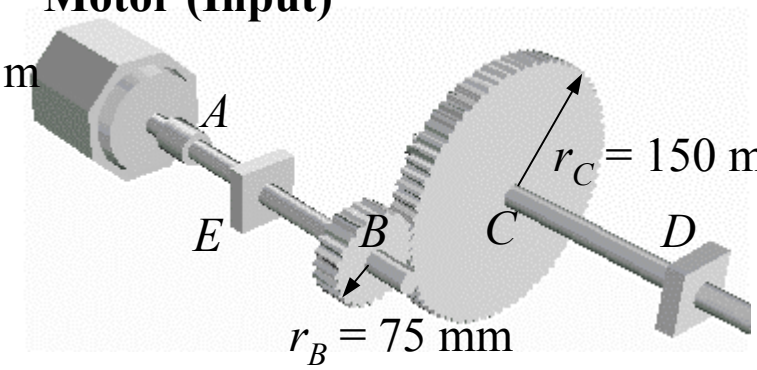
$$P_{input} = T_B(2\pi f_B)$$
$$= (278.5\ \text{N} \bullet \text{m})(2\pi \times 20\ \text{Hz})$$
$$= 35\ \text{kW}$$

Comment : $P_{input} = T_B(2\pi f_B)$

$$= (\frac{T_C}{2})(2\pi \times 2f_C) = T_C(2\pi f)$$

Motor (Input)

$T_A = T_B = 278.5\ \text{N}\bullet\text{m}$



Machine (Output)

$P_{\text{out}} = 35$ kW

$f_{\text{out}} = 10$ Hz

$T_C = T_D = 557\ \text{N}\bullet\text{m}$

allowable stress in both shaft : $\tau_{\text{allow}} = 200$ MPa

- **(b) Allowable diameter for each shaft**

- Shaft AB

$$(\tau_{AB})_{allow} = \frac{T_B c_{AB}}{J_{AB}}$$

$$200\times10^6\ \text{Pa} = \frac{(278.5\ \text{N}\bullet\text{m})c_{AB}}{(\pi/2)c_{AB}^4}$$

$$c_{AB} = \left[\frac{(278.5\ \text{N}\bullet\text{m})}{(\pi/2)(200\times10^6\ \text{Pa})}\right]^{1/3}$$

$$= 9.6\times10^{-3}\ \text{m}$$

$$d_{AB} = 19.2\times10^{-3}\ \text{m} \approx 20\ \text{mm} \Leftarrow$$

- Shaft CD

$$(\tau_{CD})_{allow} = \frac{T_C c_{CD}}{J_{CD}}$$

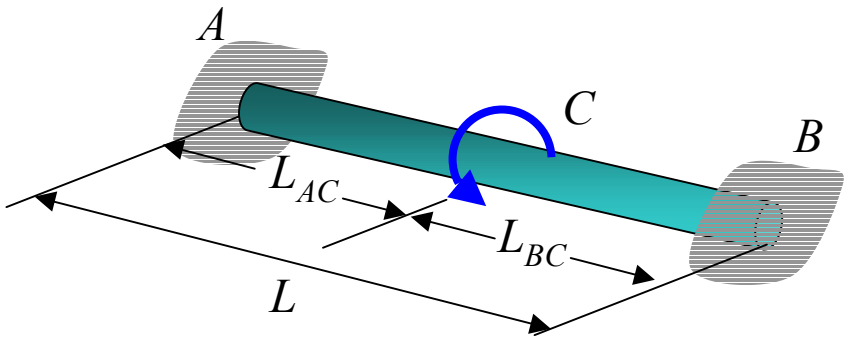
$$200\times10^6\ \text{Pa} = \frac{(557\ \text{N}\bullet\text{m})c_{CD}}{(\pi/2)c_{CD}^4}$$

