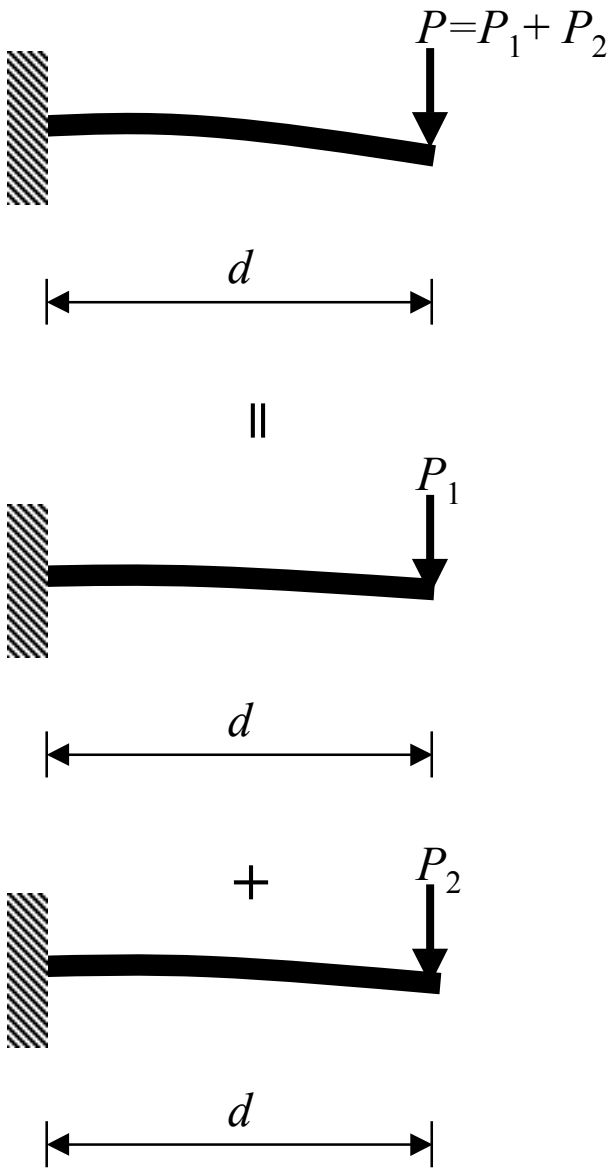
AXIAL LOAD

- Principle of Superposition
- Axial - force / Stress diagram
- Strain
  - Mechanical
  - Thermal
- Force - deformation relationship
- Deformation
- Compatibility conditions

Principle of Superposition



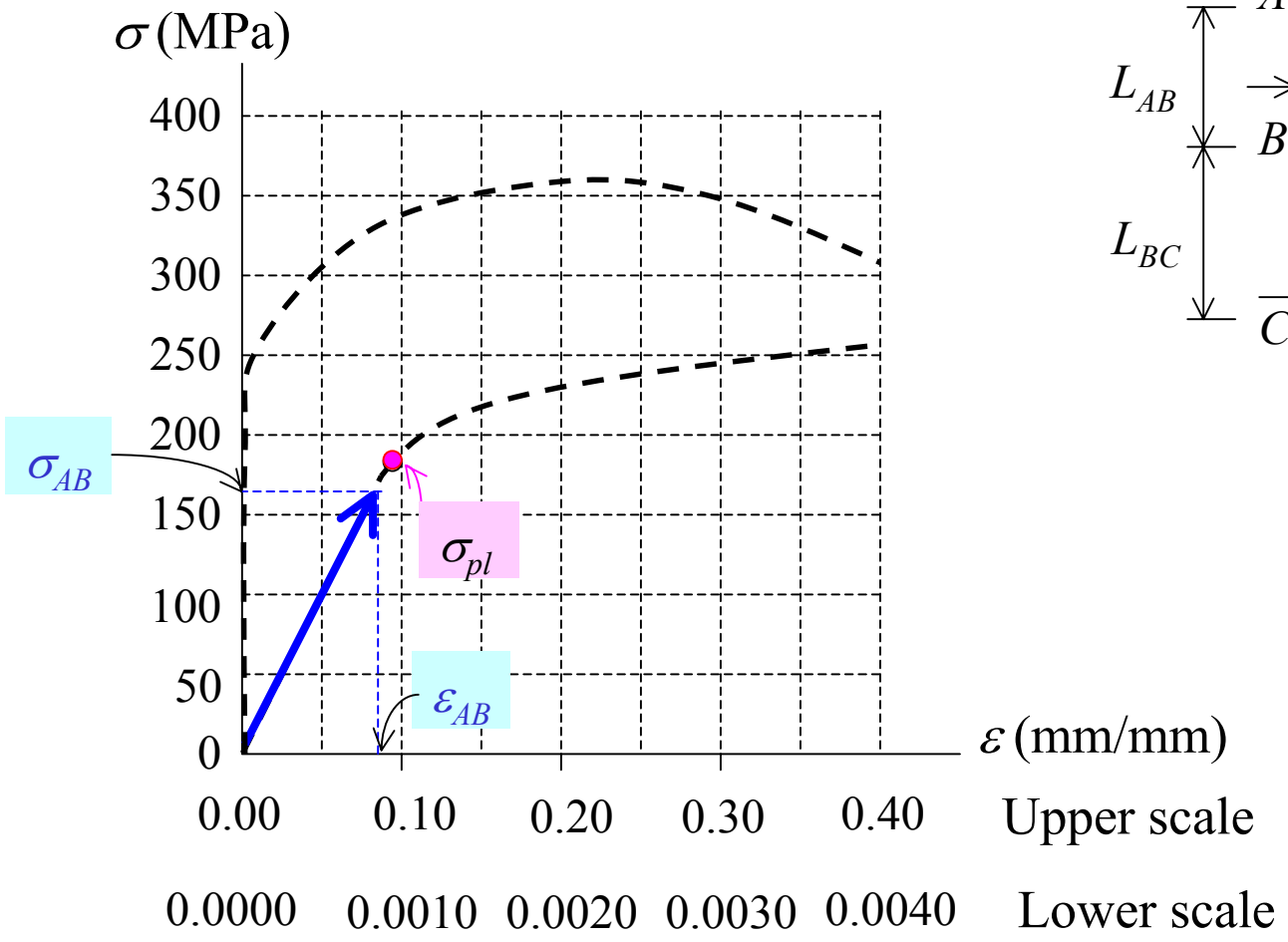
The following two conditions must be valid if the principle of superposition is to be applied.

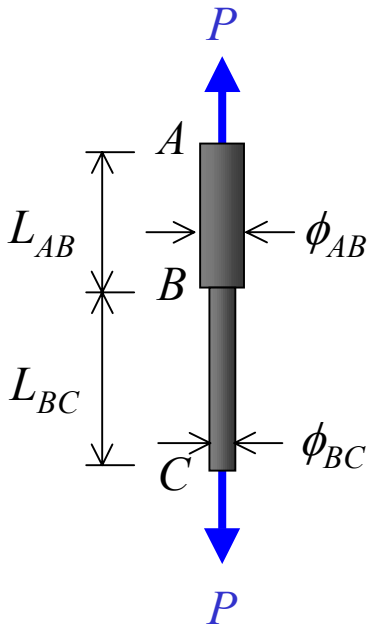
1. ***The loading must be linearly related to the stress or displacement that is to be determined***.

2. ***The loading must not significantly change the original geometry or configuration of the member***.

Strain

- **Mechanical Strain**

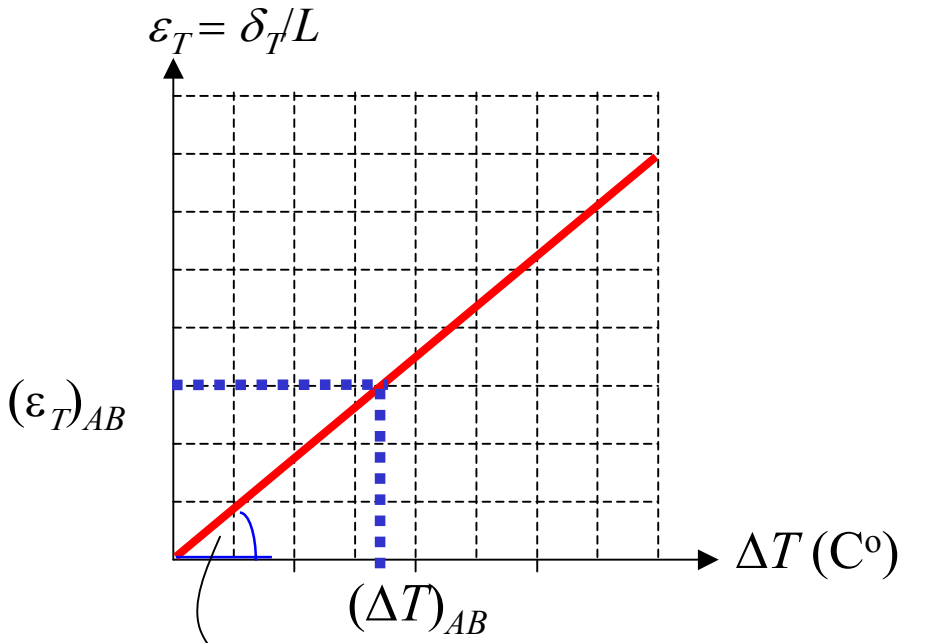




$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}}$$

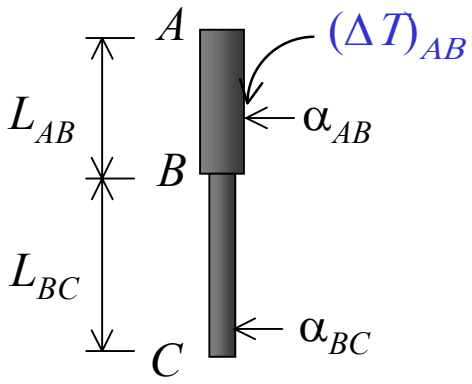
Strain

- **Thermal Strain**

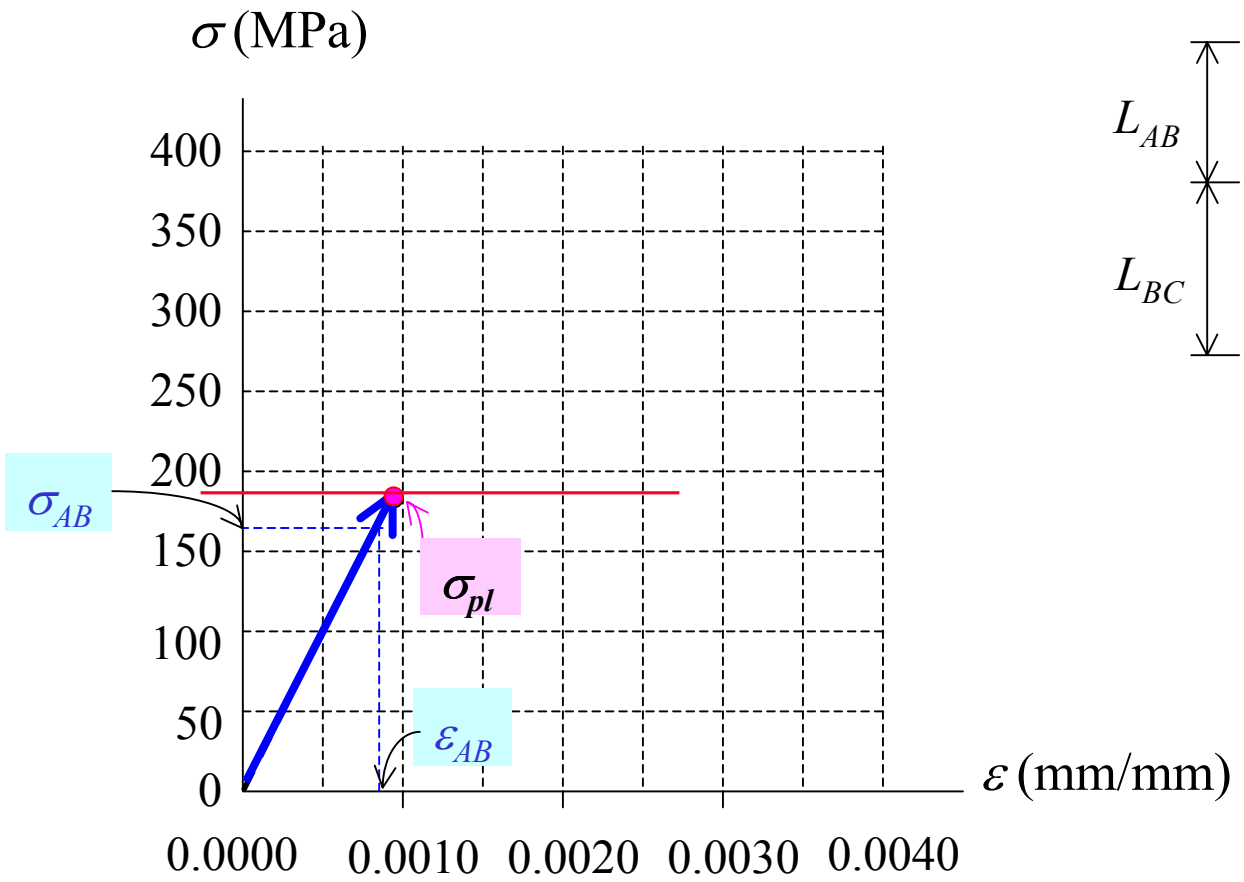


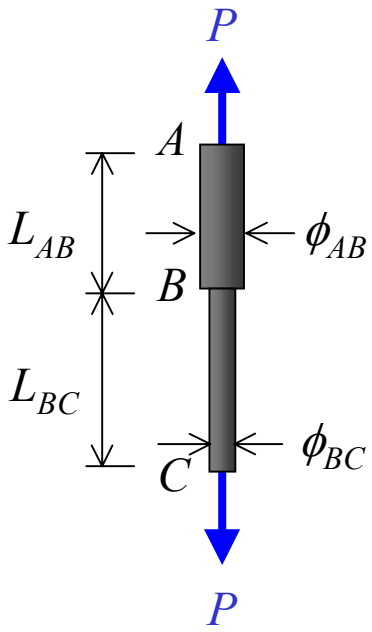
$$\alpha = \frac{(\varepsilon_T)_{AB}}{(\Delta T)_{AB}} \qquad (\mathrm{mm/mm})/^{\mathrm{o}}\mathrm{C}$$

$$(\varepsilon_T)_{AB} = \alpha (\Delta T)_{AB}$$



• **Hookean's Force - deformation relationship**

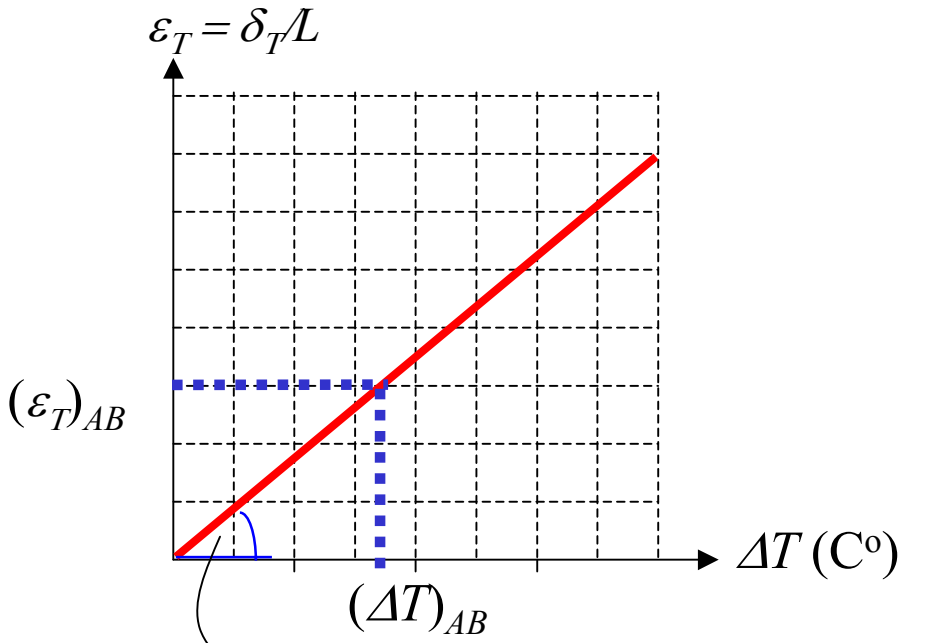


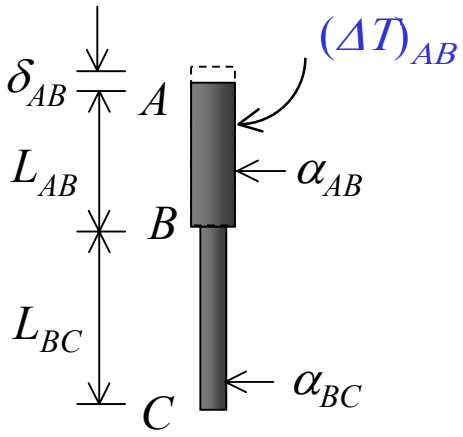


$$\delta_{AB} = \varepsilon_{AB} L_{AB} = \frac{\sigma_{AB}}{E} L_{AB}$$

$$= \frac{P L_{AB}}{AE}$$

• **Temperature - deformation relationship**

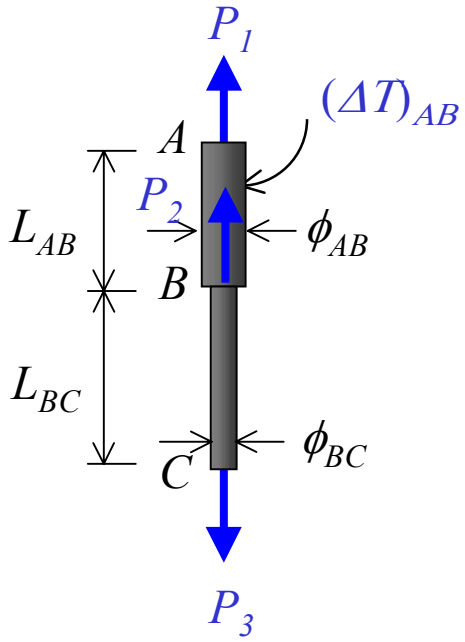




$$\alpha = \frac{(\varepsilon_T)_{AB}}{(\Delta T)_{AB}} \qquad (\mathrm{mm/mm})/^{\mathrm{o}}\mathrm{C}$$

$$(\delta_T)_{AB} = \alpha (\Delta T)_{AB} L_{AB}$$

- **Total Strains**



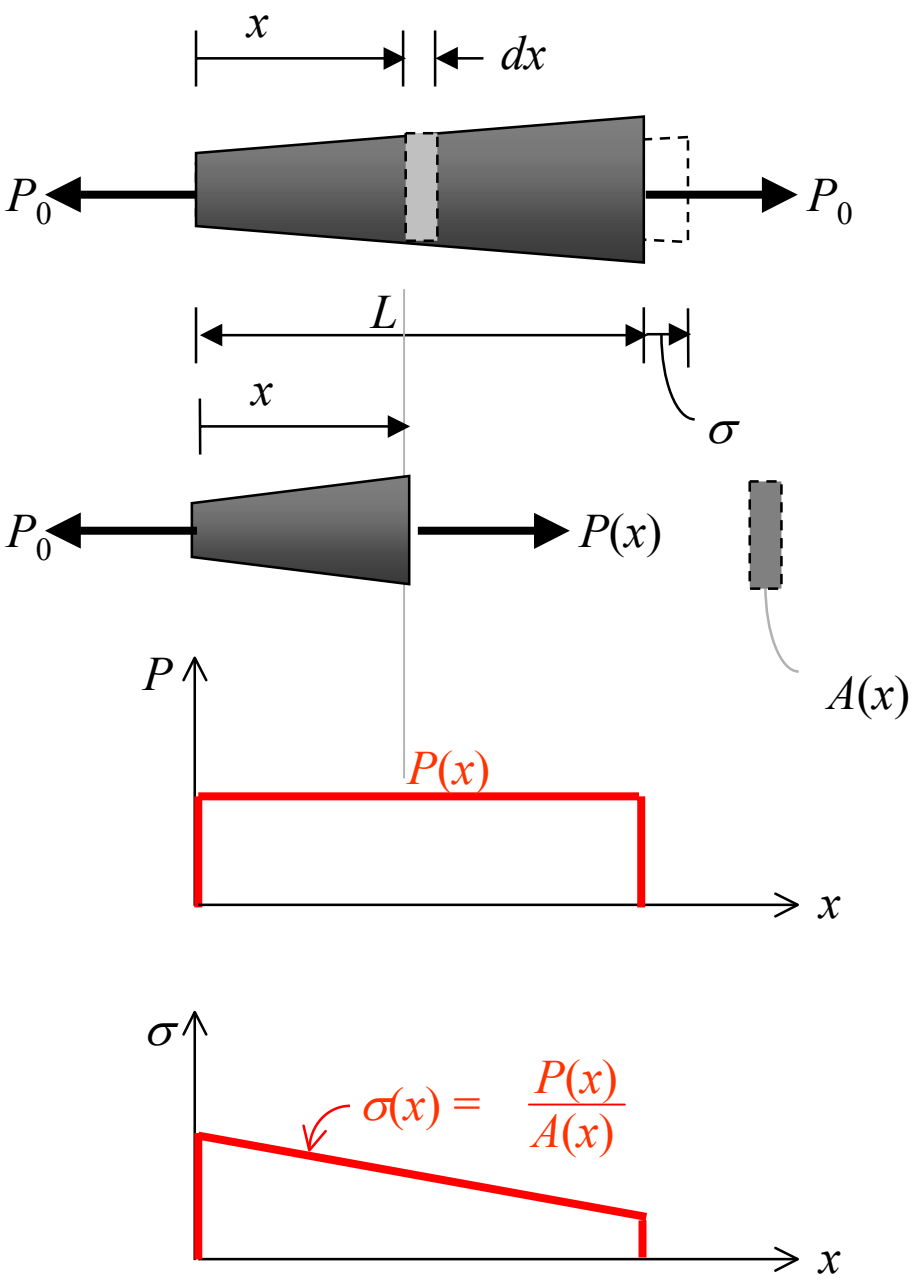
$$(\varepsilon_{Total})_{AB} = (\varepsilon_T)_{AB} + (\varepsilon_F)_{AB}$$

