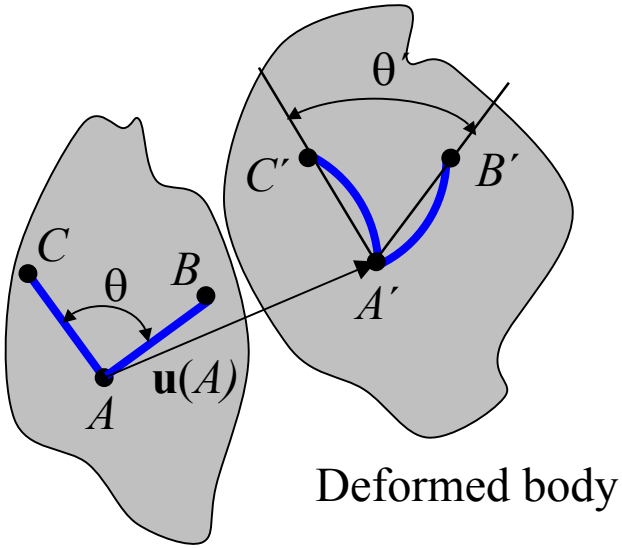
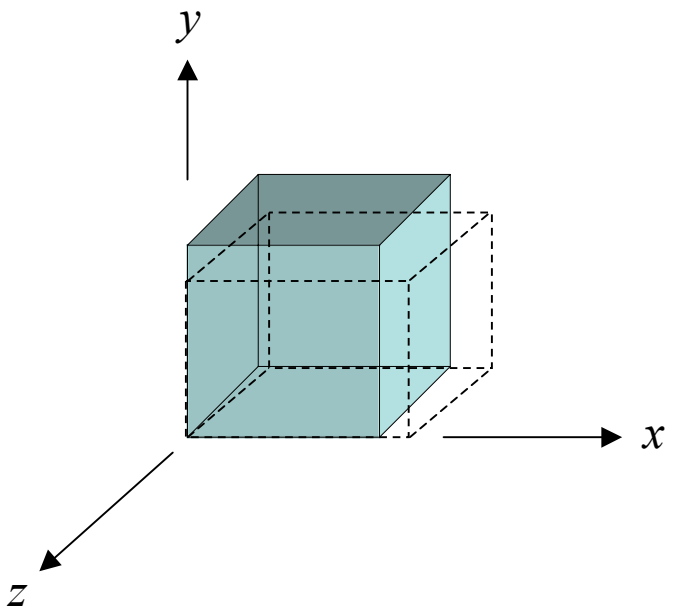
STRAIN

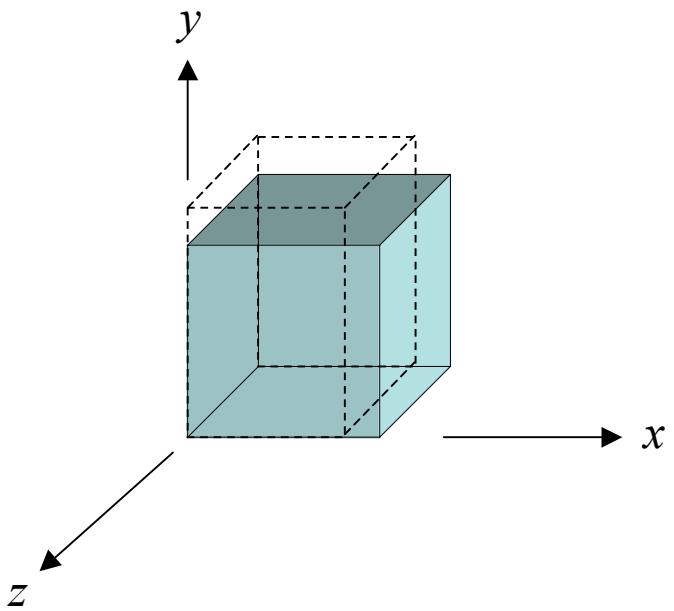
- **Deformation**
- **Strain**
  - Normal Strain
  - Shear Strain
  - Cartesian Strain Components