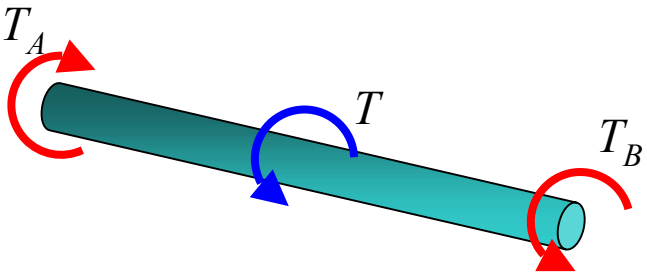
$$c_{CD} = \left[\frac{(557\ \text{N}\bullet\text{m})}{(\pi/2)(200\times10^6\ \text{Pa})}\right]^{1/3}$$

$$= 0.0121\ \text{m}$$

$$d_{CD} = 0.0242\ \text{m} \approx 25\ \text{mm} \Leftarrow$$

Statically Indeterminate Torque-Loaded Members





- Equilibrium equation

$$+\circlearrowleft \Sigma M_x = 0: \qquad -T_A + T + T_B = 0 \qquad \text{----------(1)}$$

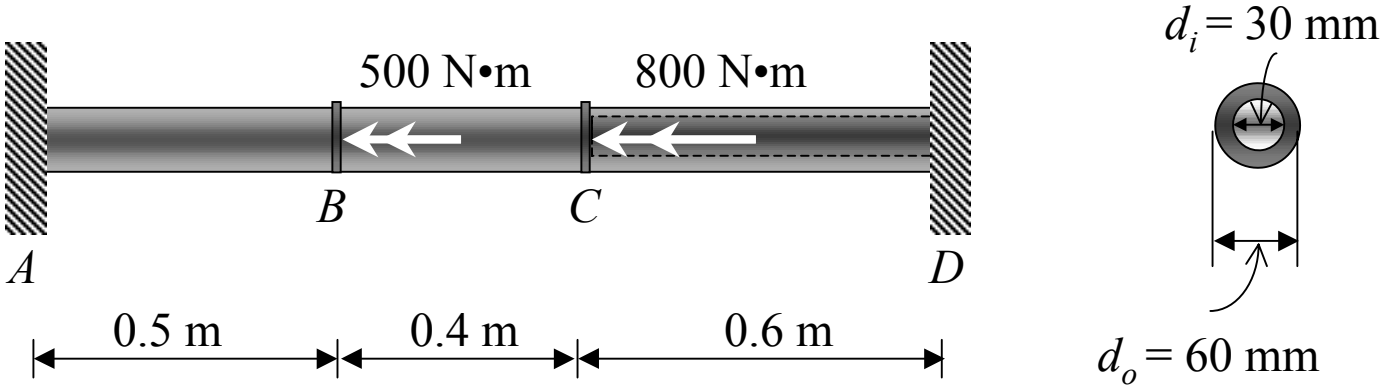
- Compatibility equation:

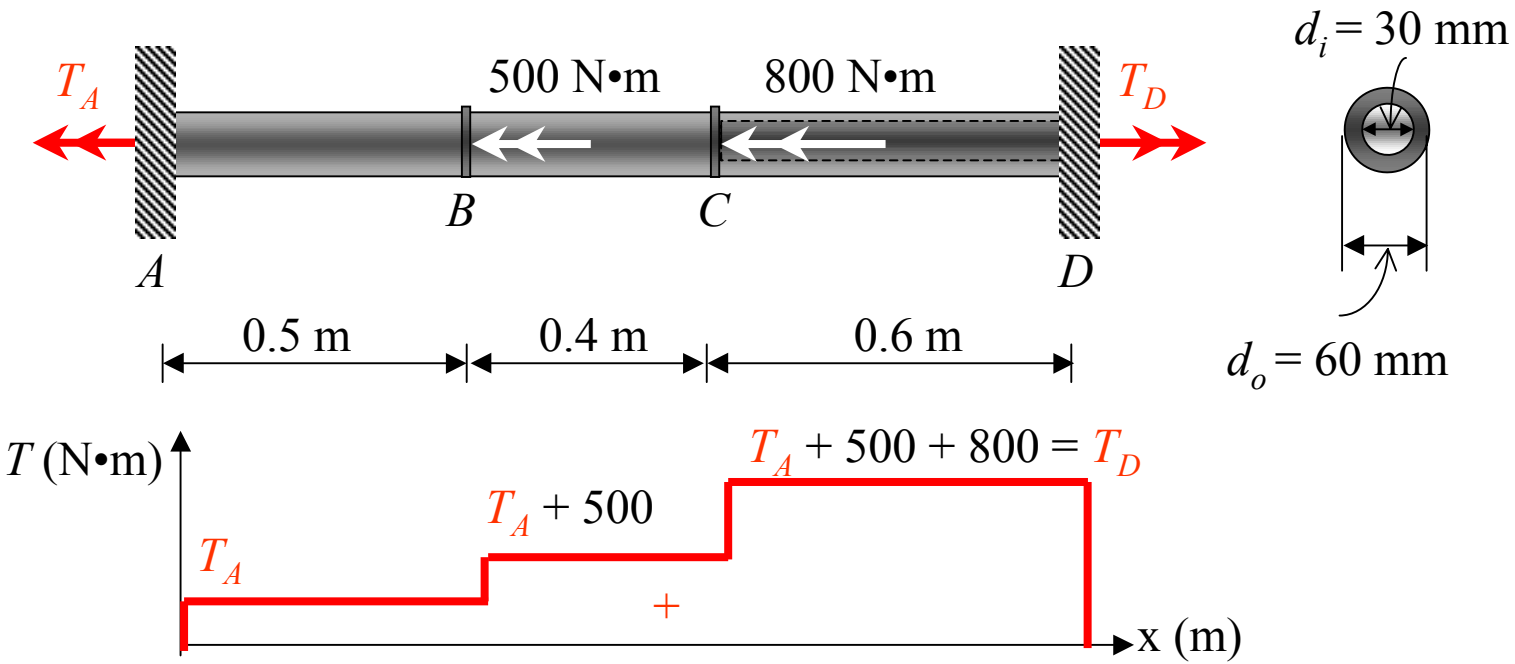
$$\phi_{A/B} = \phi_{A/C} + \phi_{B/C} = 0$$

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0 \qquad \text{------(2)}$$

Example 8

A solid steel shaft shown in the figure(G = 80 GPa), having an inner diameter of 30 mm and outer diameter of 60 mm.
(a) Determine the reactions at the fixed supports A and D.
(b) Draw the quantitative torsion,shear stress and the angle of twist diagram of the shaft.





Equilibrium equation:

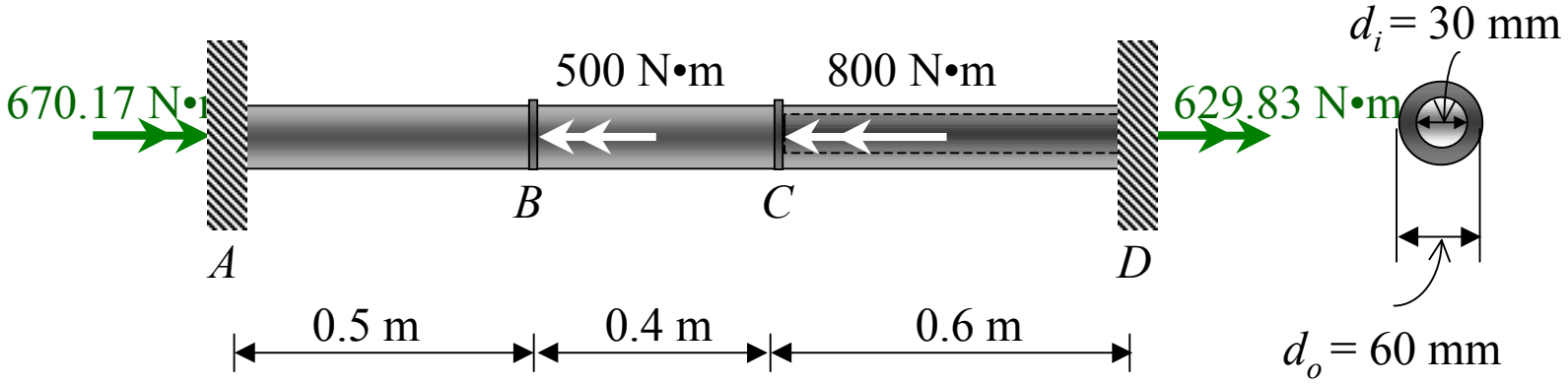
$$T_A + 500 + 800 = T_D$$

Compatibility equation:

$$\phi_{D/A} = \phi_{B/A} + \phi_{C/B} + \phi_{D/C} = 0$$

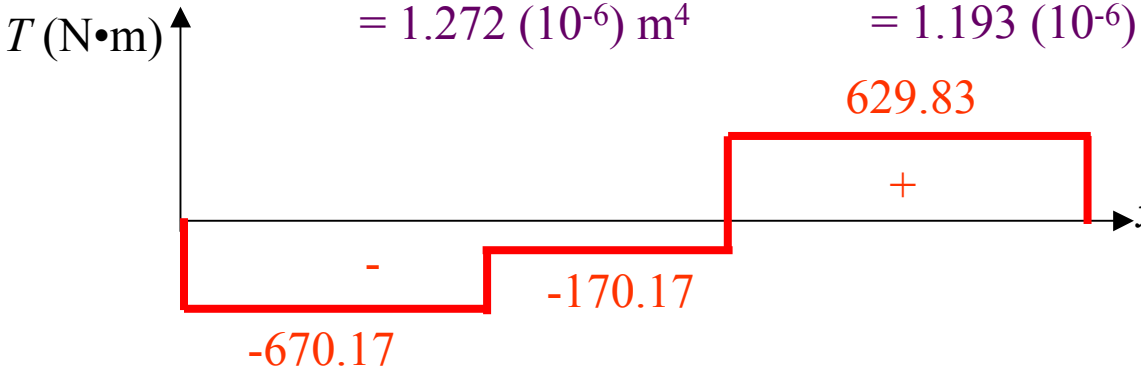
$$\frac{T_A(0.5)}{(\pi/2)(0.03^4)(80\times10^9)} + \frac{(T_A+500)(0.4)}{(\pi/2)(0.03^4)(80\times10^9)} + \frac{(T_A+1300)(0.6)}{(\pi/2)(0.03^4-0.015^4)(80\times10^9)} = 0$$