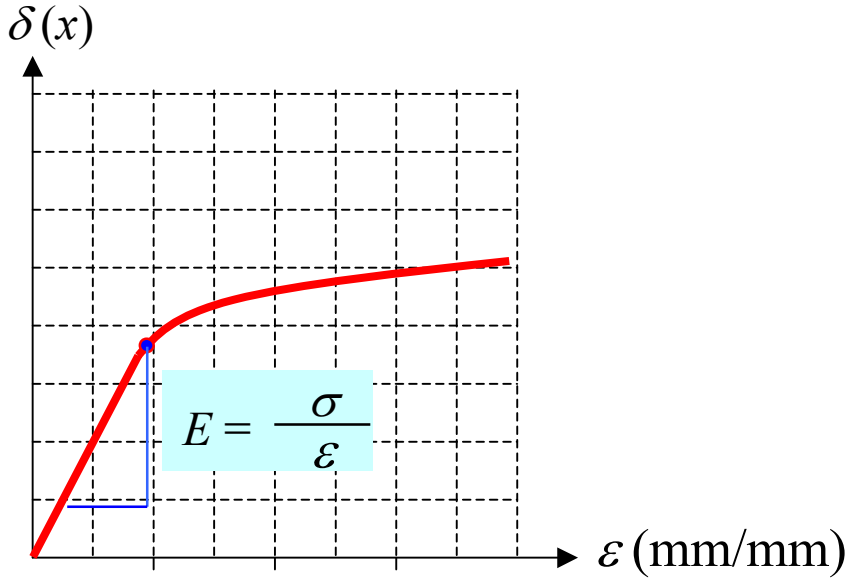
$$= \alpha(\Delta T)_{AB} + \frac{\sigma_{AB}}{E}$$

$$(\delta_{Total})_{AB} = (\delta_T)_{AB} + (\delta_F)_{AB}$$

$$= \alpha(\Delta T)_{AB} L_{AB} + \frac{P_{AB} L_{AB}}{AE}$$

Elastic Deformation





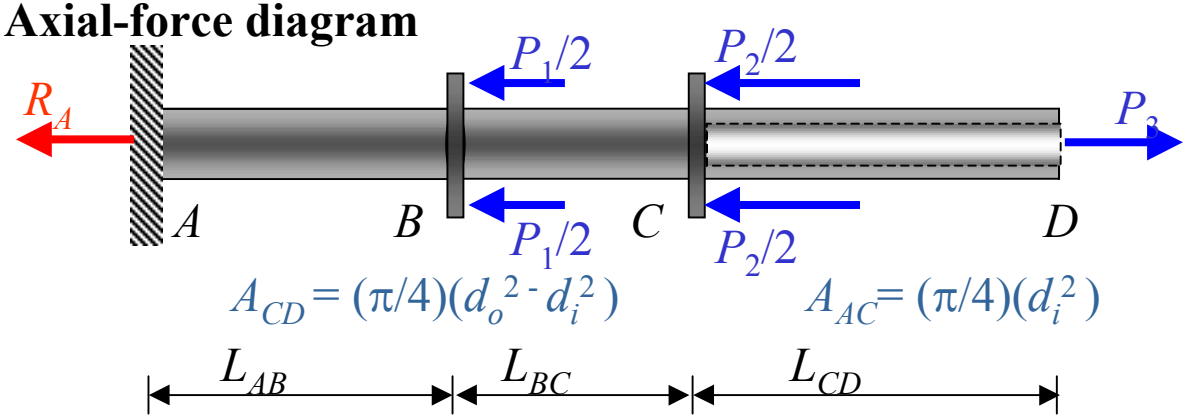
$$\sigma = \varepsilon E$$

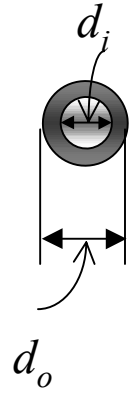
$$\frac{P(x)}{A(x)} = E\frac{d\delta}{dx}$$

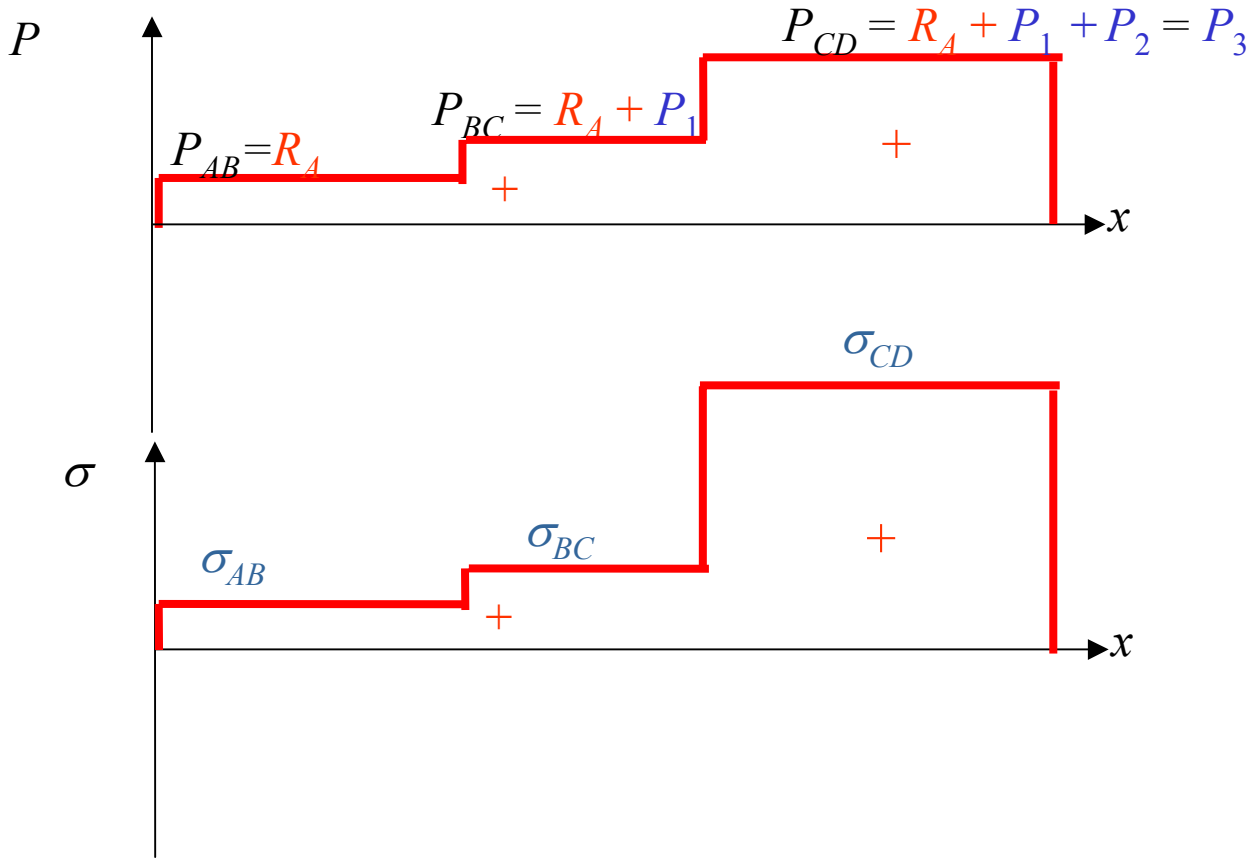
$$d\delta = \frac{P(x)dx}{A(x)E}$$

$$d\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

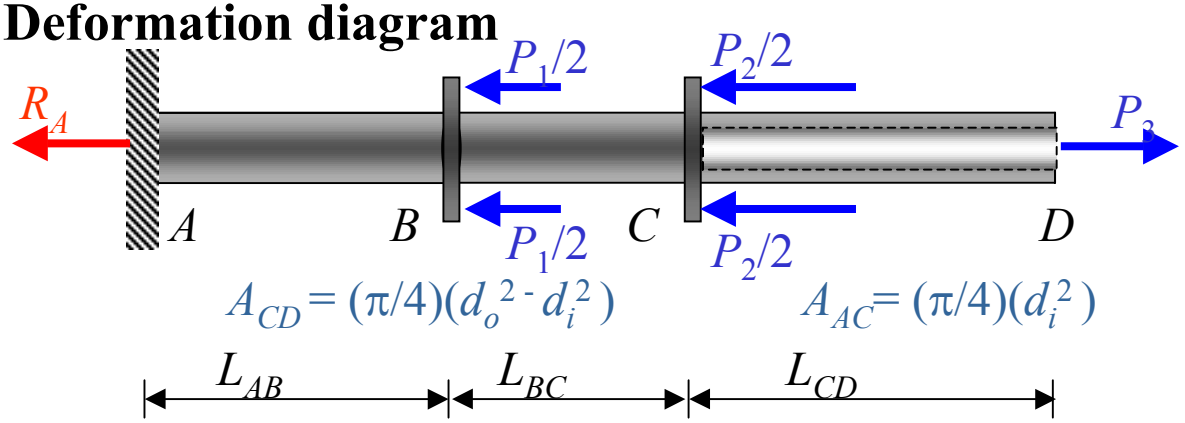
- **Axial-force diagram**

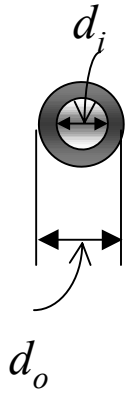


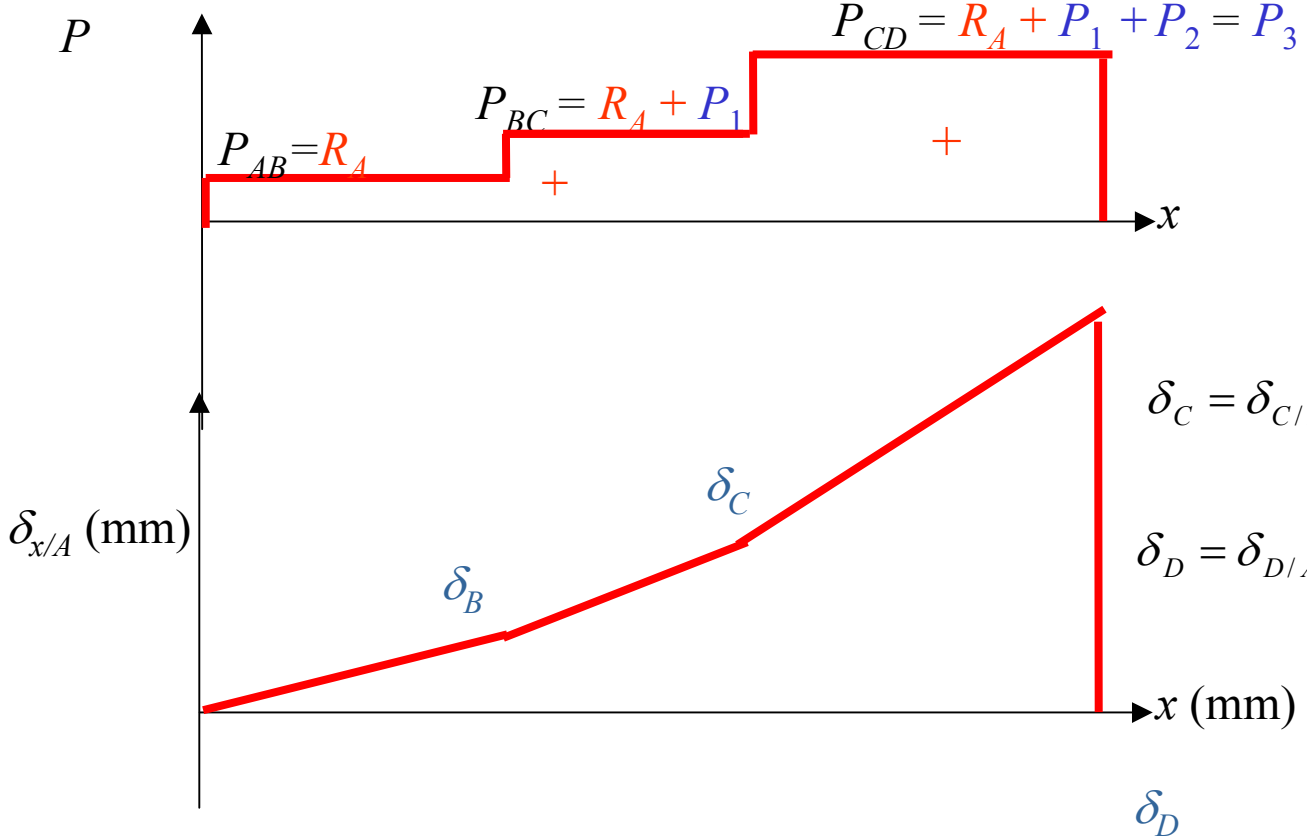




• **Deformation diagram**







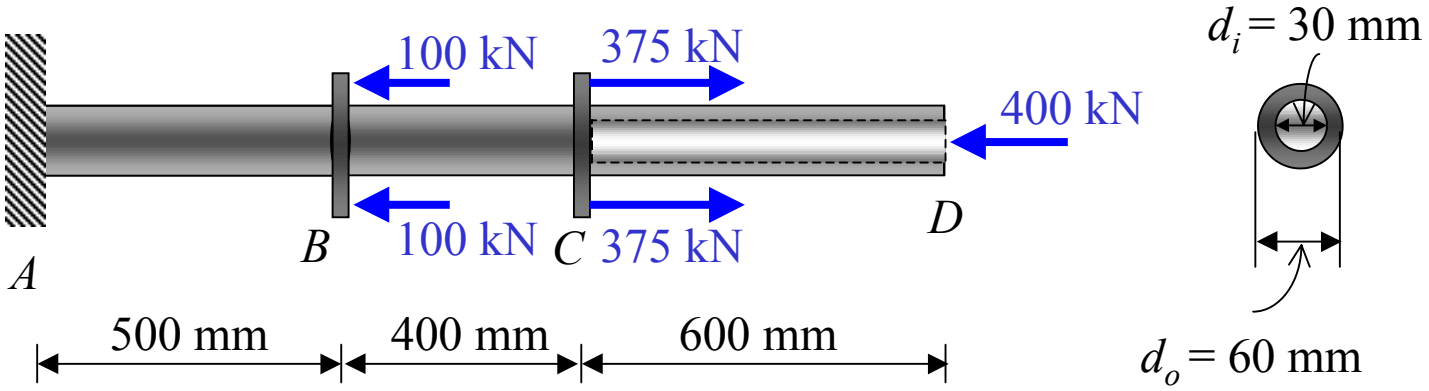
$$\delta_B = \delta_A + \delta_{B/A} = o + \frac{P_{AB}L_{AB}}{A_{AB}E_{AB}}$$

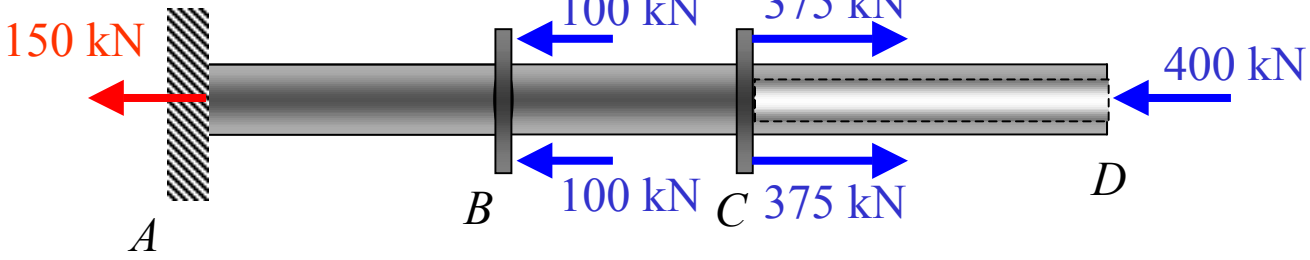
$$\delta_C = \delta_{C/A} = \delta_B + \delta_{C/B} = \delta_B + \frac{P_{BC}L_{BC}}{A_{BC}E_{BC}}$$

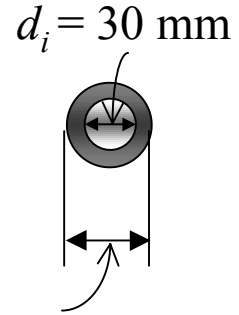
$$\delta_D = \delta_{D/A} = \delta_C + \delta_{D/C} = \delta_C + \frac{P_{CD}L_{CD}}{A_{CD}E_{CD}}$$

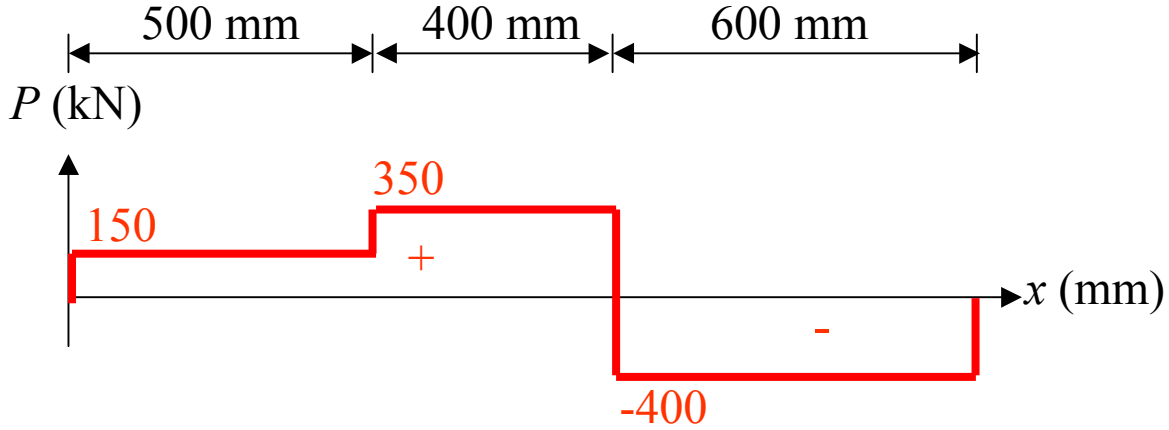
Example 1

The rod is made from a solid steel section AB and aluminum section BC. If it is fixed to a rigid support at A. A solid aluminum having an inner diameter of 30 mm and outer diameter of 60 mm. Determine (*a*) draw the quantitative stress and strain diagram of the bar (*b*) the horizontal displacement of end D and the displacement of C relative to B. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa

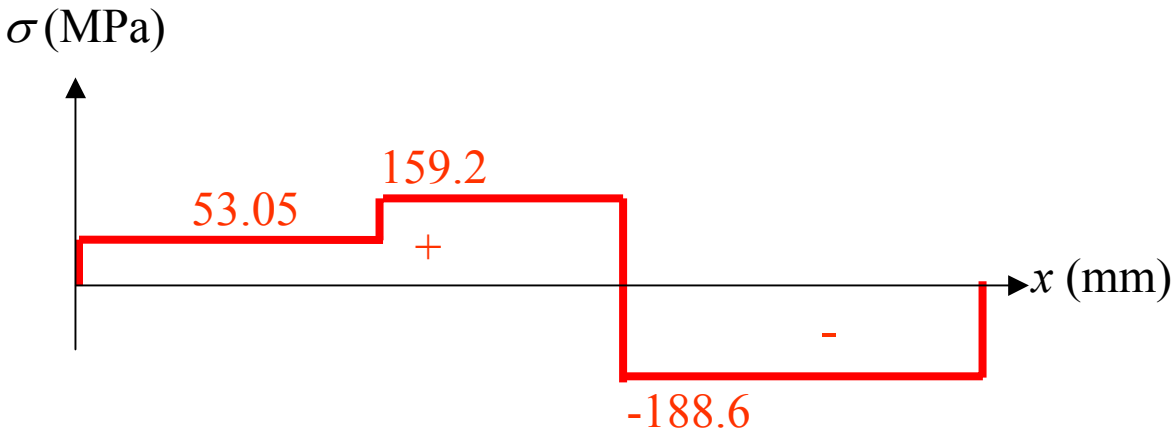








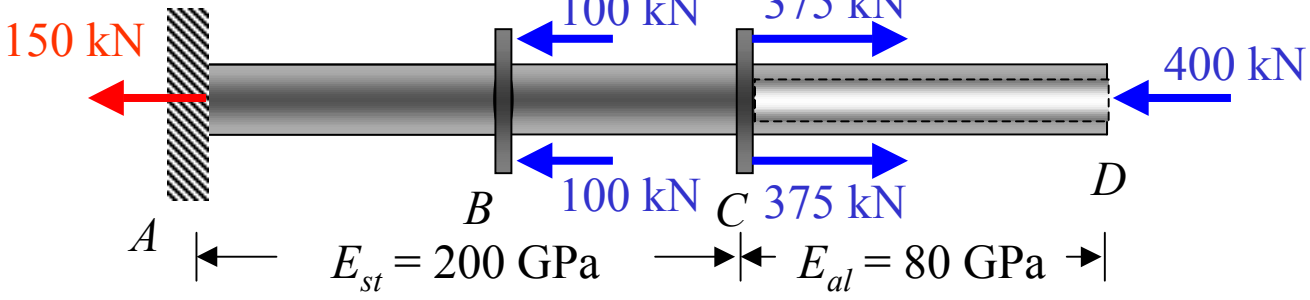
- **Stress diagram**

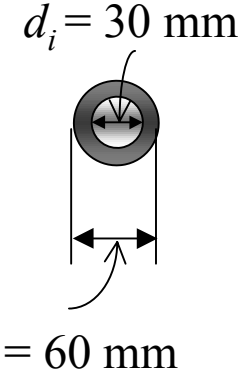


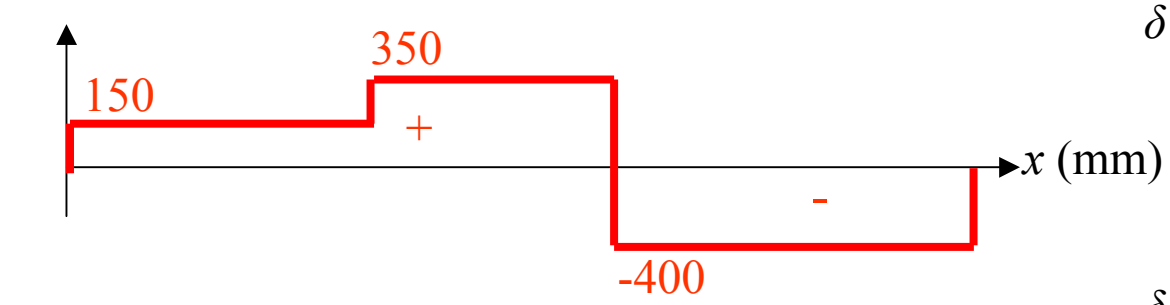
$$\sigma_{AB} = \frac{150\text{ kN}}{(\frac{\pi}{4})(0.06)^2} = 53.05\text{ MPa}$$

$$\sigma_{BC} = \frac{350\text{ kN}}{(\frac{\pi}{4})(0.06^2)} = 159.2\text{ MPa}$$

$$\sigma_{CD} = \frac{-400\text{ kN}}{(\frac{\pi}{4})(0.06^2 - 0.03^2)} = -188.6\text{ MPa}$$