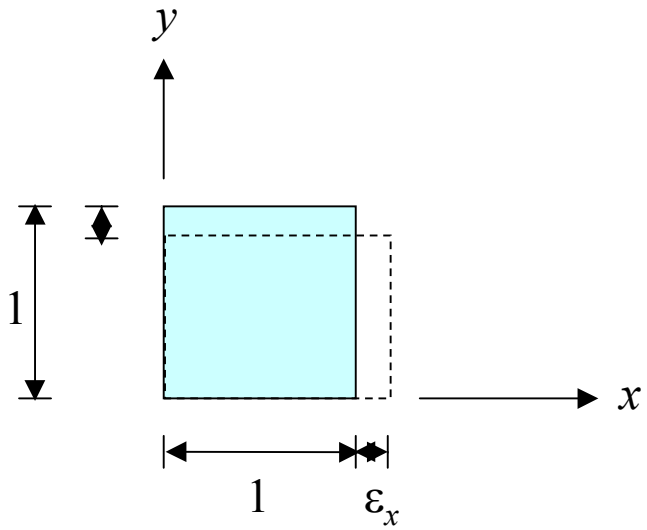
Deformation

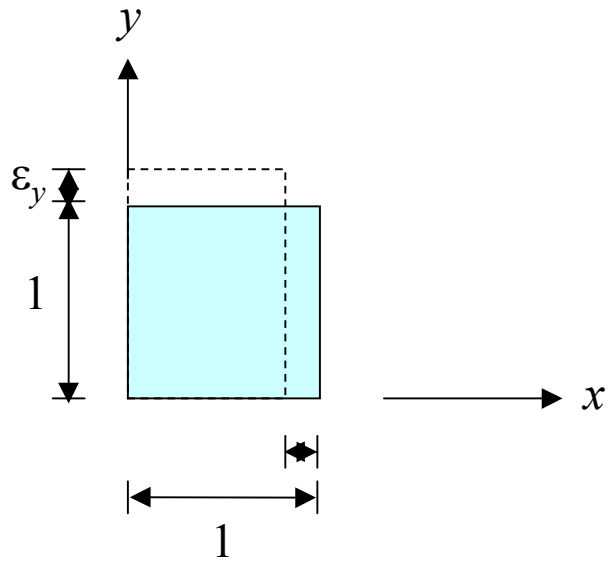


Strain • **Normal Strain**

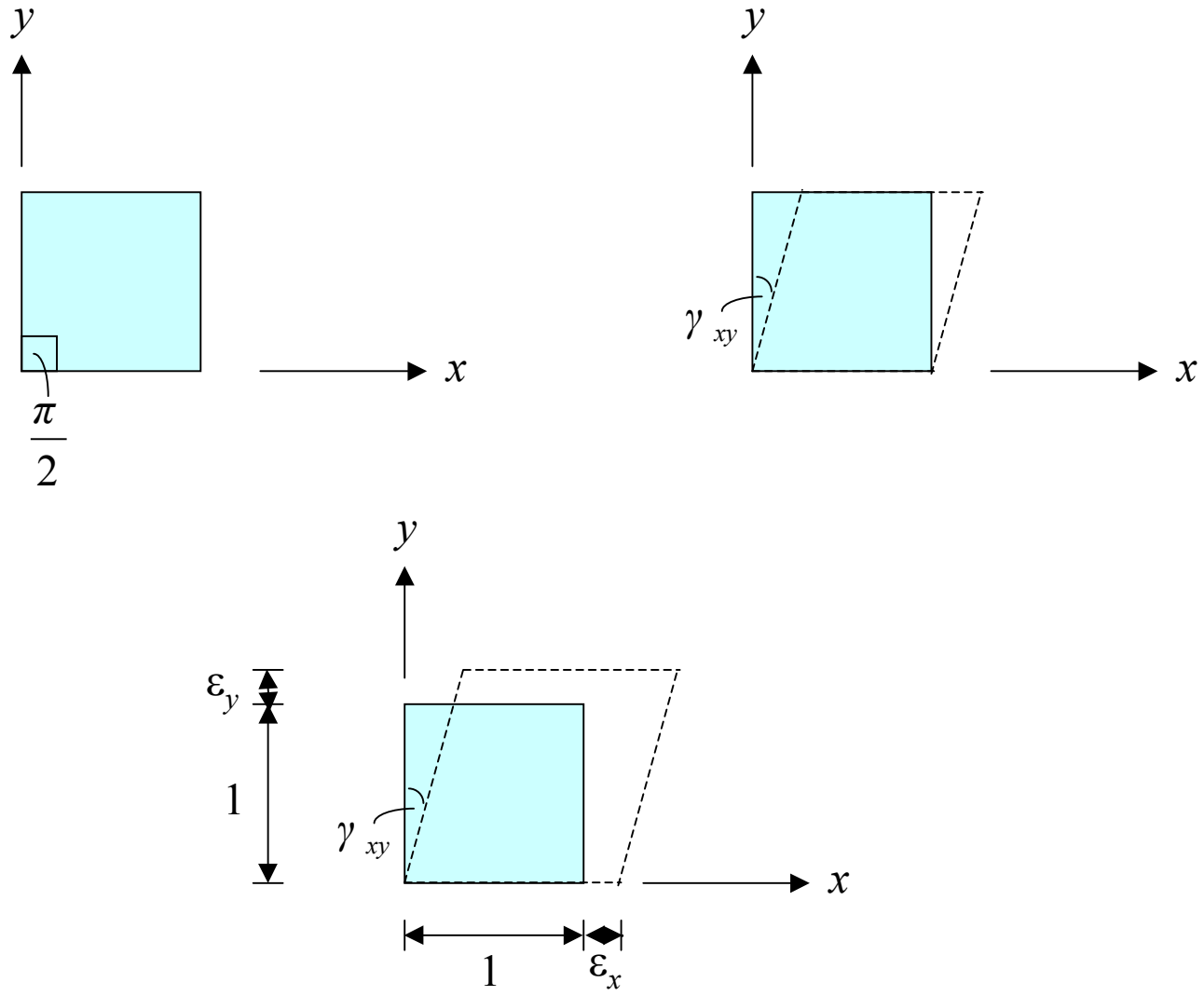






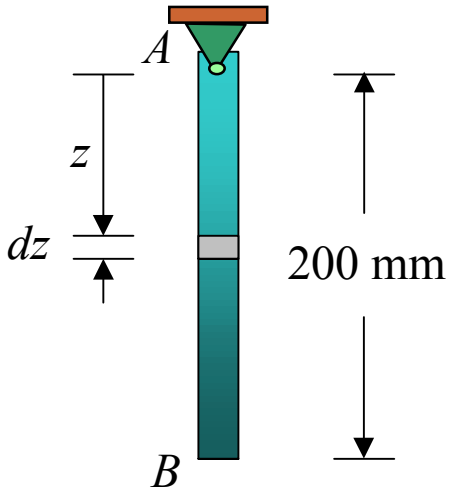


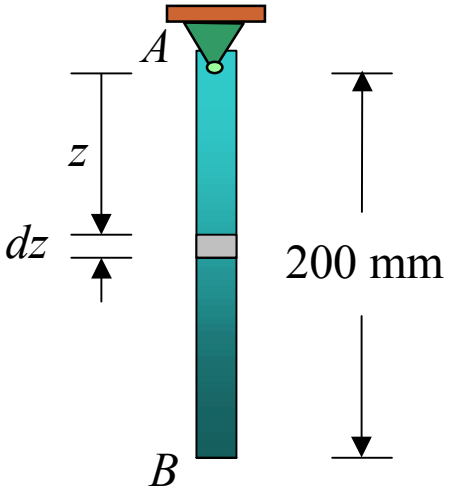
• **Shear Strain**



Example 1

The slender rod shown subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\varepsilon_z = 40(10^{-3})z^{1/2}$, where z is given in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.