T_A = -670.17 N•m, T_D = -670.17+500+800 = 629.8 N•m

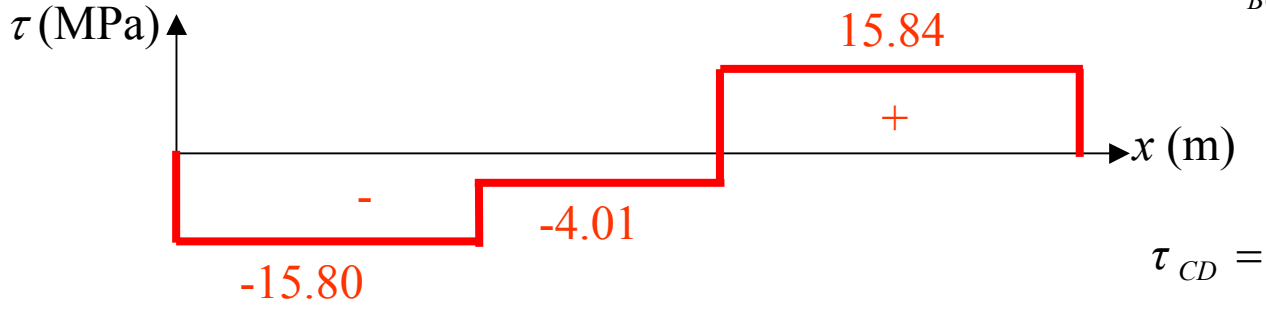


$J_{AC} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ m^4$

$J_{CD} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ m^4$

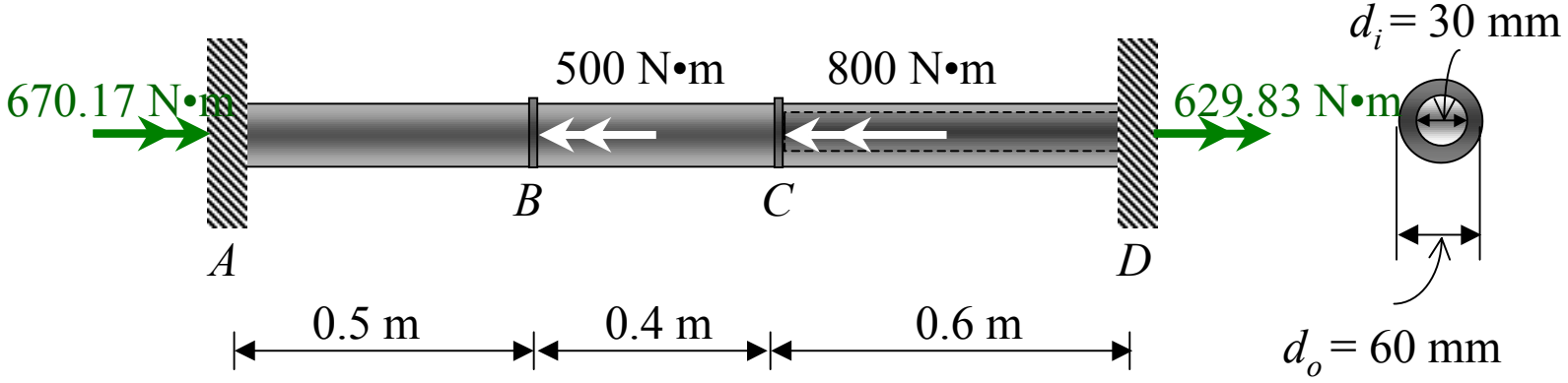


$$\tau_{AB} = \frac{(-670.17\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4)\ \text{m}^4} = -15.80\ \text{MPa}$$



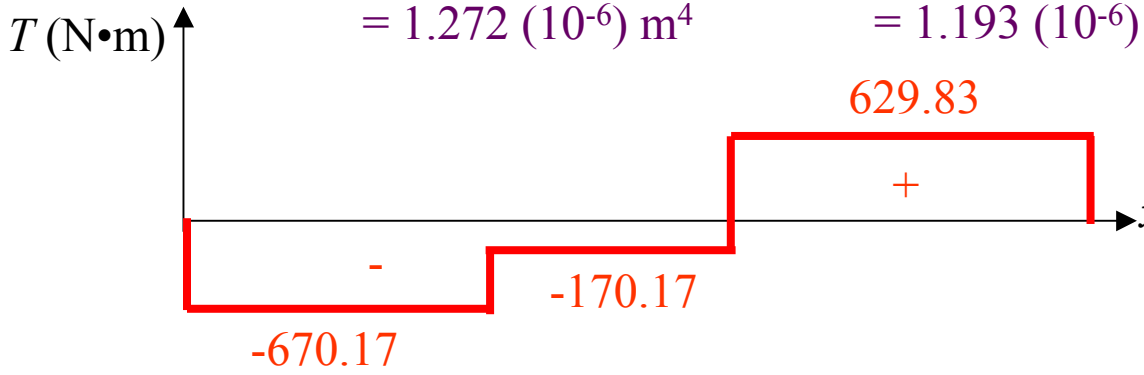
$$\tau_{BC} = \frac{(-170\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4)\ \text{m}^4} = -4.01\ \text{MPa}$$

$$\tau_{CD} = \frac{(629.83\ \text{N}\bullet\text{m})(0.03\ \text{m})}{(\pi/2)(0.03^4 - 0.015^4)\ \text{m}^4} = 15.84\ \text{MPa}$$