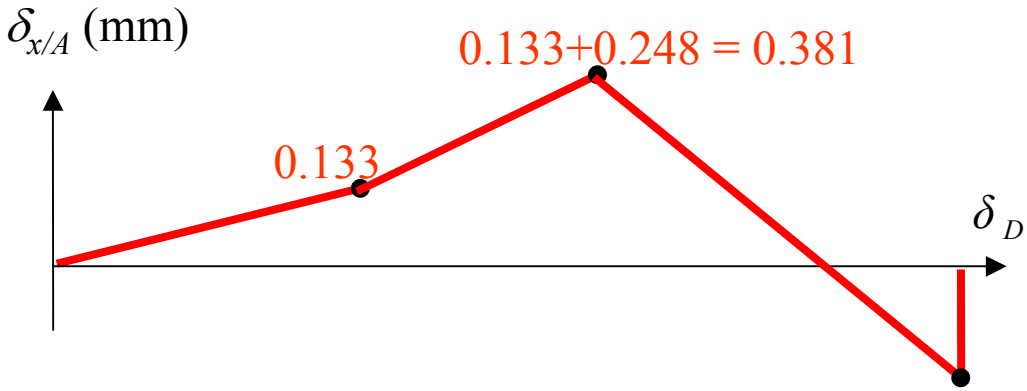




$$\delta_{B/A} = \frac{(150)(0.5)}{(\frac{\pi}{4}(0.06^2))(200\times10^6)} = 0.133 \text{ mm}$$

$$\delta_{C/B} = \frac{(350)(0.4)}{(\frac{\pi}{4}(0.06^2))(200\times10^6)} = 0.248 \text{ mm}$$

- **Displacement diagram**

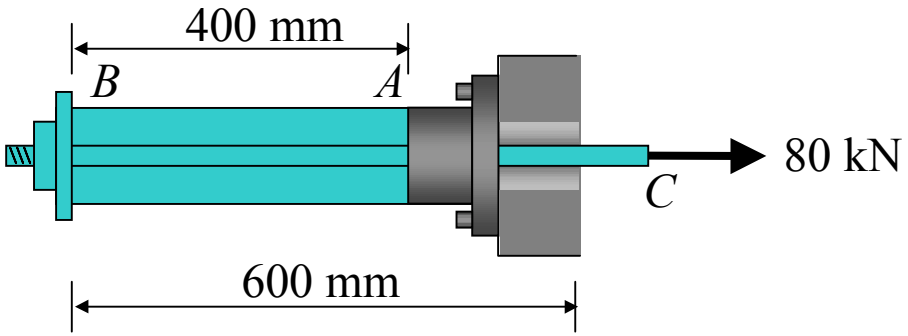


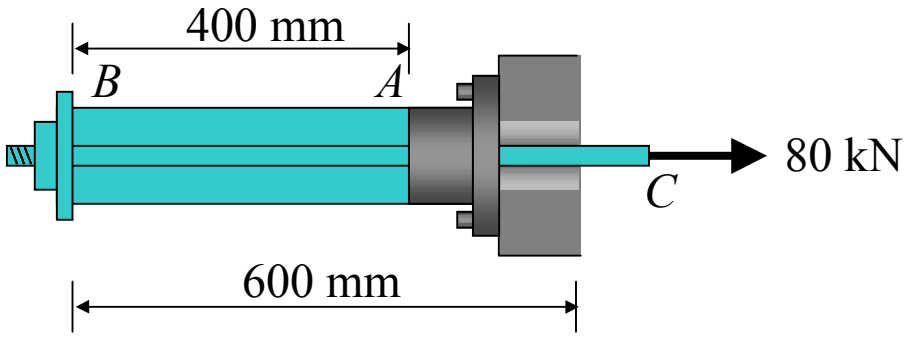
$$\delta_{D/C} = \frac{(-400)(0.6)}{(\frac{\pi}{4}(0.06^2 - 0.03^2))(70\times10^6)} = -1.617 \text{ mm}$$

$\Rightarrow \sigma_D = 0.381\text{-}1.617 = \text{-}1.236$

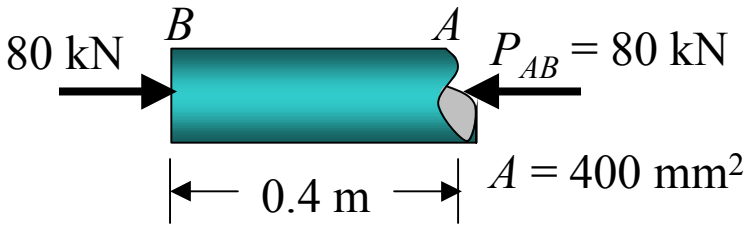
Example 2

The assembly shown consists of an aluminum tube AB having a cross-sectional area of 400 mm^2. A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end C of the rod. Take E_{st} = 200 GPa, E_{al} = 70 GPa.





- Aluminum tube *AB*

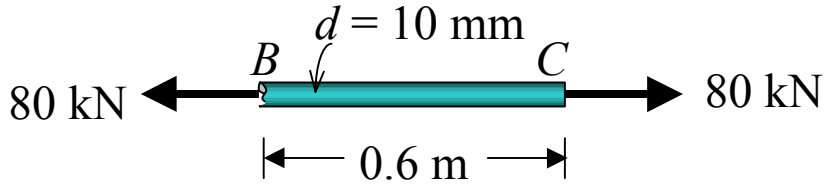


$$\delta_{B/A} = \frac{P_{AB}L_{AB}}{A_{tube}E_{al}}$$

$$= \frac{(-80\ \text{kN})(0.4\ \text{m})}{(400\times10^{-6}\ \text{m}^2)(70\times10^{6}\ \text{kN/m}^2)}$$

$$= -1.143\times10^{-3}\ \text{m} \quad = 1.143\times10^{-3}\ \text{m} \rightarrow$$

- Rod *BC*



$$\delta_{C/B} = \frac{P_{BC}L_{BC}}{A_{rod}E_{st}}$$

$$= \frac{(80\ \text{kN})(0.6\ \text{m})}{[\pi(0.005\ \text{m})^2](200\times10^{6}\ \text{kN/m}^2)}$$

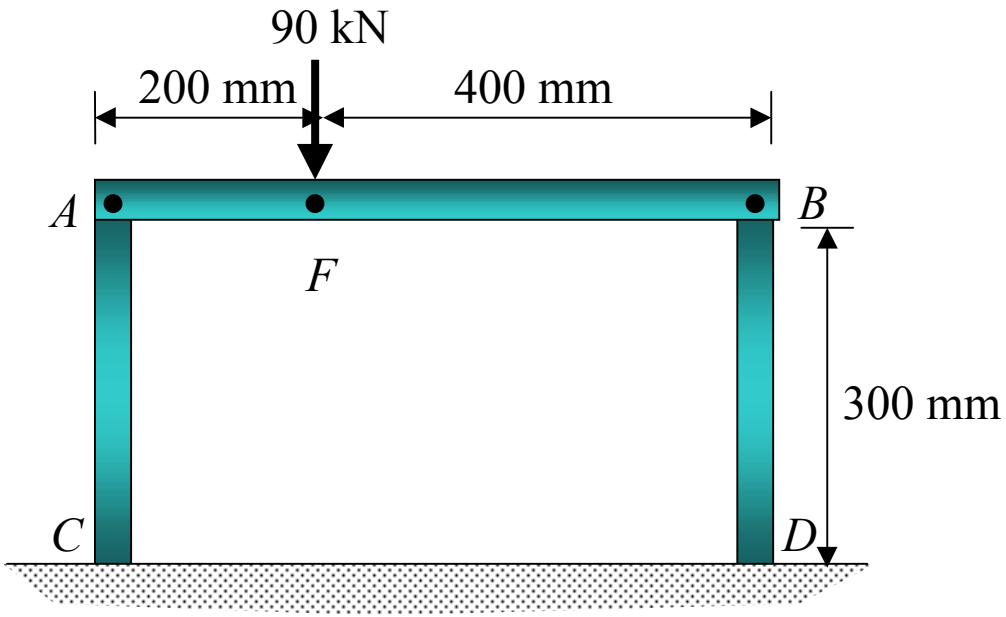
$$= 3.056\times10^{-3} \quad \text{m} \rightarrow$$

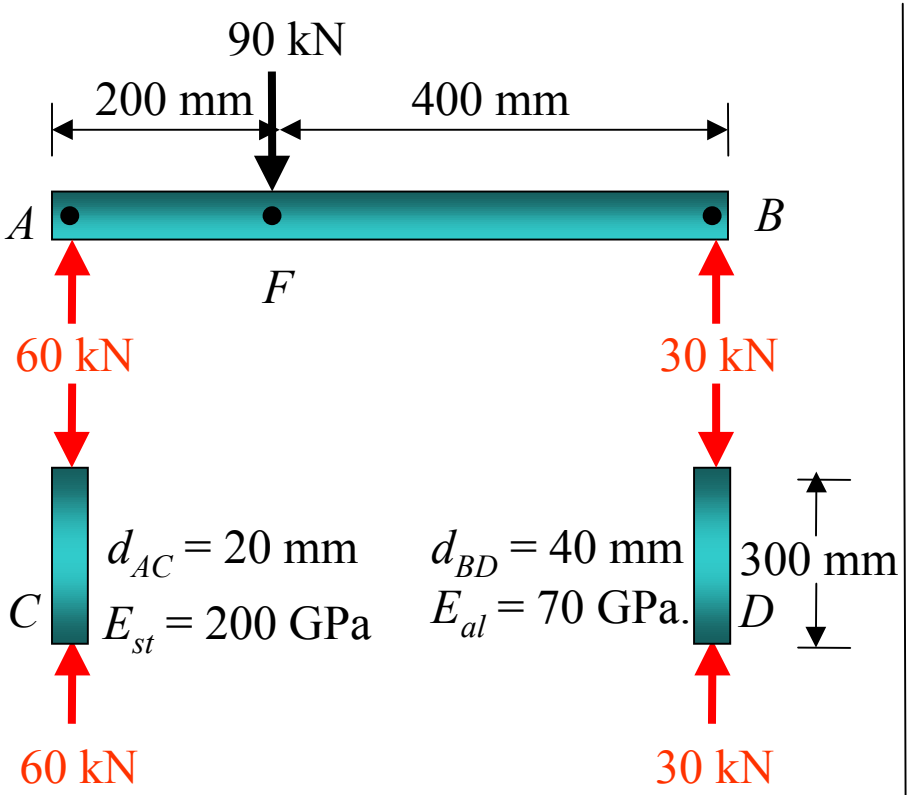
- Displacement of the end *C*

$$(\overset{+}{\longrightarrow}) \qquad \delta_C = \delta_B + \delta_{C/B} = 0.001143 + 0.003056 = 0.00420\ \text{m} = 4.20\ \text{m} \rightarrow$$

Example 3

A *rigid* beam *AB* rests on the two short posts shown. *AC* is is made of steel and has a diameter of 20 mm, and *BD* is made of aluminum and has a diameter of 40 mm. Determine the displacement of point *F* on *AB* if vertical load of 90 kN is applied over this point. Take E_{st} = 200 GPa, E_{al} = 70 GPa.





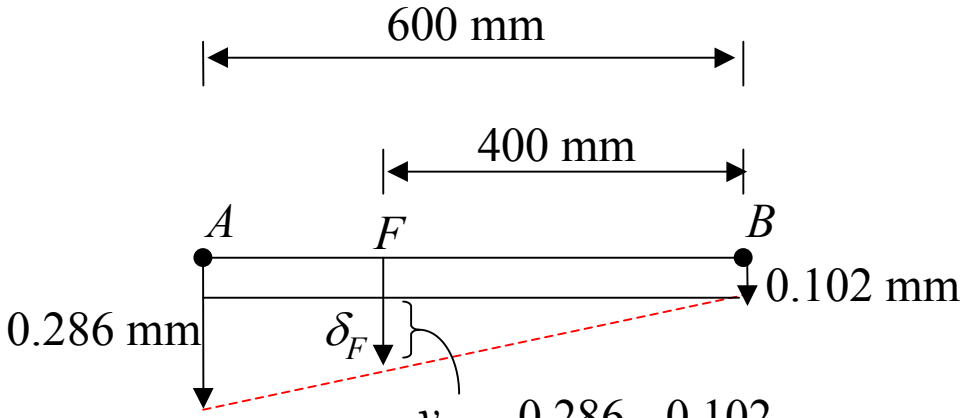
$$\delta_A = \frac{P_{AC}L_{AC}}{A_{AC}E_{st}} = \frac{(-60\text{ kN})(0.3\text{ m})}{[\pi(0.01\text{ m})^2](200\times10^6\text{ kN/m}^2)]}$$

$$= -2.86\times10^{-4}\text{ m} = 0.286\text{ mm}\downarrow$$

$$\delta_B = \frac{P_{BD}L_{BD}}{A_{BD}E_{al}} = \frac{(-30\text{ kN})(0.3\text{ m})}{[\pi(0.02\text{ m})^2](70\times10^6\text{ kN/m}^2)}$$

$$= -1.02\times10^{-4}\text{ m} = 0.102\text{ mm}\downarrow$$

- Compatibility Equation



$$\frac{y}{400} = \frac{0.286-0.102}{600}$$

$$y = (0.286-0.102)(\frac{400}{600})$$

$$y = 0.123\text{ mm}$$

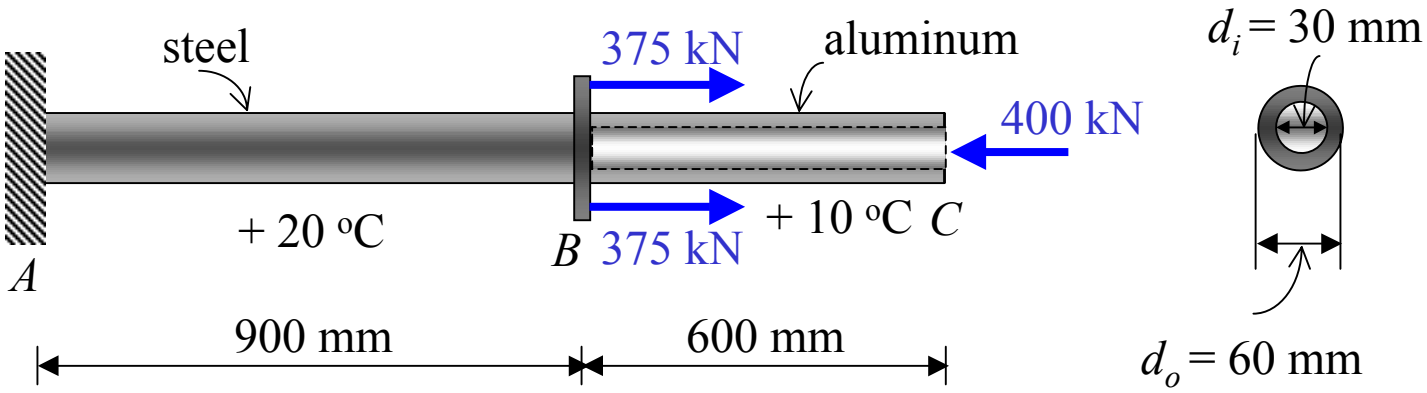
$$\delta_F = 0.102 + y$$

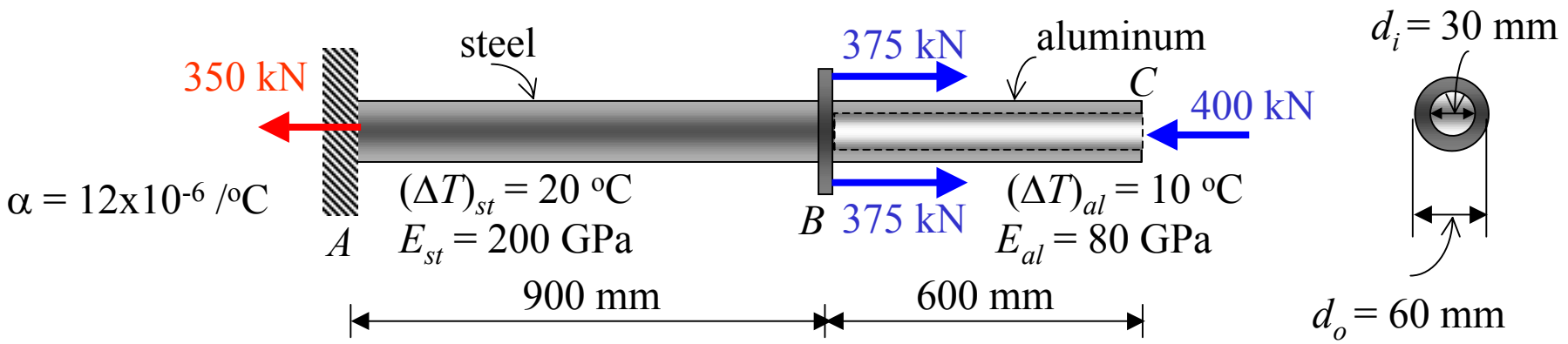
$$= 0.102 + 0.123$$

$$= 0.225\text{ mm}\downarrow$$

Example 4a

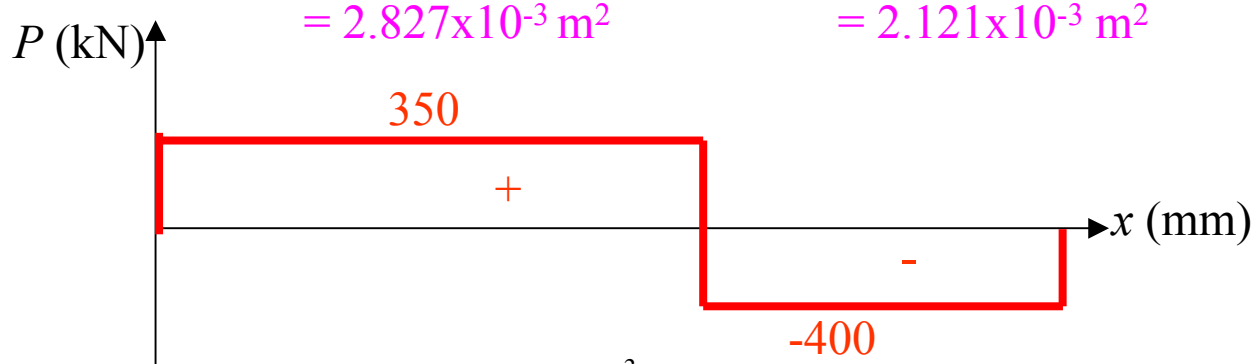
The rod is made from a solid steel section *AB* and aluminum section *BC*. If it is fixed to a rigid support at *A*. A solid aluminum having an inner diameter of 30 mm and outer diameter of 60 mm. Draw the quantitative stress diagram of the rod. Temperature in member *AB* and *CD* are increase 20 ^{o}C and 10 ^{o}C respectively prior to the loads are applied. Take E_{st} = 200 GPa, E_{al} = 70 GPa, α_{st} = 12x10^{-6} /^{o}C and α_{al} = 23x10^{-6} /^{o}C.