(a) the displacement of the end *B* of the rod due to the temperature increase

$$dz' = [40(10^{-3})z^{1/2}]dz$$

$$\Delta B = \int_0^{0.2\,\text{m}} [40(10^{-3})z^{1/2}]dz$$

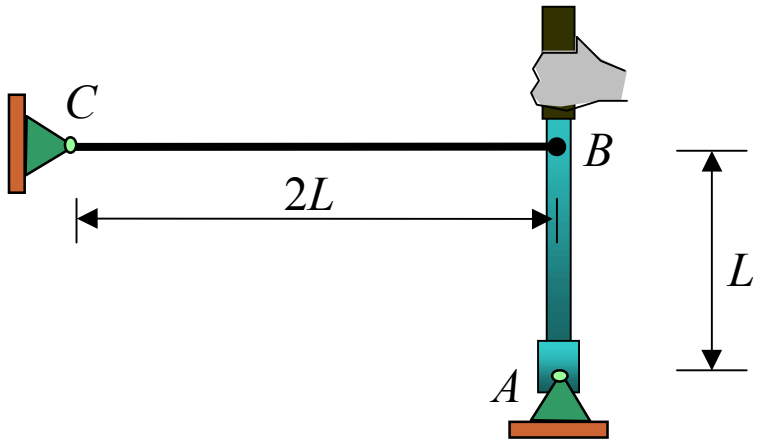
$$= 40(10^{-3})\left(\frac{2}{3}z^{3/2}\right)\Big|_0^{0.2\,\text{m}} = 0.00239\,m \downarrow$$

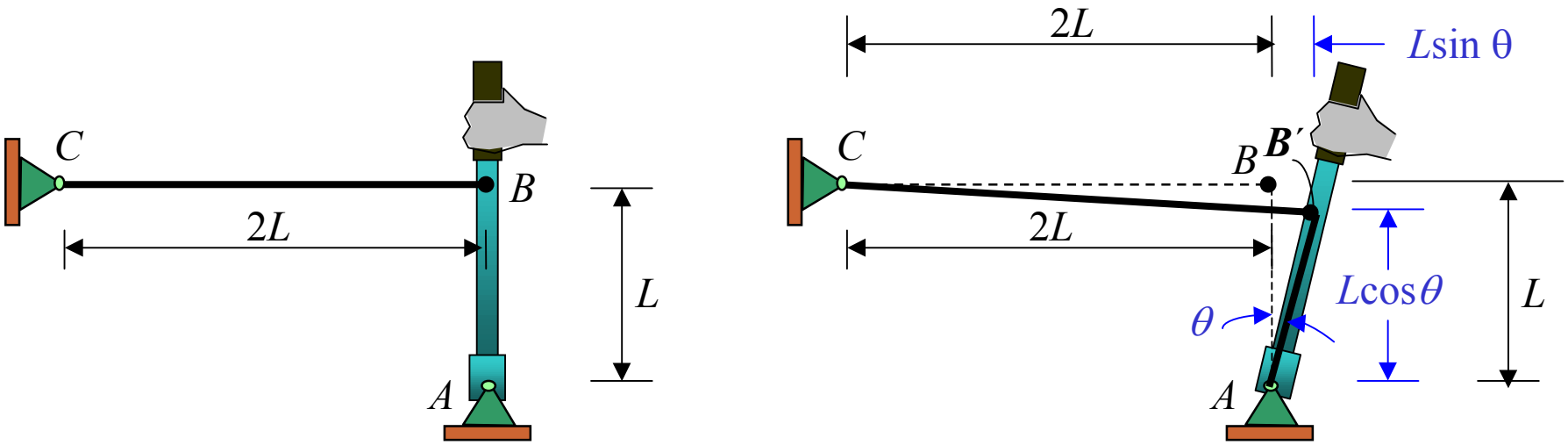
(b) the average normal strain in the rod.

$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{2.39\ \text{mm}}{200\ \text{mm}} = 0.0119\ \text{mm/mm}$$

Example 2

A force acting on the grip of the lever arm shown causes the arm to rotate clockwise through an angle of $\theta = 0.002$ rad. Determine the average normal strain developed in the wire *BC*.