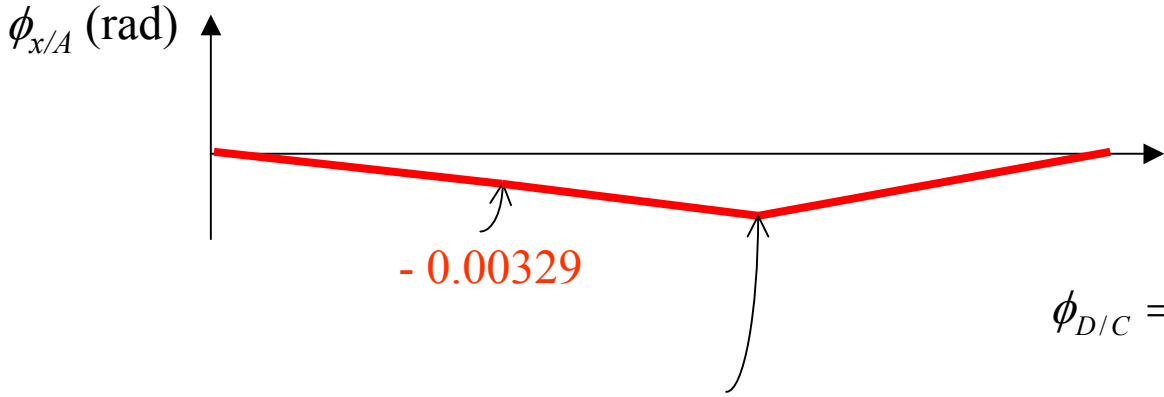


$J_{AC} = (\pi/2)(0.03^4)$
$= 1.272\ (10^{-6})\ m^4$

$J_{CD} = (\pi/2)(0.03^4 - 0.015^4)$
$= 1.193\ (10^{-6})\ m^4$



$$\varphi_{B/A} = \frac{(-670.17)(0.5)}{(\pi/2)(0.03^4)(80\times10^9)} = -0.00329\ \text{rad} = -0.593^o$$