$A_{AB} = (\pi/4)(0.06^2) = 2.827\text{x}10^{-3}\ \text{m}^2$

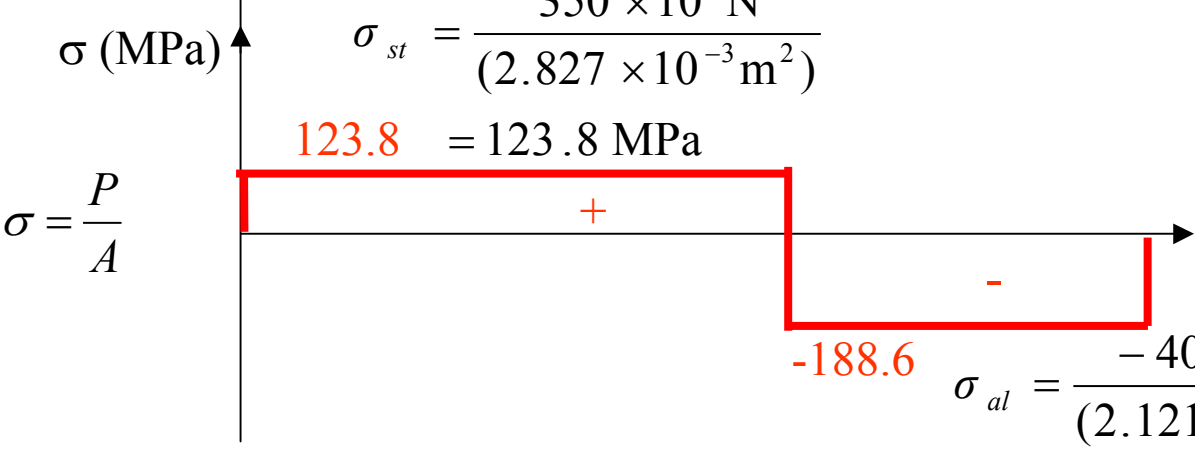
$A_{BC} = (\pi/4)(0.06^2 - 0.03^2) = 2.121\text{x}10^{-3}\ \text{m}^2$



$$\sigma_{st} = \frac{350 \times 10^3 \text{N}}{(2.827 \times 10^{-3} \text{m}^2)}$$

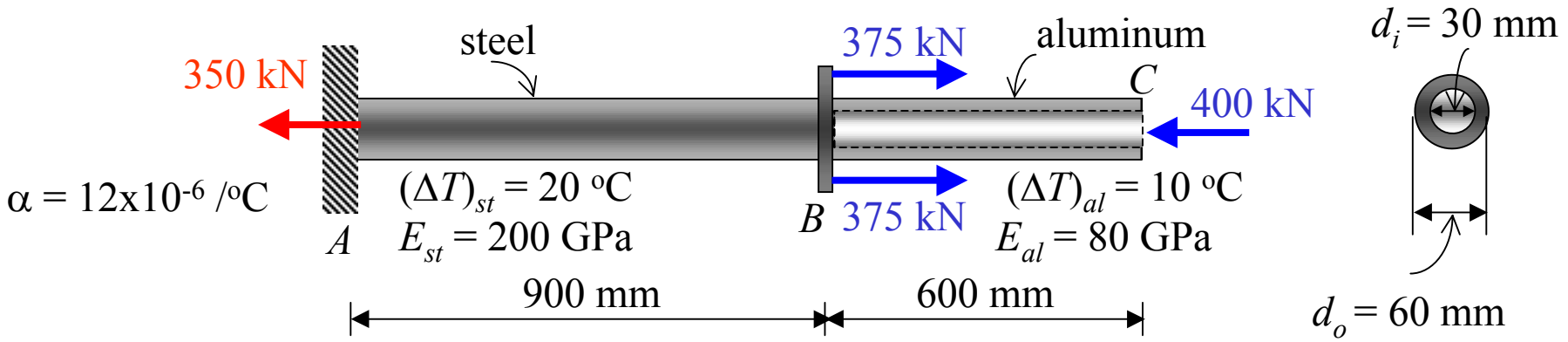
$= 123.8$ MPa

$\sigma = \frac{P}{A}$



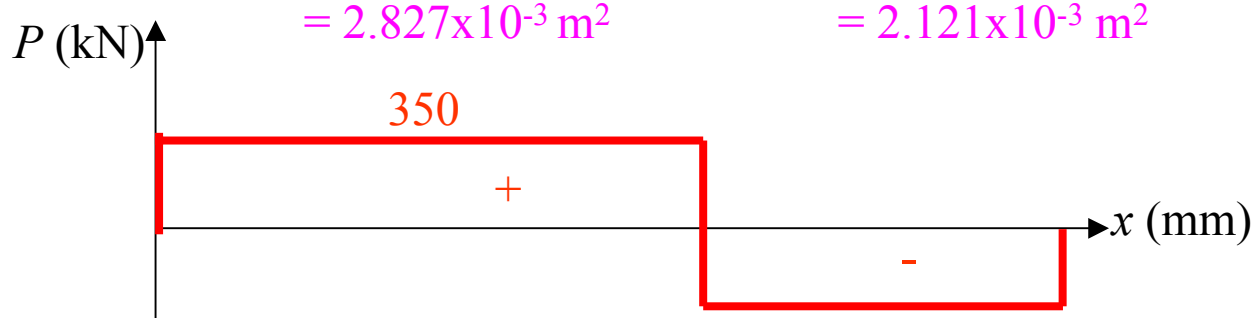
$$\sigma_{al} = \frac{-400 \times 10^3 \text{N}}{(2.121 \times 10^{-3} \text{m}^2)}$$

$= 188.6$ MPa



$A_{AB} = (\pi/4)(0.06^2) = 2.827\text{x}10^{-3}\ \text{m}^2$

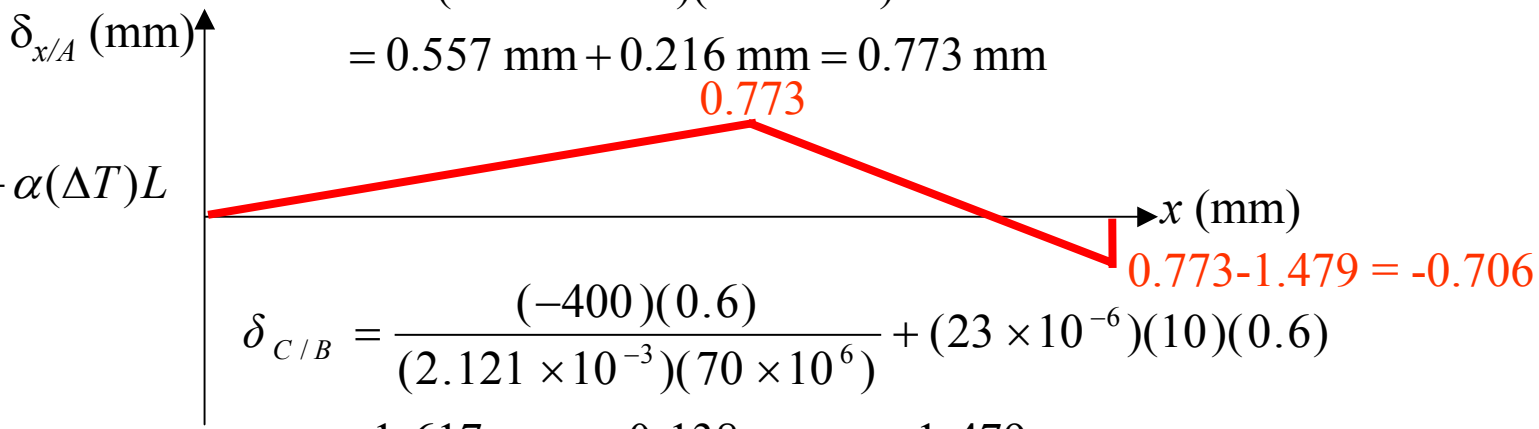
$A_{BC} = (\pi/4)(0.06^2 - 0.03^2) = 2.121\text{x}10^{-3}\ \text{m}^2$



$$\delta_{B/A} = \frac{(350)(0.9)}{(2.827\times10^{-3})(200\times10^{6})} + (12\times10^{-6})(20)(0.9)$$

$$= 0.557\ \text{mm} + 0.216\ \text{mm} = 0.773\ \text{mm}$$

$$\delta = \frac{PL}{AE} + \alpha(\Delta T)L$$

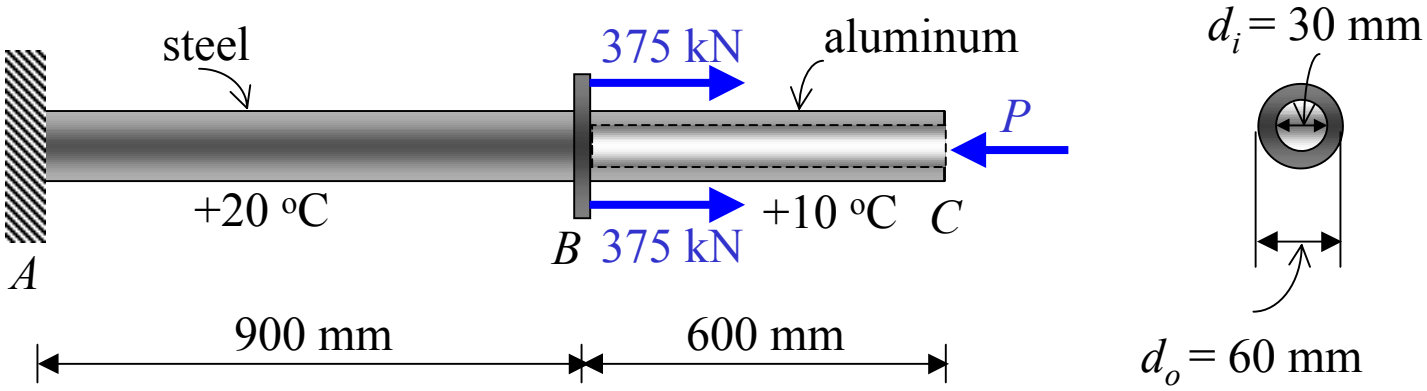


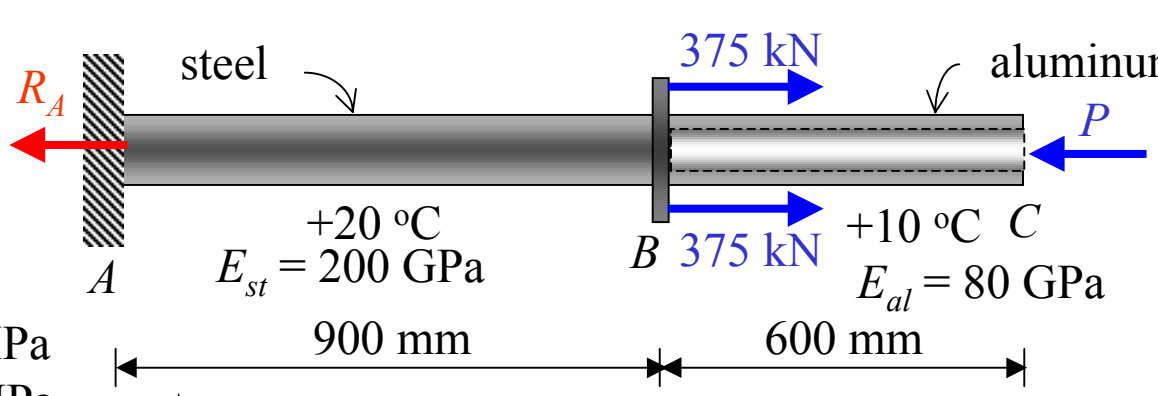
$$\delta_{C/B} = \frac{(-400)(0.6)}{(2.121\times10^{-3})(70\times10^{6})} + (23\times10^{-6})(10)(0.6)$$

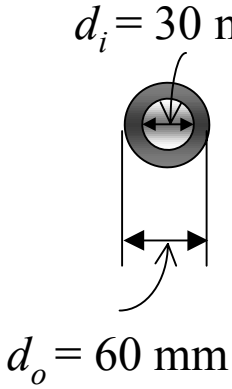
$$= -1.617\ \text{mm} + 0.138\ \text{mm} = -1.479\ \text{mm}$$

Example 4b

The rod is made from a solid steel section *AB* and aluminum section *BC*. If it is fixed to a rigid support at *A*. A solid aluminum having an inner diameter of 30 mm and outer diameter of 60 mm. Temperature in member *AB* and *CD* are increase 20 ^{o}C and 10 ^{o}C respectively. Determine the maximum load *P* the rod can take at end *C*, if the allowable normal stress in steel is $(\sigma_{st})_{allow} = 120$ MPa and the in aluminum is $(\sigma_{al})_{allow} = 180$ MPa . Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12\text{x}10^{-6}$ /^{o}C and $\alpha_{al} = 23\text{x}10^{-6}$ /^{o}C.





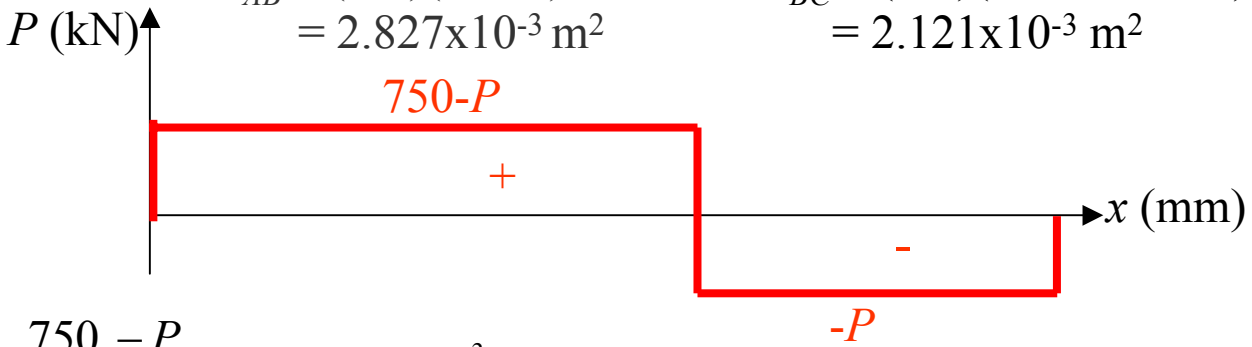


$\alpha_{st} = 12\text{x}10^{-6}$ /°C
$\alpha_{al} = 23\text{x}10^{-6}$ /°C
$(\sigma_{st})_{allow} = 120$ MPa
$(\sigma_{al})_{allow} = 180$ MPa
$\delta_C = 0$

$$\overset{+}{\rightarrow} \quad \Sigma F_x = 0: \quad -R_A + 750 - P = 0, \qquad R_A = 750 - P$$

$A_{AB} = (\pi/4)(0.06^2) = 2.827\text{x}10^{-3}\ \text{m}^2$

$A_{BC} = (\pi/4)(0.06^2 - 0.03^2) = 2.121\text{x}10^{-3}\ \text{m}^2$



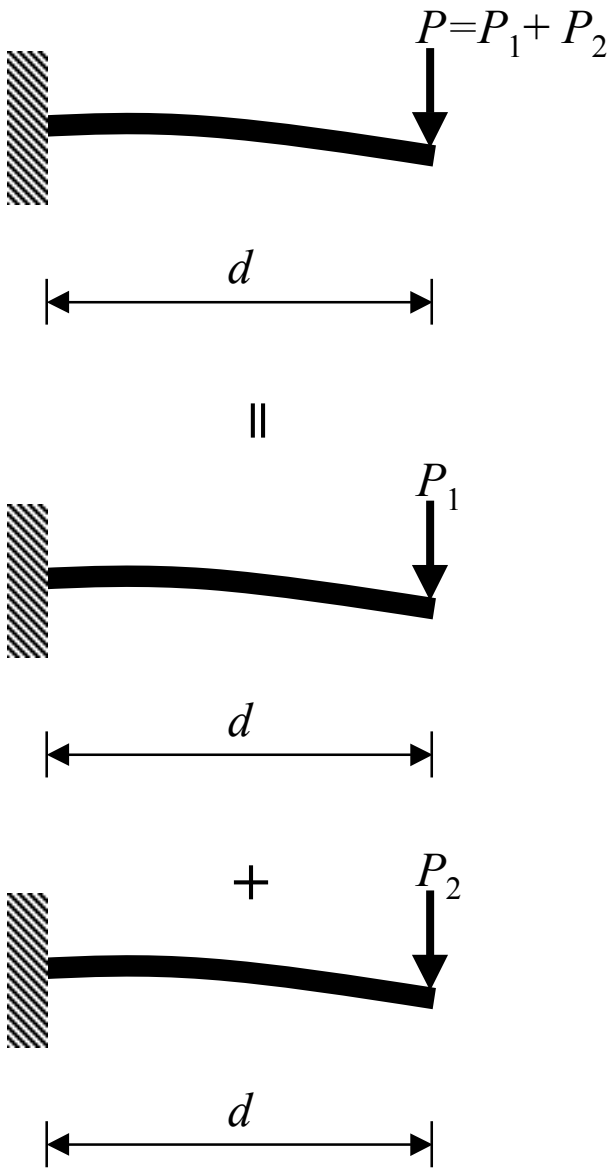
$$(\sigma_{st})_{allow}: \quad \frac{750 - P}{(2.827 \times 10^{-3})} = 120 \times 10^3, \qquad P_1 = 410.76 \text{ kN}$$

$$(\sigma_{al})_{allow}: \quad \frac{-P}{(2.121 \times 10^{-3})} = 180 \times 10^3, \qquad P_2 = 381.78 \text{ kN}$$

$$\delta_C: \quad \frac{(750 - P)(0.9)}{(2.827 \times 10^{-3})(200 \times 10^6)} + (12 \times 10^{-6})(20)(0.9) + \frac{(-P)(0.6)}{(2.121 \times 10^{-3})(70 \times 10^6)}$$

$$+ (23 \times 10^{-6})(10)(0.6) = 0, \qquad P_3 = 274.80 \text{ kN}$$

Principle of Superposition





P

The following two conditions must be valid if the principle of superposition is to be applied.

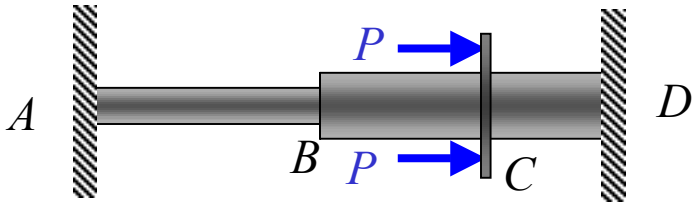
1. ***The loading must be linearly related to the stress or displacement that is to be determined***.

2. ***The loading must not significantly change the original geometry or configuration of the member***.