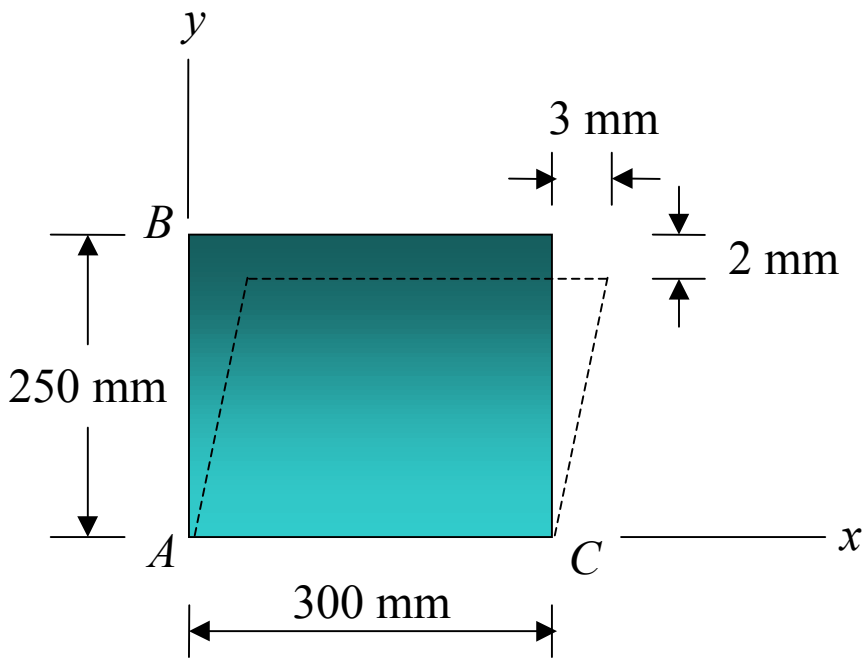
- **Solution**

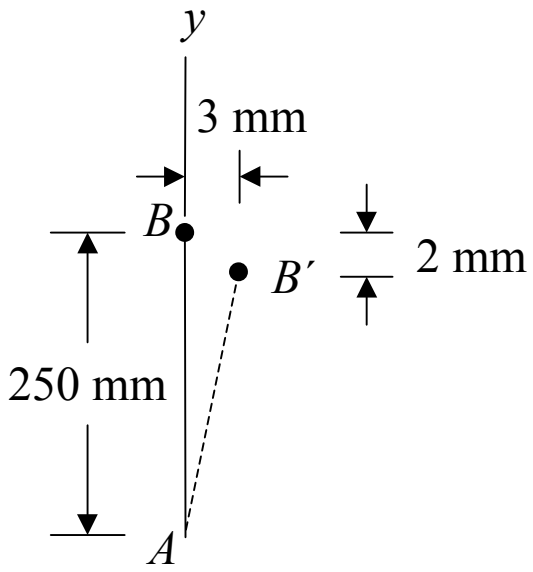
Since θ is small, the stretch in wire CB, $L\sin\theta \approx L\theta$, is using Eq.2-6. The average normal strain in the wire is then

$$\varepsilon_{avg} = \frac{CB'-CB}{CB} = \frac{L\theta}{2L} = \frac{\theta}{2} = \frac{0.002}{2} = 0.001$$

Example 3

The plate is deformed into the dashed shape shown. If in this deformed shape horizontal lines on the plate remain horizontal and do not change their length, determine (a) the average normal strain along the side *AB*, and (b) the average shear strain in the plate relative to the *x* and *y* axes.