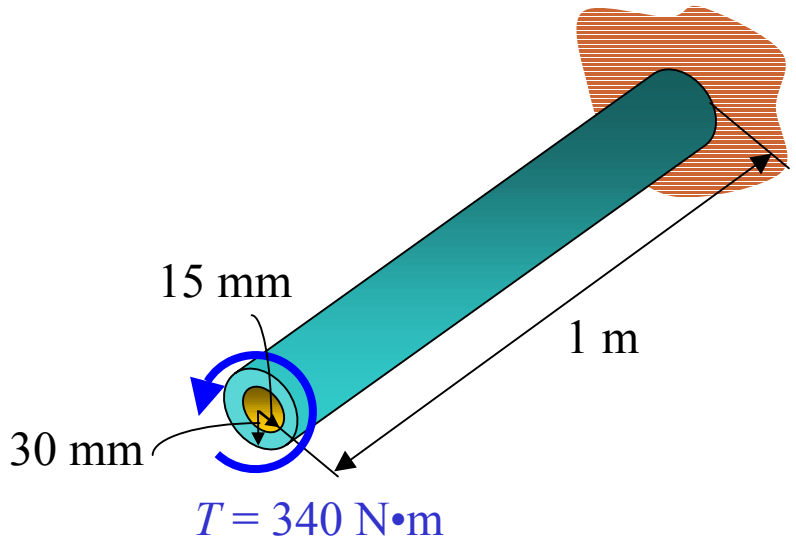


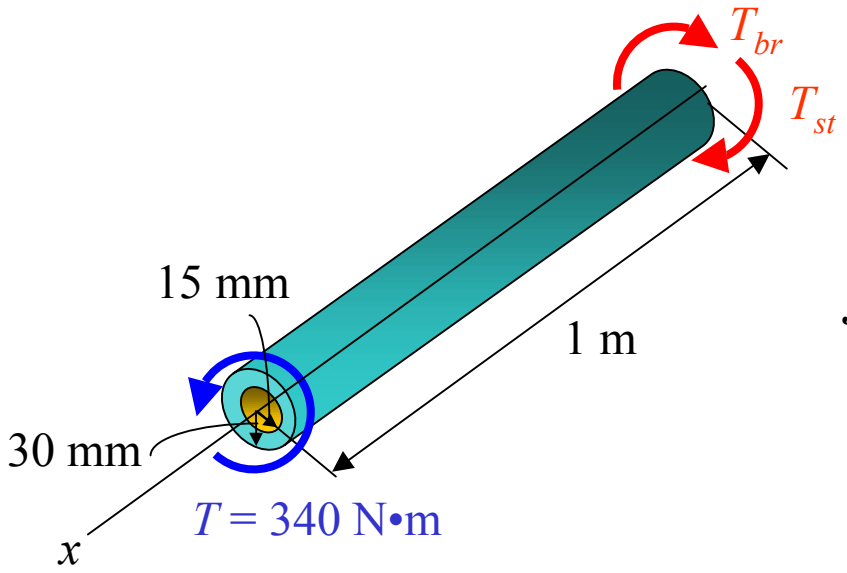
$$\varphi_{C/B} = \frac{(-170.17)(0.4)}{(\pi/2)(0.03^4)(80\times10^9)} = -0.000669\ \text{rad} = -0.12^o$$

$$\phi_{D/C} = \frac{(629.83)(0.6)}{(\pi/2)(0.03^4 - 0.015^4)(80\times10^9)} = 0.00396\ \text{rad} = 0.713^\circ$$

Example 9

The shaft shown is made form a steel tube, which is bonded to a brass core. If a torque of $T = 340$ N•m is applied at its end, plot the shear-stress distribution along a radial line of its cross sectional area. Take $G_{st} = 80$ GPa, $G_{br} = 35$ Gpa.





• **Section Properties**

$$J_{br} = \frac{\pi}{2} c_{br}^{\ 4} = \frac{\pi}{2}(0.015\ \text{m})^4 = 7.952 \times 10^{-8}\ \text{m}^4$$

$$J_{st} = \frac{\pi}{2}(c_{st}^{\ 4} - c_{br}^{\ 4}) = \frac{\pi}{2}[(0.03\ \text{m})^4 - (0.015\ \text{m})^4]$$