Statically Indeterminate Axially Loaded Members

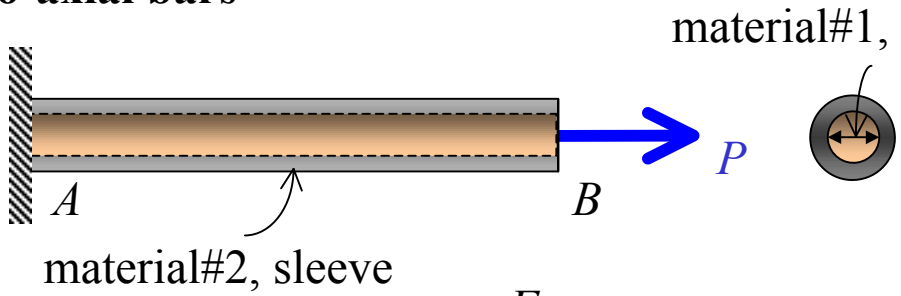
Compatibility Conditions

- **End to end bars**



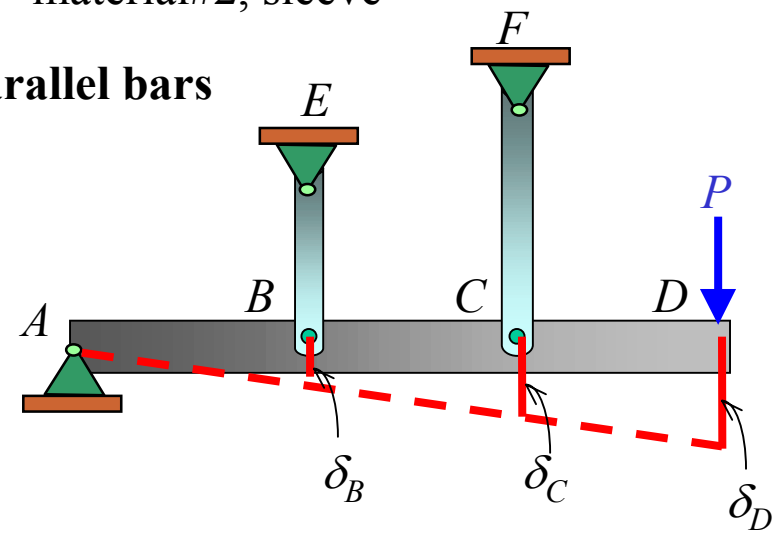
$$\delta_{D/A} = \delta_{B/A} + \delta_{C/B} + \delta_{DC} = 0$$

- **Co-axial bars**



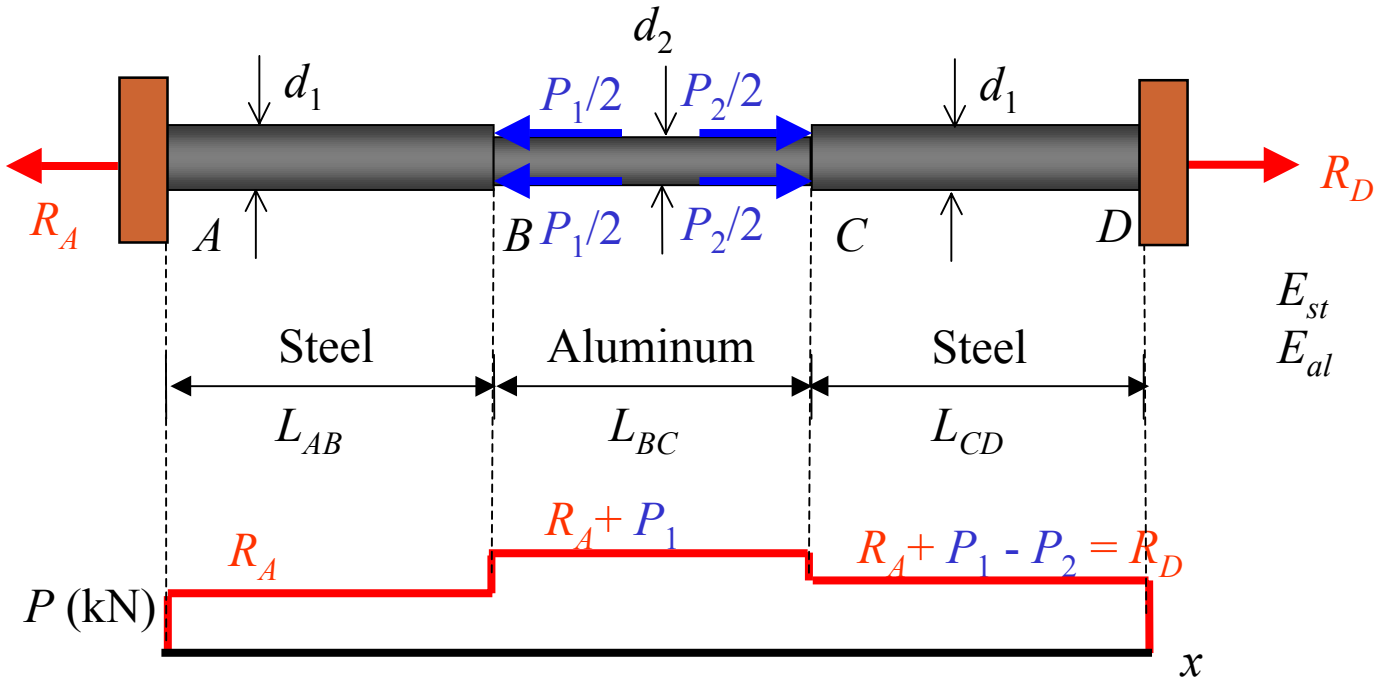
$$(\delta_B)_1 = (\delta_B)_2$$

- **Parallel bars**



$$\frac{\delta_B}{L_{AB}} = \frac{\delta_C}{L_{AC}} = \frac{\delta_D}{L_{AD}}$$

Statically Indeterminate Axially Loaded Members: End to End Bars



- Equilibrium Equation : $\Sigma F_x = 0$

$$-R_A - P_1 + P_2 + R_D = 0 \qquad \text{-----(1)}$$

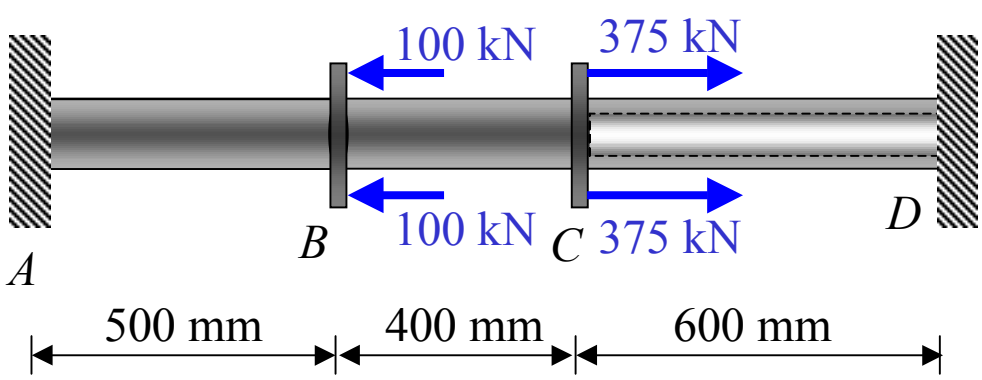
- Compatibility Equation :

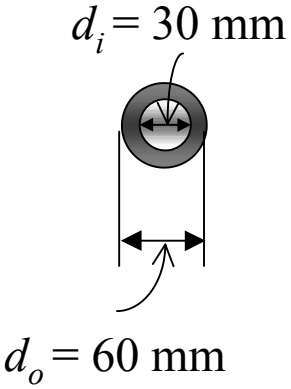
$$\delta_{D/A} = \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = 0$$

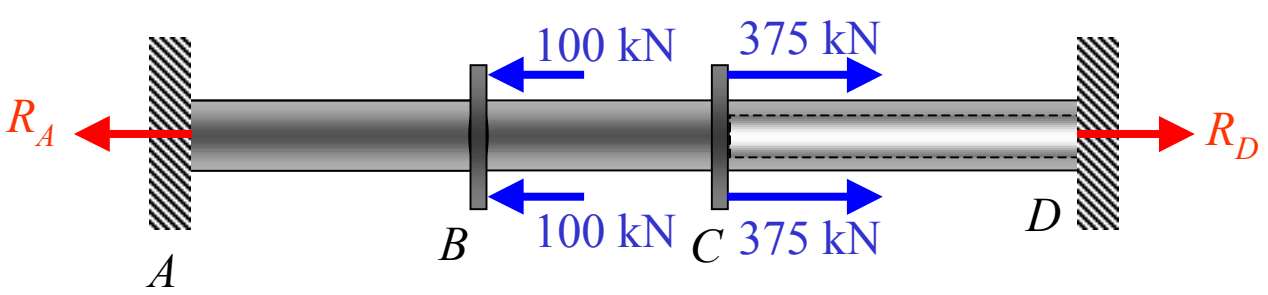
$$\frac{R_A L_{AB}}{A_{AB} E_{st}} + \frac{(R_A + P_1) L_{BC}}{A_{BC} E_{al}} + \frac{R_D L_{CD}}{A_{CD} E_{st}} = 0 \qquad \text{-----(2)}$$

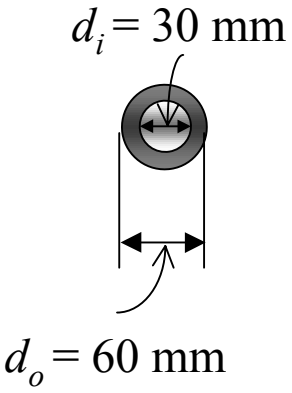
Example 5

A solid steel rod shown in the figure, having an inner diameter of 30 mm and outer diameter of 60 mm. Determine the reactions at supports. Draw the quantitative normal stress and deformation diagrams of the rod. Take E = 200 GPa.



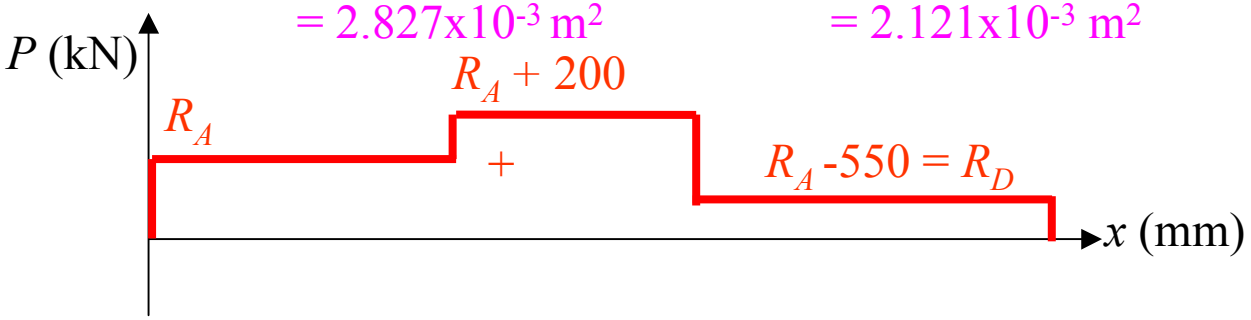






$A_{AC} = (\pi/4)(0.06^2) = 2.827\text{x}10^{-3}\ \text{m}^2$

$A_{CD} = (\pi/4)(0.06^2 - 0.03^2) = 2.121\text{x}10^{-3}\ \text{m}^2$



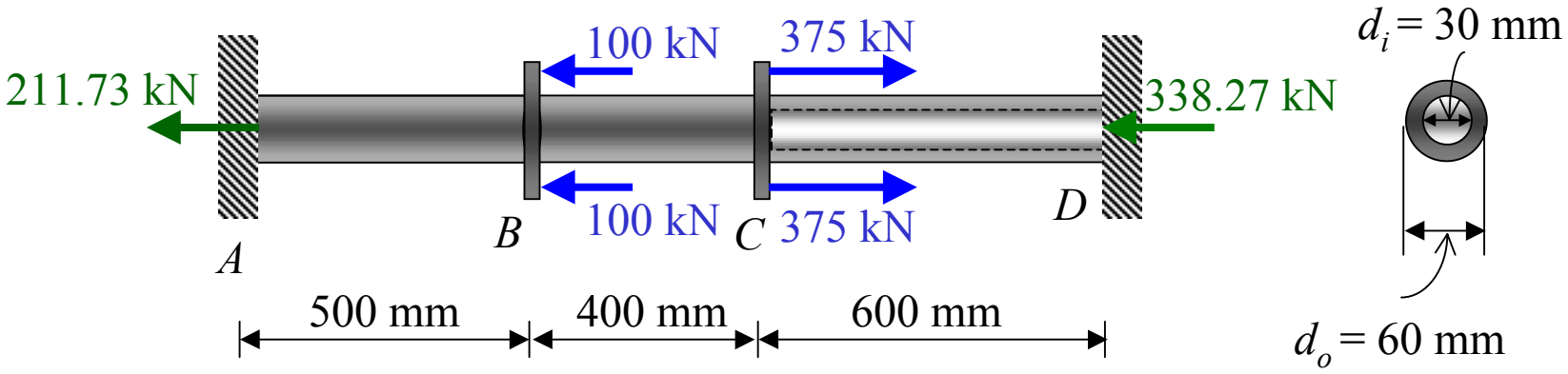
- Equilibrium equation: $R_A - 550 = R_D$ -----(1)

- Compatibility equation: $\delta_{D/A} = \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = 0$

$$\frac{R_A(0.5)}{(2.827\times10^{-3})(200\times10^{6})} + \frac{(R_A+200)(0.4)}{(2.827\times10^{-3})(200\times10^{6})} + \frac{(R_A-550)(0.6)}{(2.121\times10^{-3})(200\times10^{6})} = 0 \text{ --(2)}$$

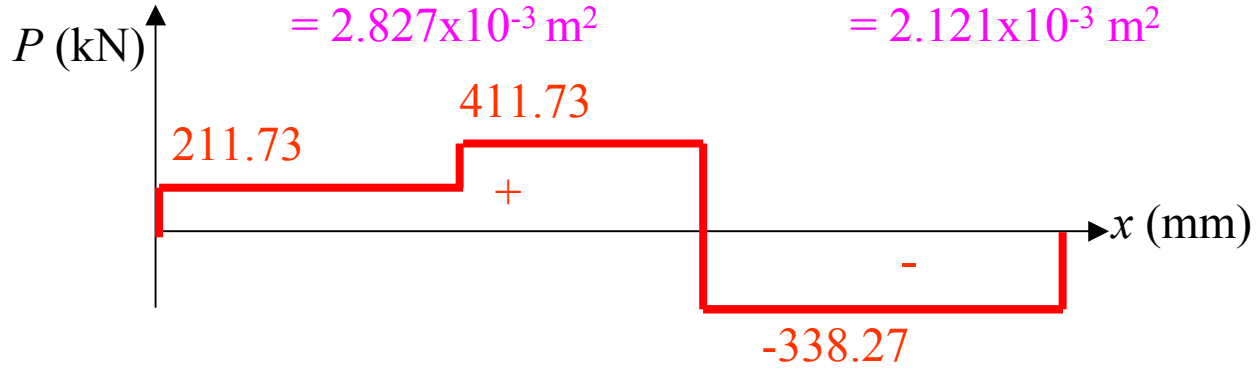
$R_A = 211.73$ kN ⇐

$R_D = R_A - 550 = 211.73 - 550 = -338.27$ kN ⇐

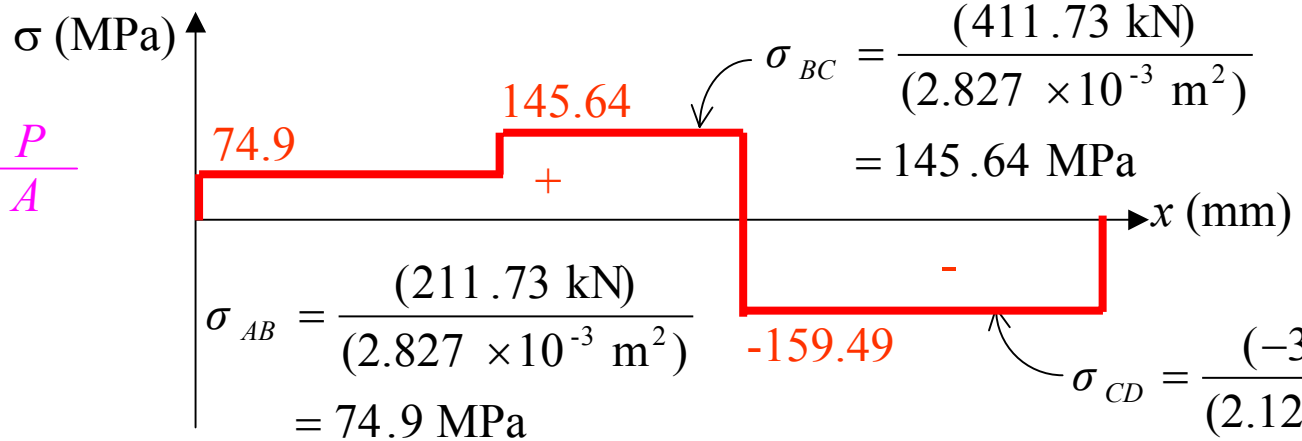


$A_{AC} = (\pi/4)(0.06^2)$
$= 2.827 \times 10^{-3}\ m^2$

$A_{CD} = (\pi/4)(0.06^2 - 0.03^2)$
$= 2.121 \times 10^{-3}\ m^2$



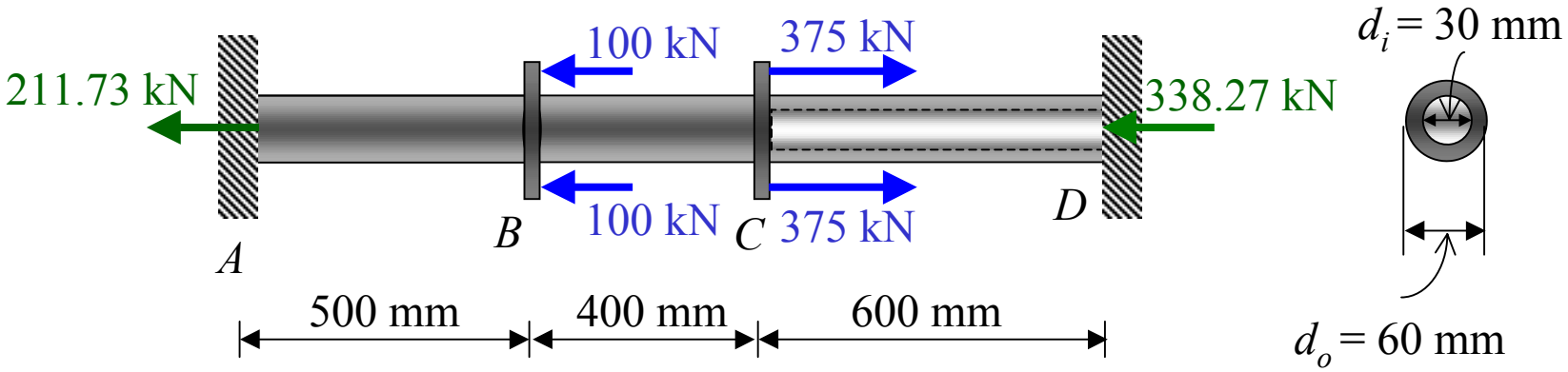
$\sigma = \frac{P}{A}$



$$\sigma_{AB} = \frac{(211.73\ \text{kN})}{(2.827 \times 10^{-3}\ \text{m}^2)} = 74.9\ \text{MPa}$$

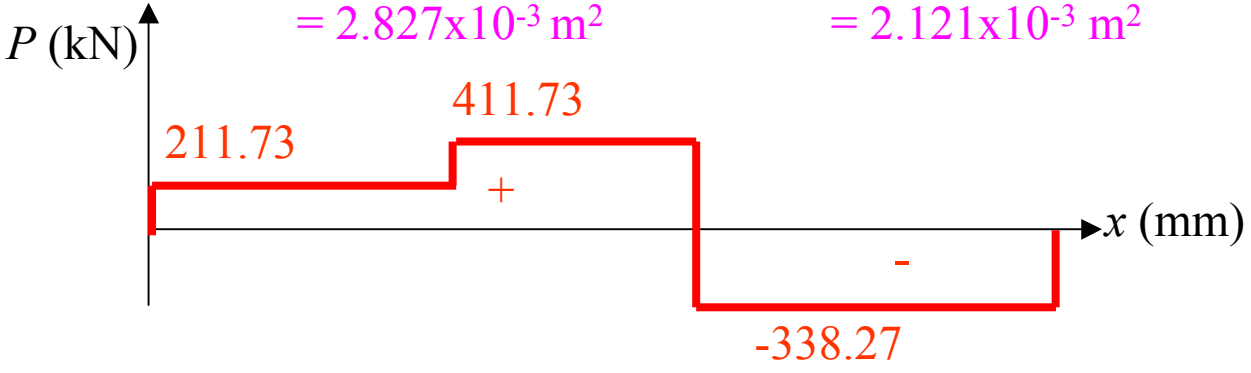
$$\sigma_{BC} = \frac{(411.73\ \text{kN})}{(2.827 \times 10^{-3}\ \text{m}^2)} = 145.64\ \text{MPa}$$

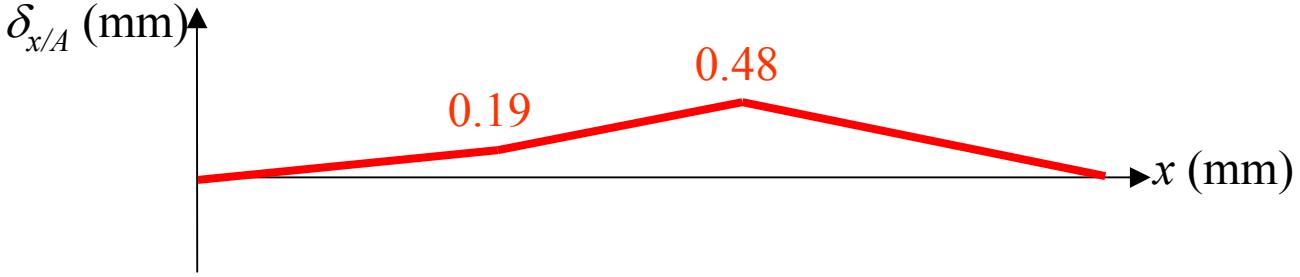
$$\sigma_{CD} = \frac{(-338.27\ \text{kN})}{(2.121 \times 10^{-3}\ \text{m}^2)} = -159.49\ \text{MPa}$$