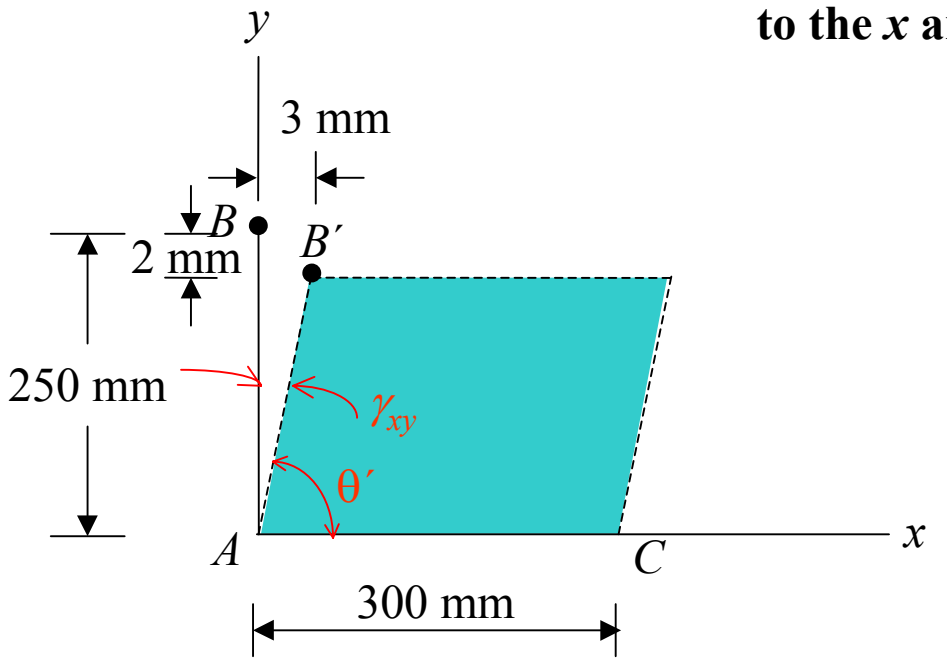


(a) the average normal strain along the side *AB*

$$AB' = \sqrt{(250-2)^2 + (3)^2} = 248.018 \text{ mm}$$

$$(\varepsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018 - 250}{250}$$

$$= -7.93(10^{-3}) \text{ mm/mm}$$



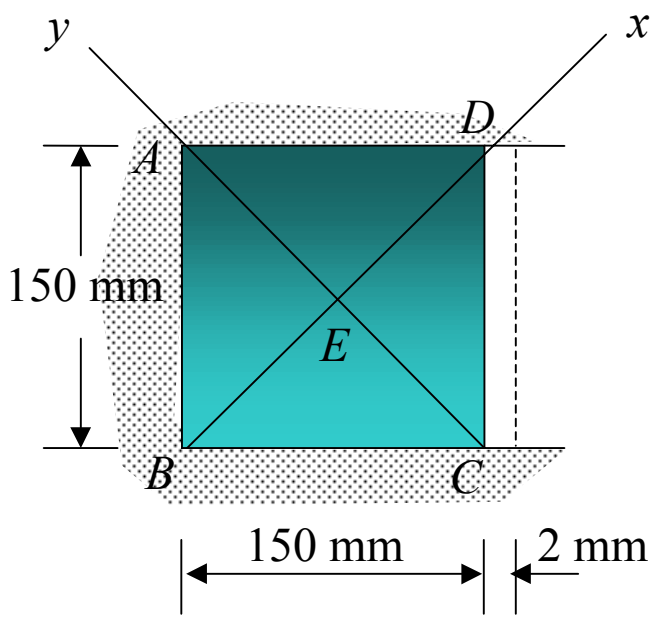
(b) the average shear strain in the plate relative to the *x* and *y* axes.

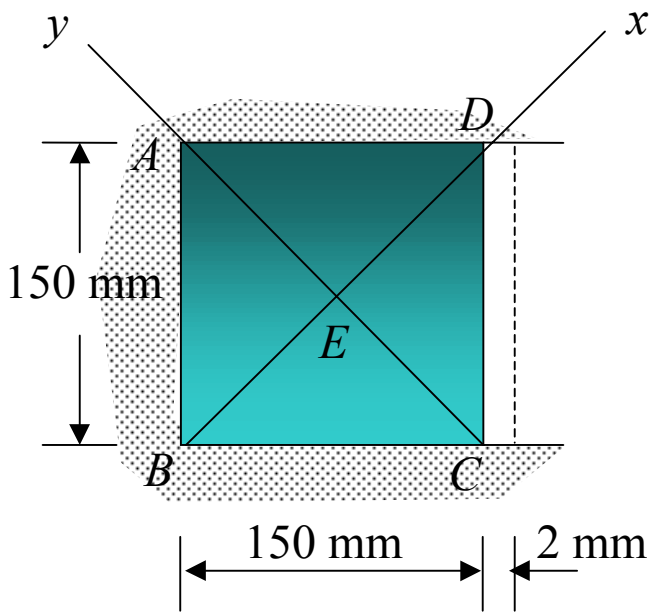
$$\gamma_{xy} = \frac{\pi}{2} - \theta'$$