$$= 1.193(10^{-6})\ \text{m}^4$$

• **Equilibrium**

$$T_{st} + T_{br} = 340 \qquad \text{----------(1)}$$

• **Compatibility**

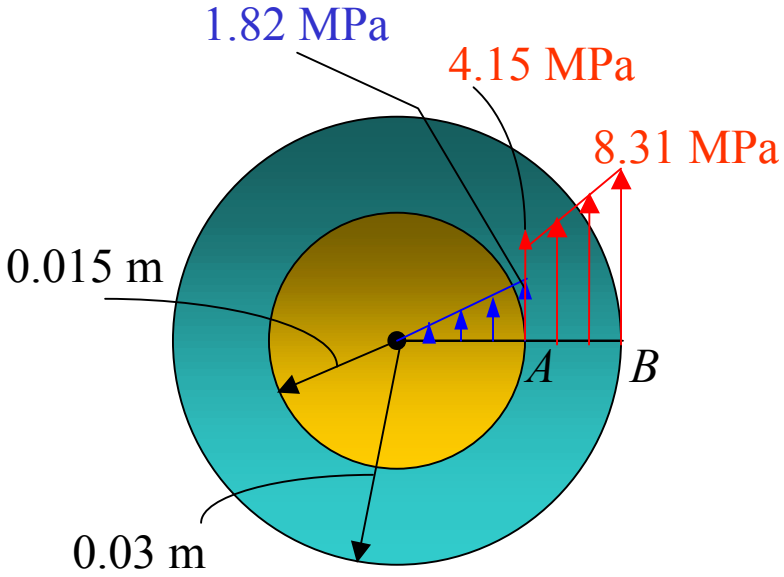
$$\phi = \phi_{st} = \phi_{br}$$

$$\frac{T_{st} L}{J_{st} G_{st}} = \frac{T_{br} L}{J_{br} G_{st}}$$

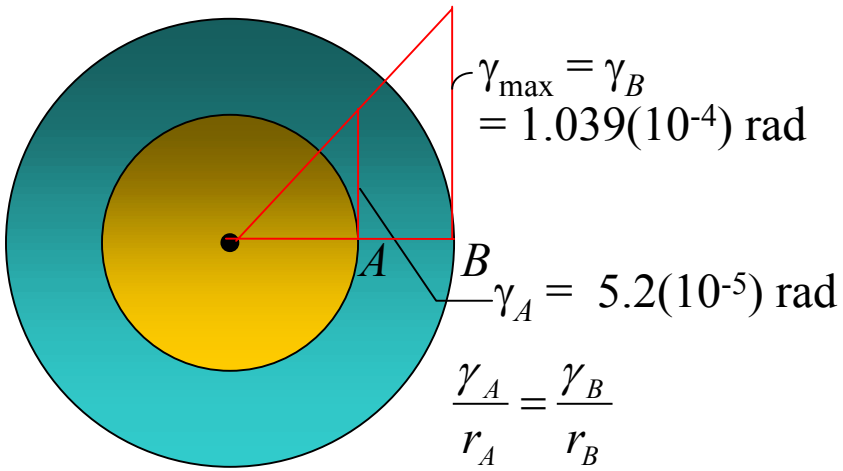
$$\frac{T_{st} L}{(1.193 \times 10^{-6}\text{m}^4)(80 \times 10^9 \text{Pa})} = \frac{T_{br} L}{(7.952 \times 10^{-8}\text{m}^4)(35 \times 10^9 \text{Pa})}$$

$$T_{st} = 34.29\ T_{br} \qquad \text{----------(2)}$$

Solving (1) and (2) , $T_{st} = 330.4$ N•m, $T_{br} = 9.63$ N•m



Shear stress distribution



Shear strain distribution

$T_{st} = 330.4$ N•m, $T_{br} = 9.63$ N•m

$J_{br} = 7.952(10^{-8})$ m^4, $J_{st} = 1.193(10^{-6})$ m^4

- **Shear Stress**

$$\tau = \frac{T\rho}{J}$$

$$(\tau_{br})_{max} = \frac{(9.63 \text{ N}\bullet\text{m})(0.015 \text{ m})}{(7.952 \times 10^{-8} \text{ m}^4)} = 1.82 \text{ MPa}$$

$$(\tau_{st})_{min} = \frac{(330.4 \text{ N}\bullet\text{m})(0.015 \text{ m})}{(1.193 \times 10^{-6} \text{ m}^4)} = 4.15 \text{ MPa}$$

$$(\tau_{st})_{max} = \frac{(330.4 \text{ N}\bullet\text{m})(0.03 \text{ m})}{(1.193 \times 10^{-6} \text{ m}^4)} = 8.31 \text{ MPa}$$

- **Shear Strain**

$$\gamma_A = \frac{(\tau_{br})_A}{G_{br}} = \frac{(\tau_{st})_A}{G_{st}}$$

$$= \left(\frac{1.82 \text{ MPa}}{35 \times 10^3 \text{ MPa}}\right)_{br} = 5.2 \times 10^{-5} \text{ rad}$$

$$\gamma_{max} = \gamma_B = \frac{8.31 \text{ MPa}}{80 \times 10^3 \text{ MPa}} = 1.039 \times 10^{-4} \text{ rad}$$