$A_{AC} = (\pi/4)(0.06^2)$
$= 2.827 \times 10^{-3}$ m^2

$A_{CD} = (\pi/4)(0.06^2 - 0.03^2)$
$= 2.121 \times 10^{-3}$ m^2

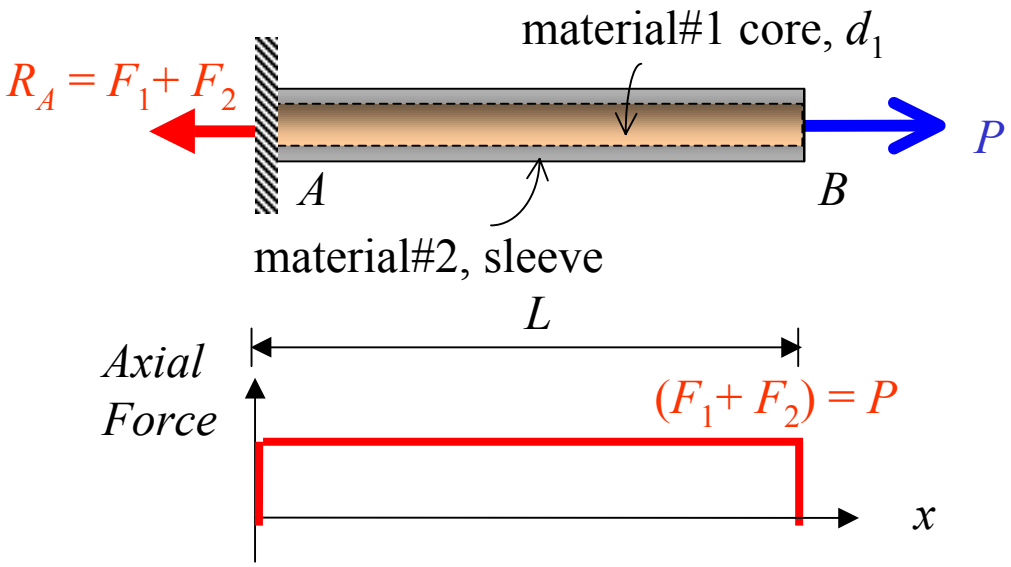


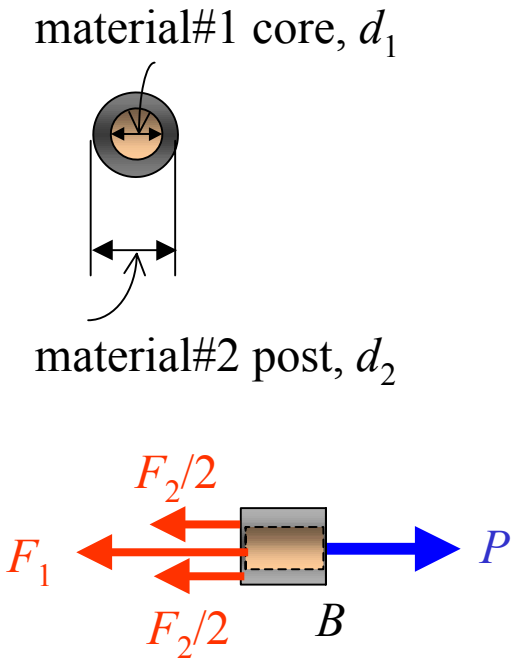


$$\delta_{D/A} = \frac{(211.73)(0.5)}{(2.827 \times 10^{-3})(200 \times 10^{6})} + \frac{(411.73)(0.4)}{(2.827 \times 10^{-3})(200 \times 10^{6})} + \frac{(-338.27)(0.6)}{(2.121 \times 10^{-3})(200 \times 10^{6})}$$

$$= 0.19 + 0.29 - 0.48 = 0 \text{ mm}$$

Statically Indeterminate Axially Loaded Members: Coaxial Bars





- Equilibrium Equation :

$$-(F_1 + F_2) + P = 0 \qquad \text{------(1)}$$

- Compatibility Equation :

$$(\delta_{B/A})_1 = (\delta_{B/A})_2$$

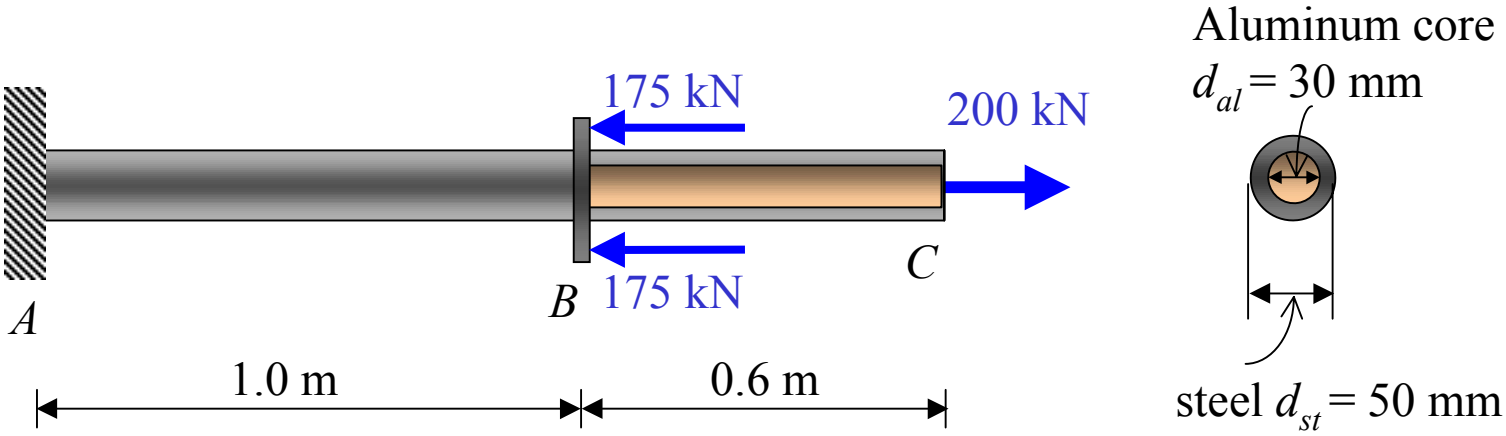
$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad -----(2)$$

Example 6

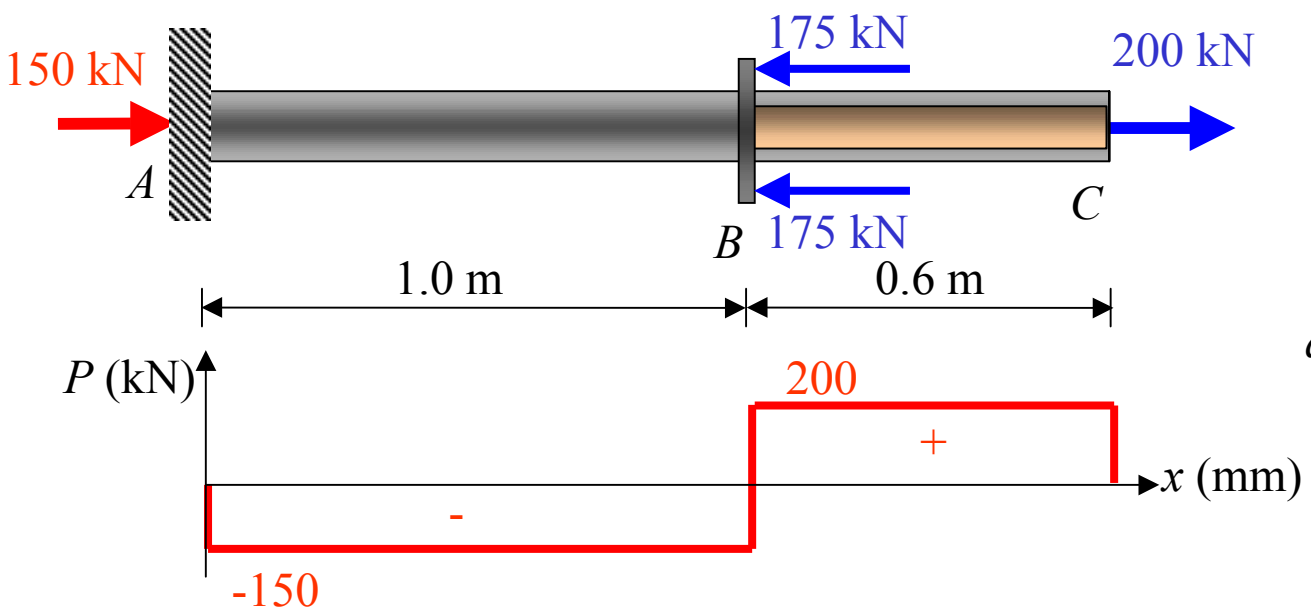
The rod is made from a solid steel section *AB* and a tubular portion made of steel and having a aluminum core section *BC*. If it is fixed to a rigid support at A. Take E_{st} = 200 GPa, E_{al} = 80 GPa

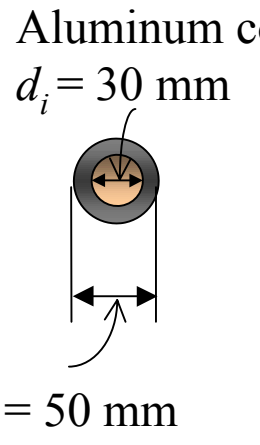
(a) Compute the normal stress in steel and aluminum.

(b) Determine the displacement of the end *C* of the rod.



(a) Compute the normal stress in steel and aluminum.





- Compatibility equation

$$(\delta_{C/B})_{st} = (\delta_{C/B})_{al}$$

$$\left(\frac{PL}{AE}\right)_{st} = \left(\frac{PL}{AE}\right)_{al}$$

$$\frac{P_{st}(0.6)}{(\pi/4)(0.05^2 - 0.03^2)(200\times10^6)} = \frac{P_{al}(0.6)}{(\pi/4)(0.03^2)(80\times10^6)} \quad ---(1)$$

$$P_{st} = (4.444)P_{al}$$

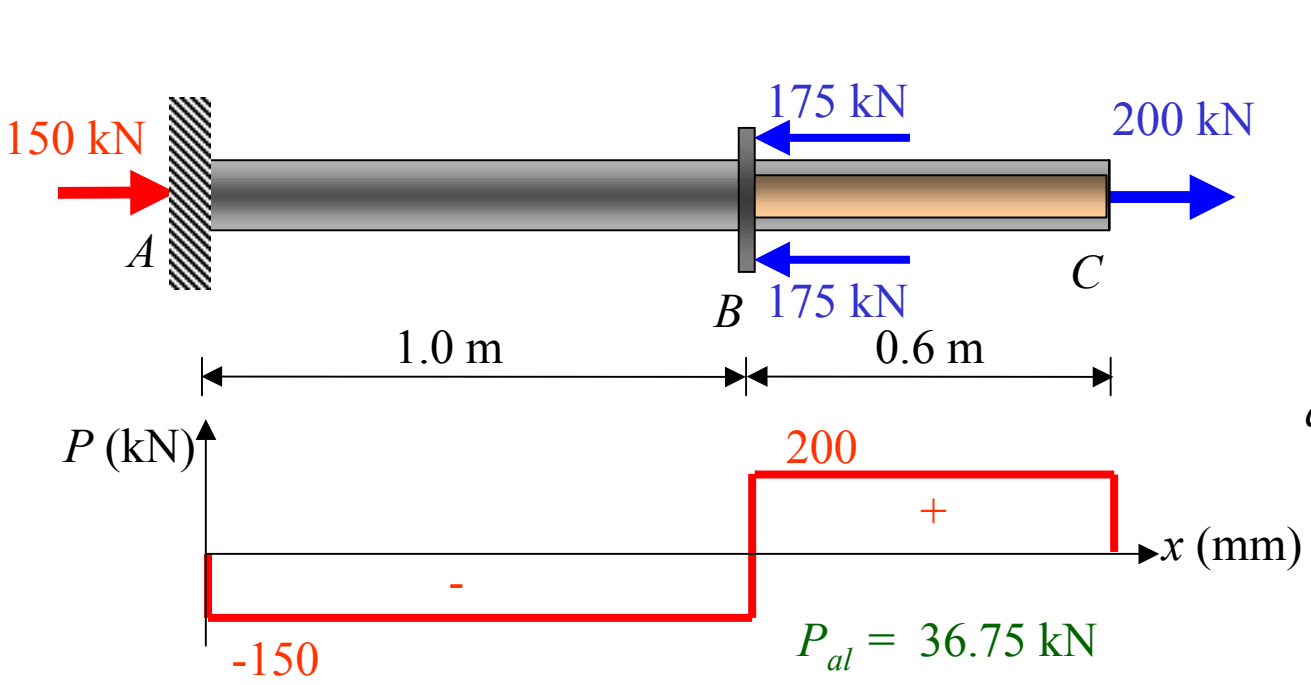
- Equilibrium equation

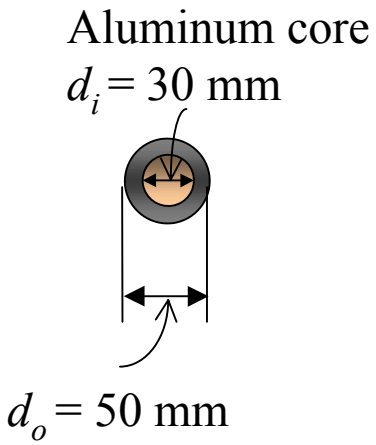
$$P_{st} + P_{al} = 200$$

$$4.444\,P_{al} + P_{al} = 200 \quad ---(2)$$

$$P_{al} = 36.75 \text{ kN}$$

$$P_{st} = 163.25 \text{ kN}$$



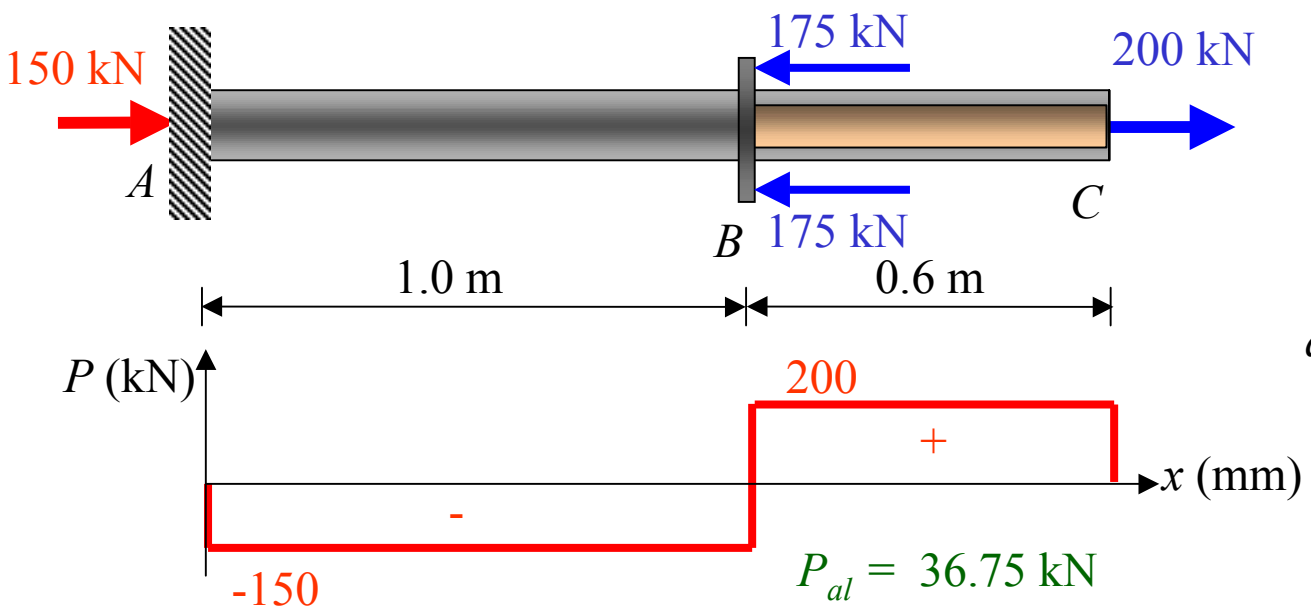


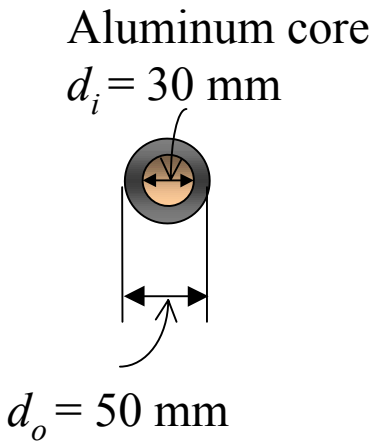
- Normal stress

$$\bullet\ (\sigma_{BC})_{st} = \frac{(P_{BC})_{st}}{A_{st}} = \frac{(163.25)}{(\pi/4)(0.05^2 - 0.03^2)} = 129.91 \text{ MPa} \Leftarrow$$

$$\bullet\ (\sigma_{BC})_{al} = \frac{(P_{BC})_{al}}{A_{al}} = \frac{(36.75)}{(\pi/4)(0.03^2)} = 51.28 \text{ MPa} \Leftarrow$$

(b) Determine the displacement of the end C of the rod.





$P_{al} = 36.75$ kN

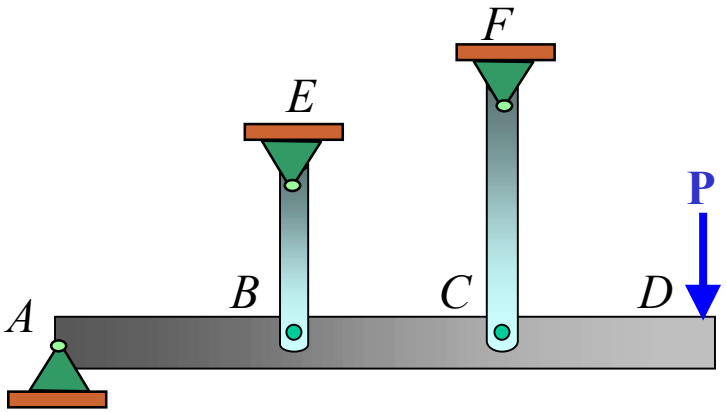
$P_{st} = 163.25$ kN

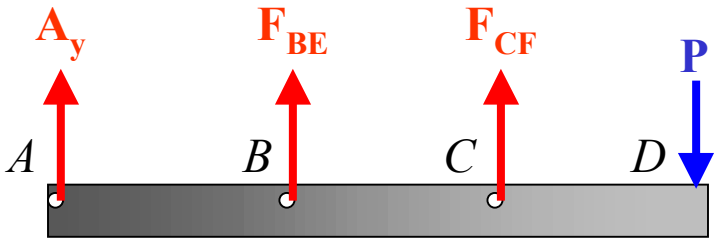
$$\delta_{C/A} = \delta_{B/A} + \delta_{C/B}$$

$$\bullet\ \delta_{C/A} = \frac{(-150)(1.0)}{(\pi/4)(0.05^2)(200\times10^6)} + \frac{(36.75)(0.6)}{(\pi/4)(0.03^2)(80\times10^6)} = -0.115 \text{ mm} \Leftarrow$$

Statically Indeterminate Axially Loaded Members: Parallel Bars

- Equilibrium Equation





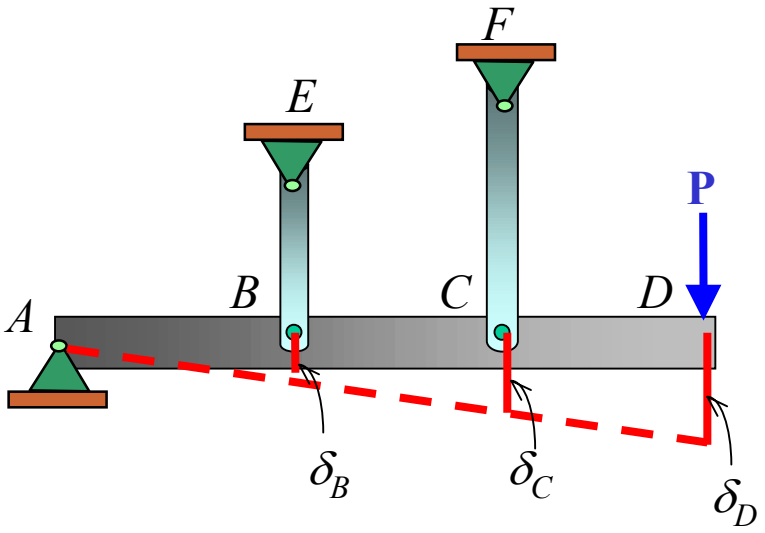
$+\uparrow \Sigma F_y = 0$:

$$A_y + F_{BE} + F_{CF} - P = 0 \quad \text{-----(1)}$$

$+\circlearrowleft \Sigma M_A = 0$:

$$F_{BE}(L_{AB}) + F_{CF}(L_{AC}) + P(L_{AD}) = 0 \quad \text{-----(2)}$$

- Compatibility Equation

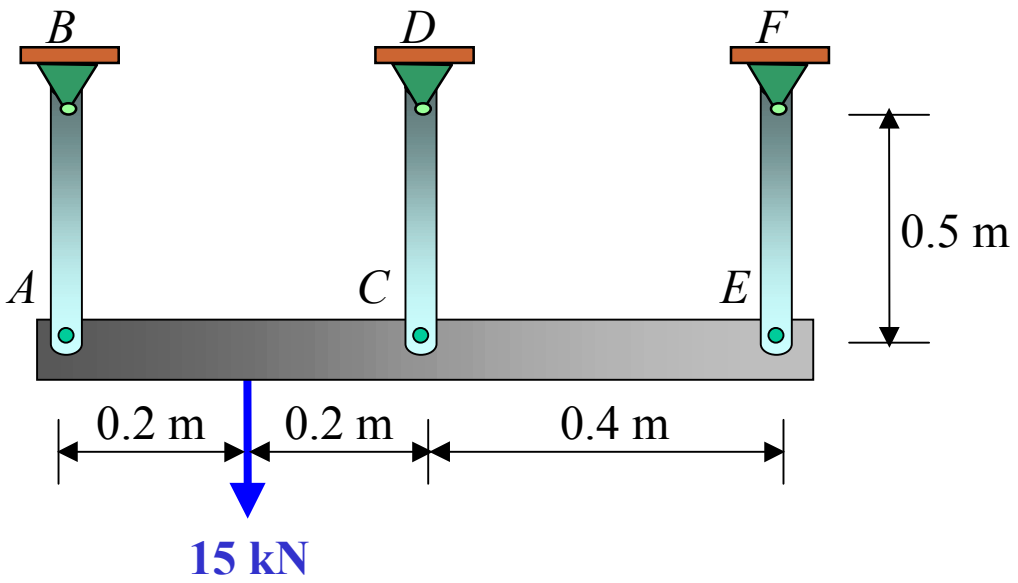


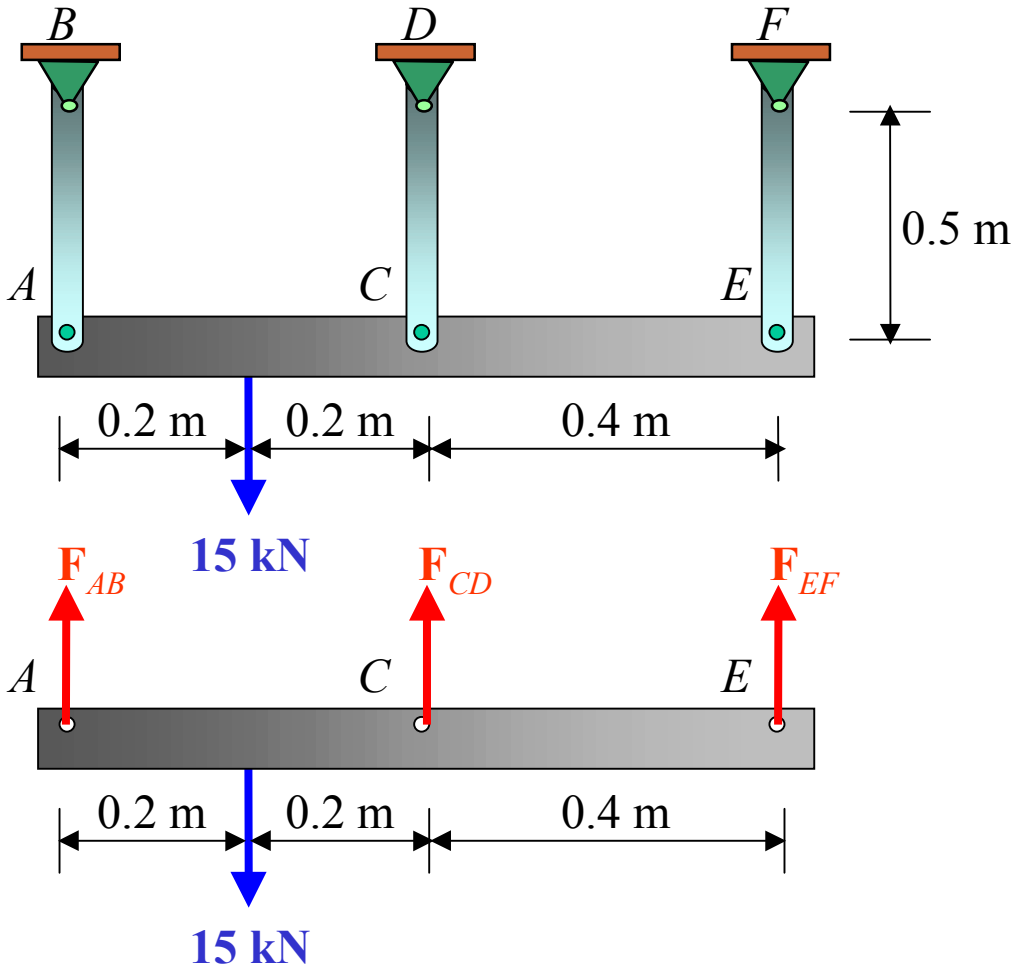
$$\frac{\delta_B}{L_{AB}} = \frac{\delta_C}{L_{AC}} = \frac{\delta_D}{L_{AD}}$$

$$\frac{F_{BE}L_{BE}}{A_{BE}E_{BE}} = \frac{F_{CF}}{A_{CF}E_{CF}} \qquad -----(3)$$