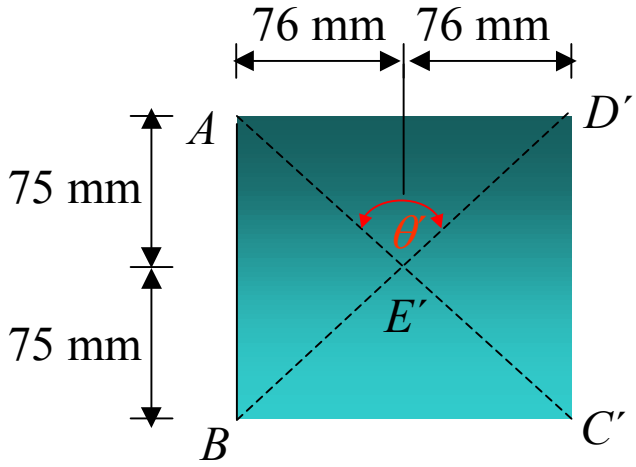
$$\gamma_{xy} = \tan^{-1}\left(\frac{3}{250-2}\right) = 0.0121 \text{ rad}$$

Example 4

The plate shown is held in the rigid horizontal guides at its top and bottom, *AD* and *BC*. If its right side *CD* is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal *AC*, and (b) the shear strain at *E* relative to the *x*, *y* axes.





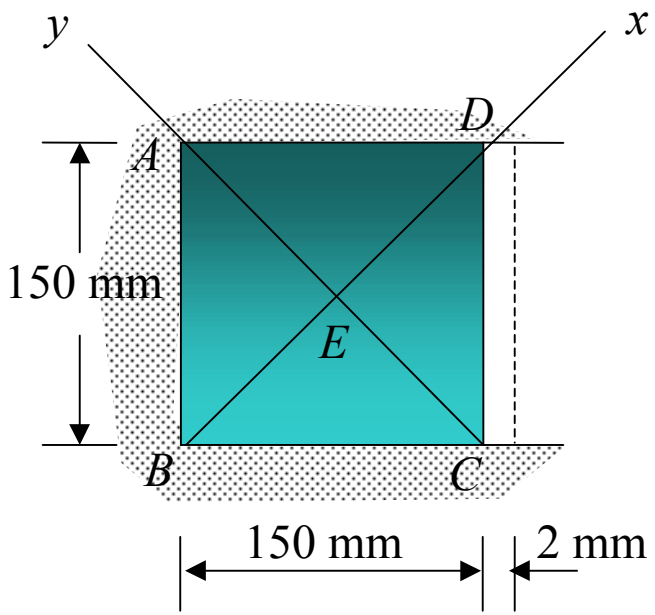


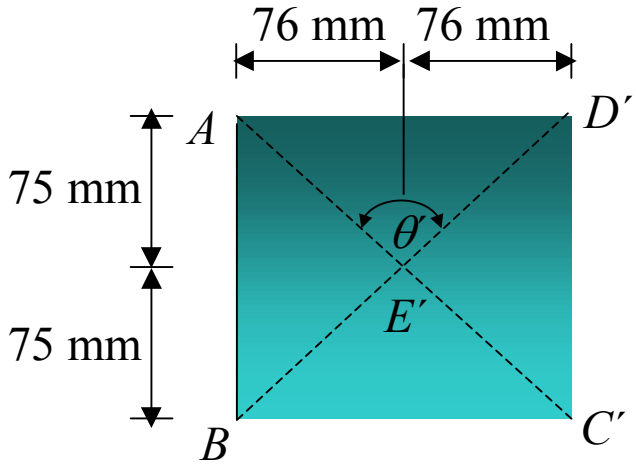
(a) the average normal strain along the diagonal *AC*

$$AC = \sqrt{(0.150)^2 + (0.150)^2} = 0.21213 \text{ m}$$

$$AC' = \sqrt{(0.150)^2 + (0.152)^2} = 0.21355 \text{ m}$$

$$(\varepsilon_{AC})_{avg} = \frac{AC' - AC}{AC} = \frac{0.21355 - 0.21213}{0.21213} = 0.00669 \text{ mm/mm}$$





(b) the shear strain at *E* relative to the *x*, *y* axes.

$$\tan\left(\frac{\theta'}{2}\right) = \frac{76}{75}$$

$$\theta' = 90.759^o = \frac{\pi}{180^o}(90.759^o) = 1.58404 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 1.58404 = -0.0132 \text{ rad}$$