Example 8

The three A-36 steel bar shown are pin-connected to a *rigid* member. If the applied load on the member is 15 kN, determine the fore developed in each bar. Bars *AB* and *EF* each have a cross-sectional area of 25 mm^2 , and bar *CD* has a cross-sectional area of 15 mm^2

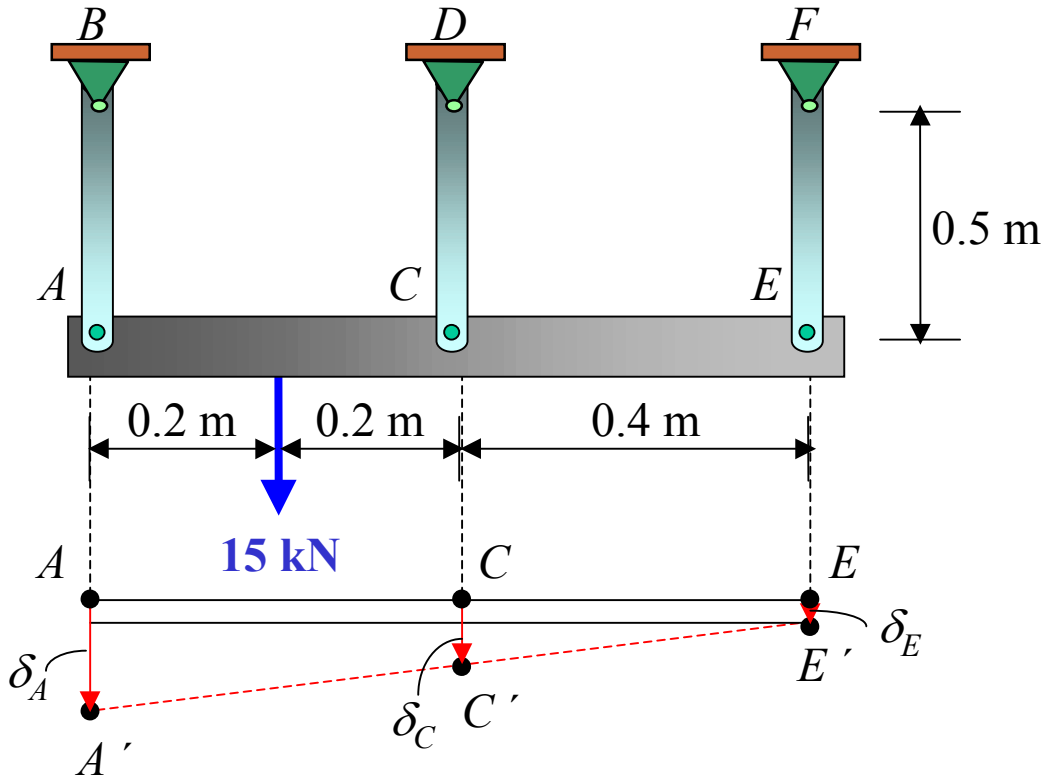




- Equilibrium Equation

$+\uparrow \Sigma F_y = 0$: $F_{AB} + F_{CD} + F_{EF} - 15\ \text{kN} = 0$ -----(1)

$+\circlearrowleft \Sigma M_C = 0$: $-F_{AB}(0.4\ \text{m}) + 15\ \text{kN}(0.2\ \text{m}) + F_{EF}(0.4\ \text{m}) = 0$ -----(2)



- Compatibility Equation

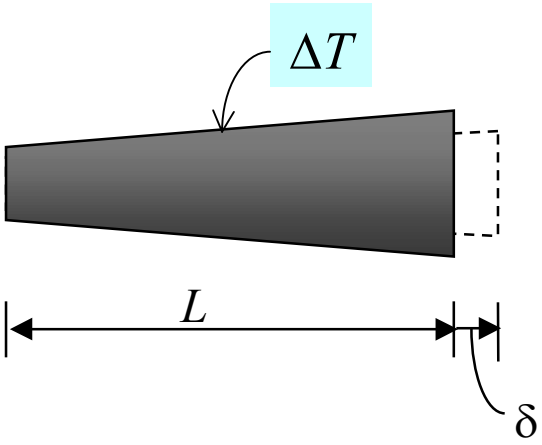
$$\frac{\delta_A - \delta_E}{0.8\ \text{m}} = \frac{\delta_C - \delta_E}{0.4\ \text{m}}$$

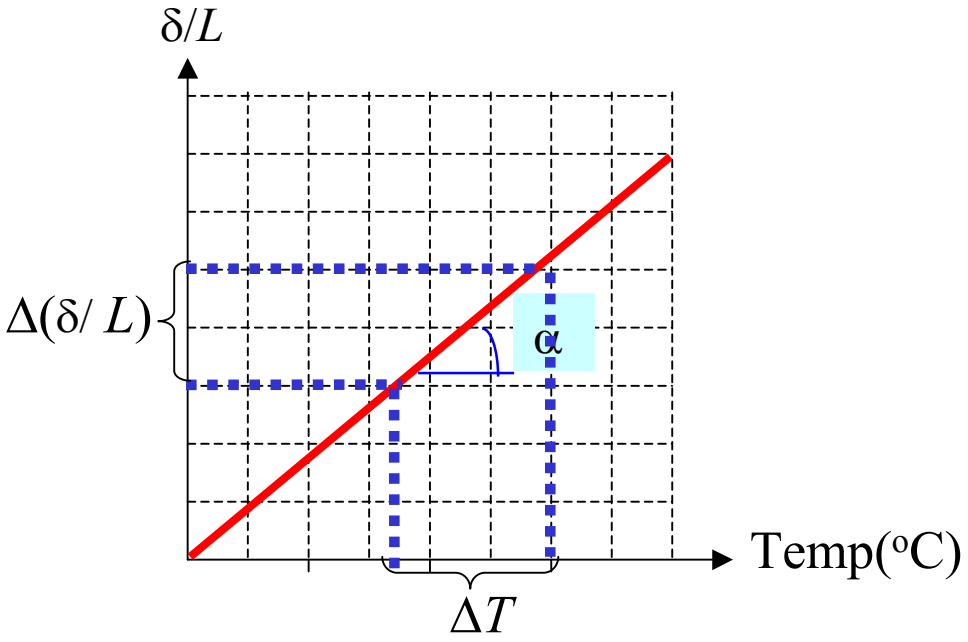
$$\delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

$$\frac{F_{CD}(0.5\ \text{m})}{(15\ \text{mm}^2)E_{st}} = \frac{1}{2}\left[\frac{F_{AB}(0.5\ \text{m})}{(25\ \text{mm}^2)E_{st}}\right] + \frac{1}{2}\left[\frac{F_{EF}(0.5\ \text{m})}{(25\ \text{mm}^2)E_{st}}\right] \qquad -----(3)$$

Solving Eqs. 1-3 : $F_{AB} = 9.52$ kN, $F_{CD} = 3.46$ kN, $F_{EF} = 2.02$ kN,

Thermal Stress



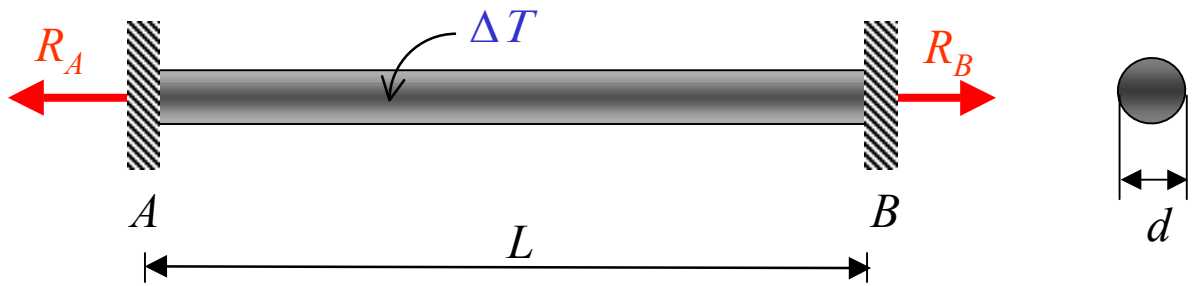


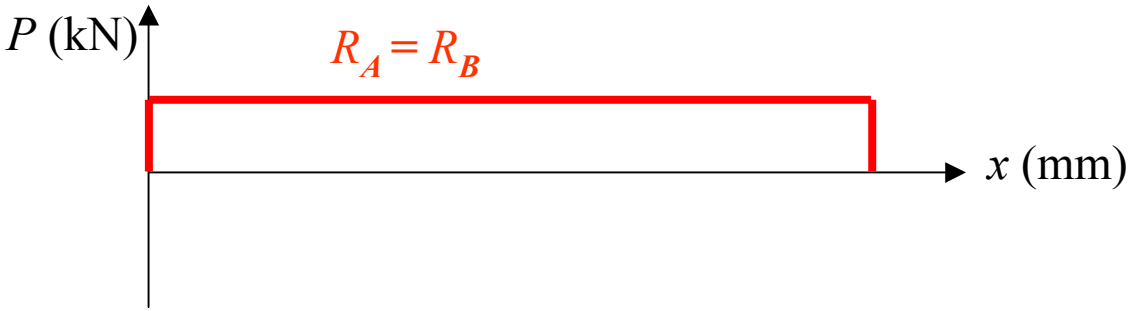
$$\delta_T = \int_0^L \varepsilon_T dx = \int_0^L \alpha(\Delta T) dx$$

$$\delta_T = \alpha(\Delta T)L$$

$$\varepsilon_T = \alpha(\Delta T)$$

$$\alpha = \Delta(\delta/L)/(\Delta T) \qquad (\text{mm/mm})/^{o}\text{C}$$

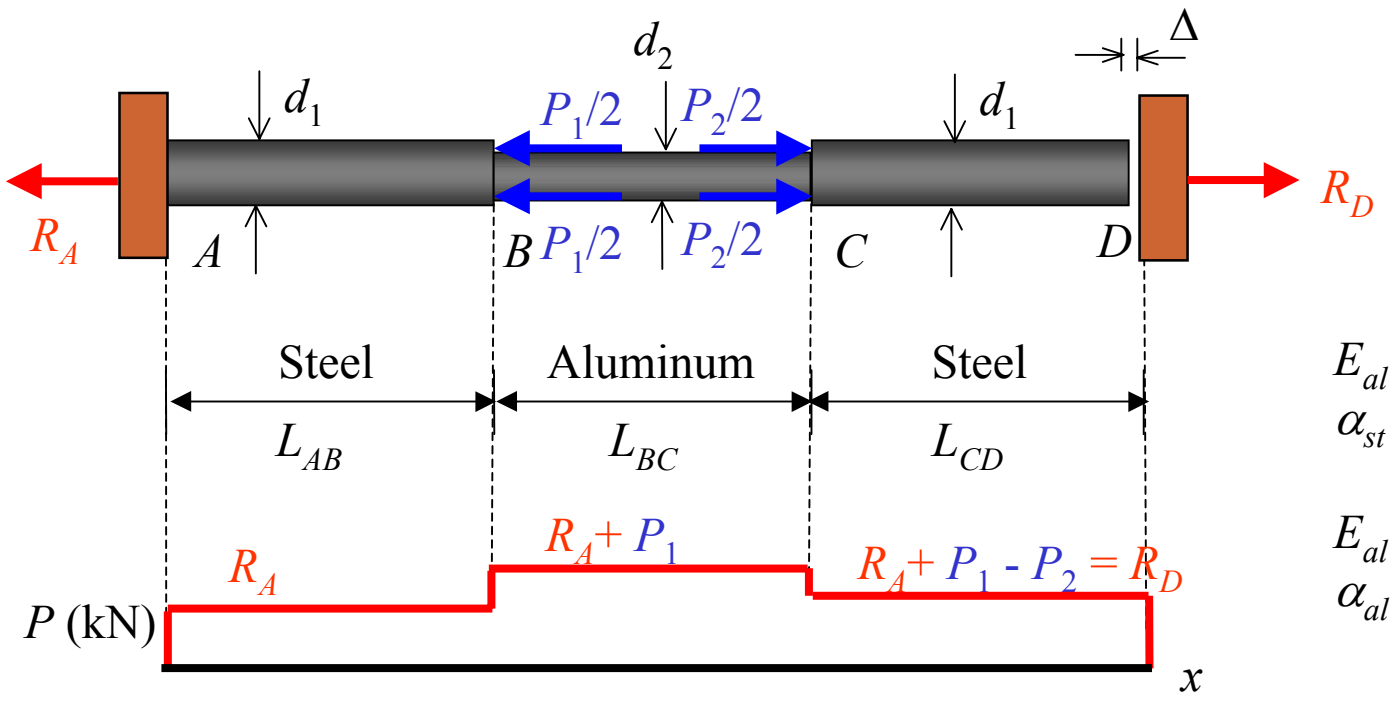




$$\delta_{B/A} = 0 = \frac{R_A L}{AE} + \alpha(\Delta T)L$$

$$R_A = -\alpha(\Delta T)AE = R_B$$

• **Compatibility condition with thermal strain**



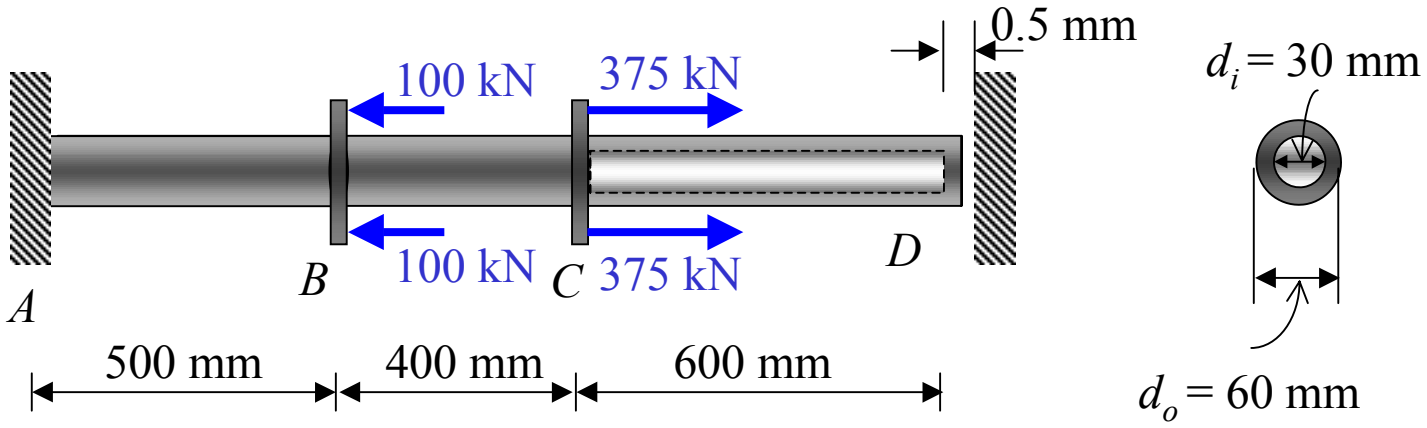
$$\delta = \delta_{\text{Force}} + \delta_{\Delta T} = \Sigma[(\varepsilon_{\text{Force}} L) + (\varepsilon_{\Delta T} L)] = \Sigma[(PL/AE)] + \Sigma[\alpha(\Delta T) L]$$

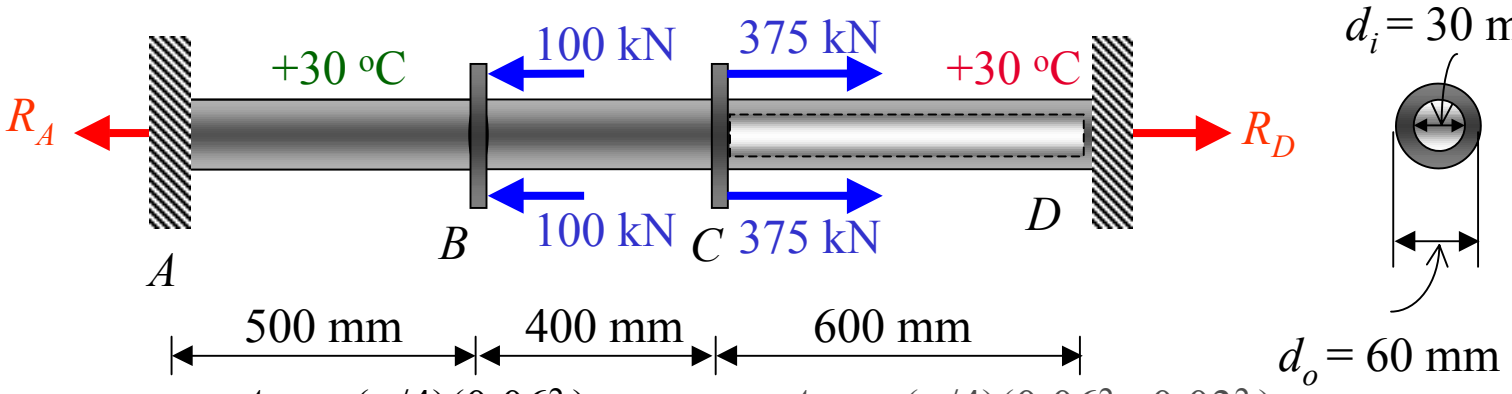
Compatibility equation:

$$\delta_{D/A} = \Delta \quad \rightarrow \quad \delta_{B/A} + \delta_{C/B} + \delta_{D/C} = \Delta$$

Example 7

A solid steel rod shown in the figure, having an inner diameter of 30 mm and outer diameter of 60 mm. Draw the quantitative normal stress and the Elongation of the rod. Temperature in member *AB* and *CD* are increase 30 ^{o}C. Take E = 200 GPa, $\alpha = 12 \times 10^{-6}$ /^{o}C.

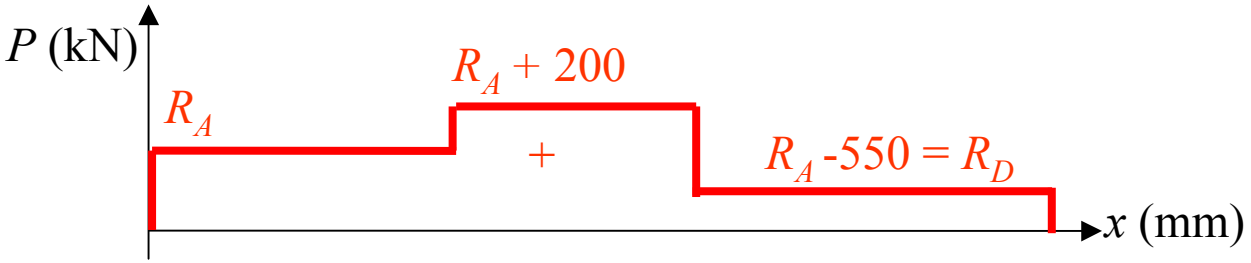




$A_{AC} = (\pi/4)(0.06^2) = 2.827\text{x}10^{-3}\ \text{m}^2$

$A_{CD} = (\pi/4)(0.06^2 - 0.03^2) = 2.121\text{x}10^{-3}\ \text{m}^2$

- Equilibrium equation:

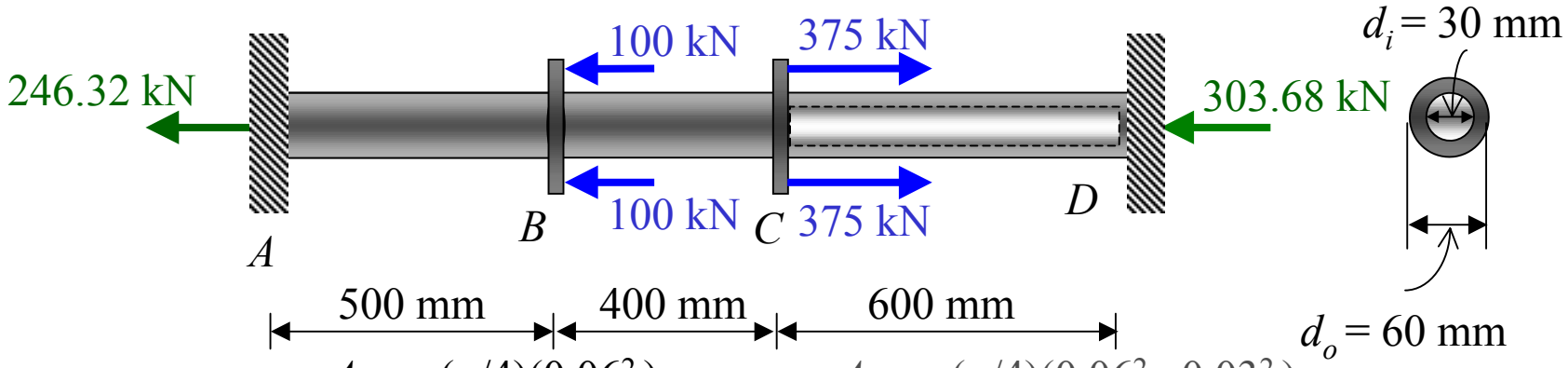


- Compatibility equation: $\delta_{D/A} = 0.5\text{x}10^{-3}\ \text{m} = \delta_{\text{Force}} + \delta_{\Delta T} = \Sigma[(PL/AE) + \alpha(\Delta T)L]$

$$0.5\times10^{-3} = \frac{R_A(0.5)}{(2.827\times10^{-3})(200\times10^{6})} + (12\times10^{-6})(+30)(0.5) + \frac{(R_A+200)(0.4)}{(2.827\times10^{-3})(200\times10^{6})}$$

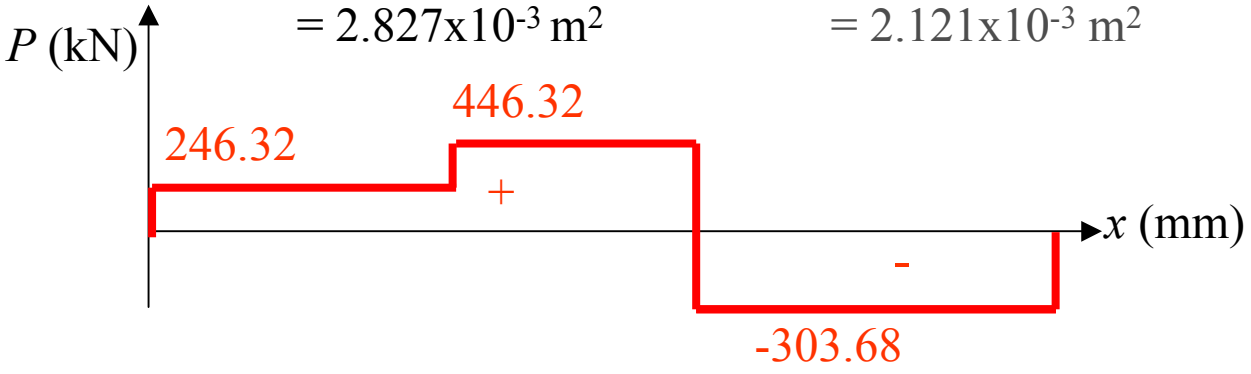
$$+\frac{(R_A-550)(0.6)}{(2.121\times10^{-3})(200\times10^{6})} + (12\times10^{-6})(+30)(0.6)$$

$R_A = 246.32$ kN , $R_D = R_A - 550 = 246.32 - 550 = -303.68$ kN

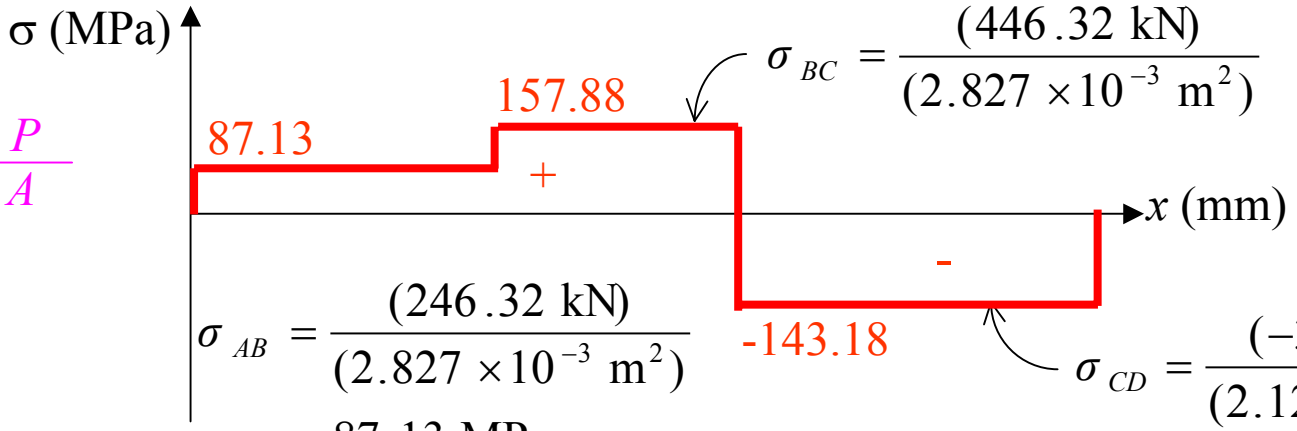


$A_{AC} = (\pi/4)(0.06^2)$
$= 2.827 \times 10^{-3}\ m^2$

$A_{CD} = (\pi/4)(0.06^2 - 0.03^2)$
$= 2.121 \times 10^{-3}\ m^2$



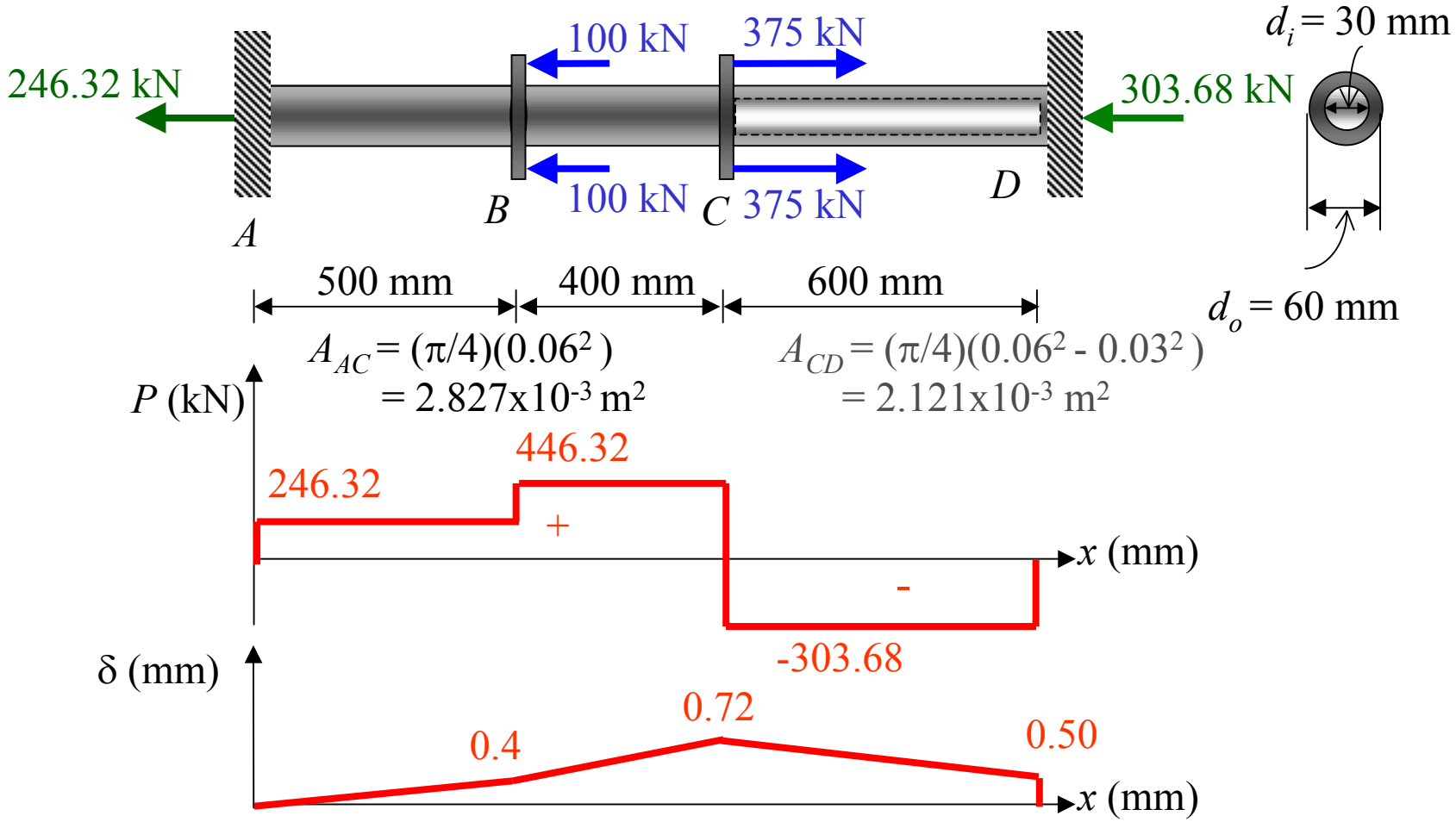
$$\sigma = \frac{P}{A}$$



$$\sigma_{AB} = \frac{(246.32\ \text{kN})}{(2.827 \times 10^{-3}\ \text{m}^2)} = 87.13\ \text{MPa}$$

$$\sigma_{BC} = \frac{(446.32\ \text{kN})}{(2.827 \times 10^{-3}\ \text{m}^2)}$$

$$\sigma_{CD} = \frac{(-303.68\ \text{kN})}{(2.121 \times 10^{-3}\ \text{m}^2)} = -143.18\ \text{MPa}$$



$$\delta_{D/A} = \frac{(246.32)(0.5)\times 10^3}{(2.827\times 10^{-3})(200\times 10^6)} + (12\times 10^{-6})(30)(.500) + \frac{(446.32)(0.4)\times 10^3}{(2.827\times 10^{-3})(200\times 10^6)}$$

$$+ \frac{(-303.68)(0.6)\times 10^3}{(2.121\times 10^{-3})(200\times 10^6)} + (12\times 10^{-6})(30)(.600)$$

$$= 0.218 + 0.18 + 0.316 - 0.43 + 0.216 = 0.5 \text{ mm}$$

Example 8

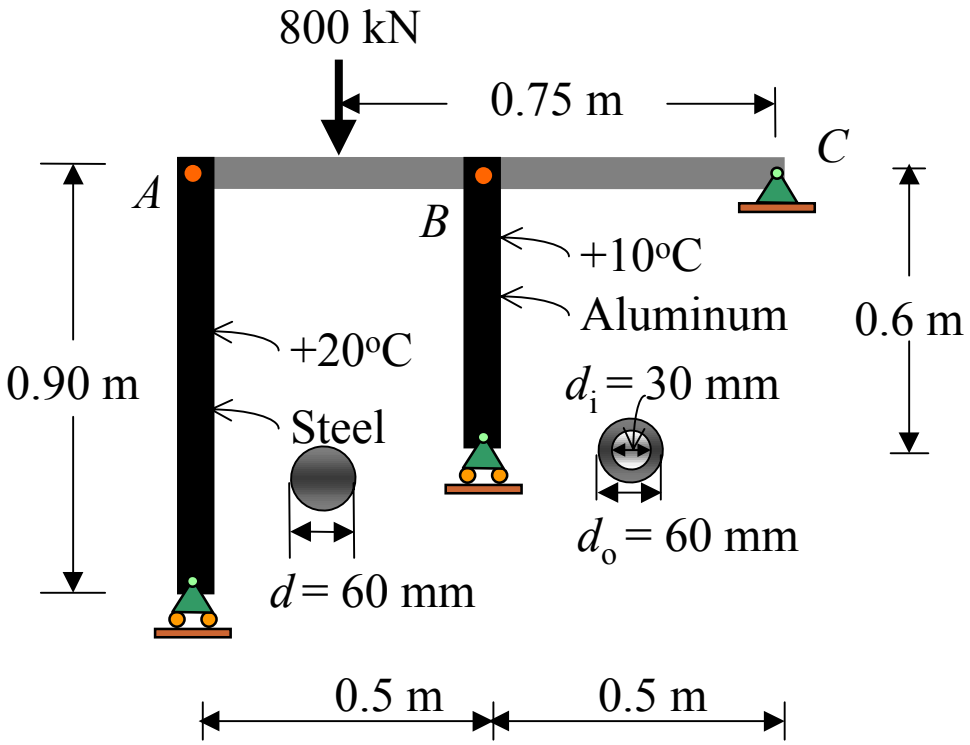
From the frame shown, determine:

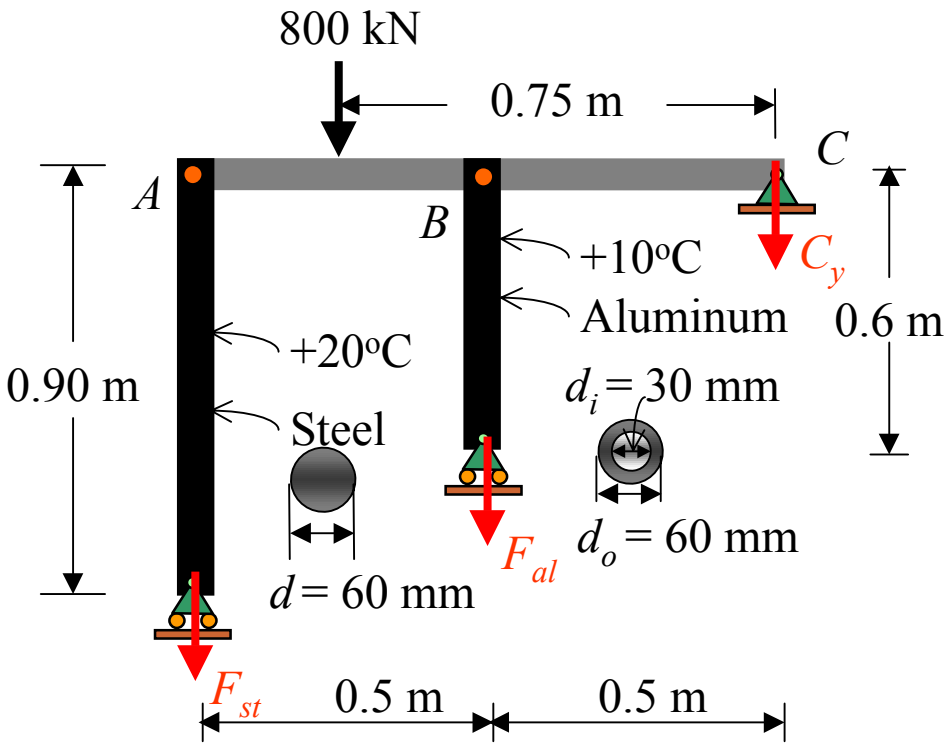
(a) Reactions at all support

(b) Normal stress in steel and aluminum

(c) Displacement at *A* and *B*

Take E_{st} = 200 GPa, E_{al} = 70 GPa, α_{st} = 12x10^{-6} /^{o}C and α_{al} = 23x10^{-6} /^{o}C.





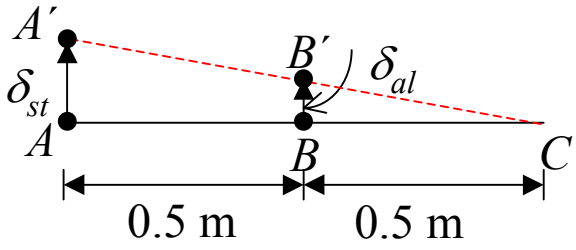
(a) Reactions at all support

- **Equilibrium equation**

$$(+\circlearrowleft \quad \Sigma M_C = 0: \quad 800(0.75) + 0.5F_{al} + 1F_{st} = 0$$

$$F_{al} = -1200 - 2F_{st} \quad ---(1)$$

- **Compatibility**



$$\frac{\delta_{st}}{1} = \frac{\delta_{al}}{0.5}$$

$$\delta_{st} = 2\delta_{al} \quad ---(2)$$

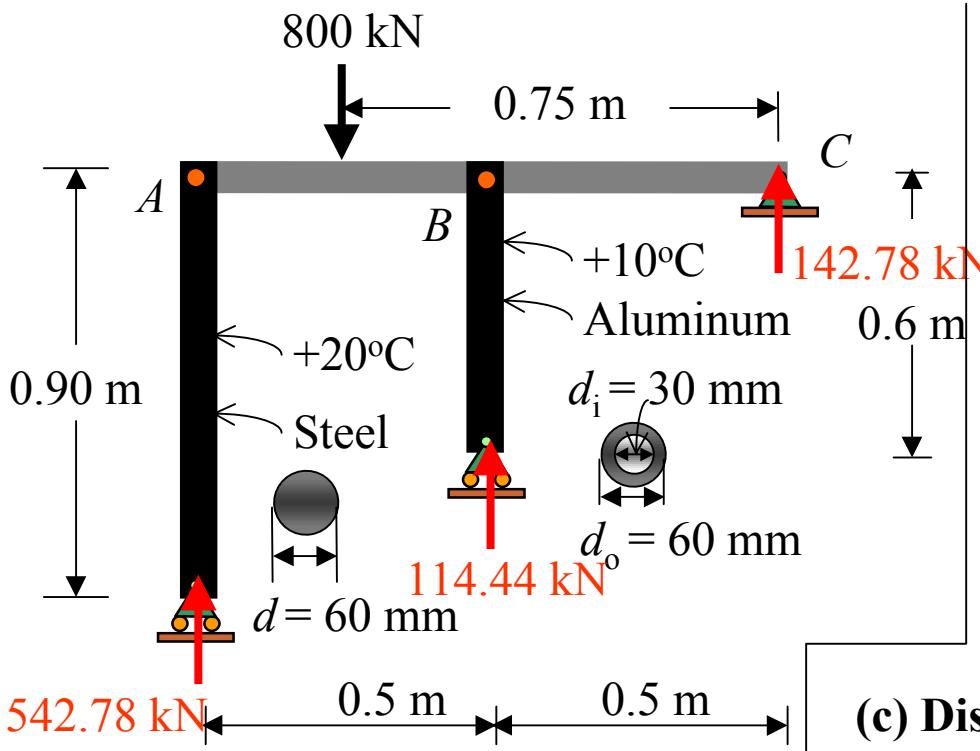
$$\delta_{st} = \frac{F_{st}(0.9)}{(\frac{\pi}{4}(0.06^2))(200\times10^6)} + (12\times10^{-6})(20)(0.9)$$

$$\delta_{al} = \frac{F_{al}(0.6)}{(\frac{\pi}{4}(0.06^2 - 0.03^2))(70\times10^6)} + (23\times10^{-6})(10)(0.6)$$

Substitute δ_{st} and δ_{al} in eq.(2) from eq.(1) and (2) can solve

F_{st} = -542.78 kN (C) F_{al} = -114.44 kN (C)

$$+\uparrow \quad \Sigma F_y = 0: \quad 542.78 + 114.44 - 800 - C_y = 0, \quad C_y = -142.78 \text{ kN}, \uparrow$$



$E_{st} = 200$ GPa
$E_{al} = 70$ GPa
$\alpha_{st} = 12\text{x}10^{-6}$ /°C
$\alpha_{al} = 23\text{x}10^{-6}$ /°C.

(b) Normal stress in steel and aluminum

$$\sigma_{st} = \frac{-542.78 \text{ kN}}{\frac{\pi}{4}(0.06^2)} = -191.97 \text{ MPa}\,(C)$$

$$\delta_{al} = \frac{-114.44 \text{ kN}}{\frac{\pi}{4}(0.06^2 - 0.03^2)} = -53.97 \text{ MPa}(C)$$

(c) Displacement at *A* and *B*

$$\delta_{st} = \frac{-(542.78)(0.9)}{(\frac{\pi}{4}(0.06^2))(200 \times 10^6)} + (12 \times 10^{-6})(20)(0.9)$$

$$= -0.864 \text{ mm} + 0.216 \text{ mm} = -0.648 \text{ mm}$$

$$\delta_{al} = \frac{(-114.44 \text{ kN})(0.6)}{(\frac{\pi}{4}(0.06^2 - 0.03^2))(70 \times 10^6)} + (23 \times 10^{-6})(10)(0.6)$$

$$= -0.463 \text{ mm} + 0.138 \text{ mm} = -0.325 \text{ mm}$$