DESIGN OF BEAMS AND SHAFTS

- Basis for Beam Design
- Stress Variations Throughout a Prismatic Beam
- Design of pristmatic beams
  - Steel beams
  - Wooden beams
- Design of Shaft
  - Combined bending
  - Torsion

Basis for Beam Design

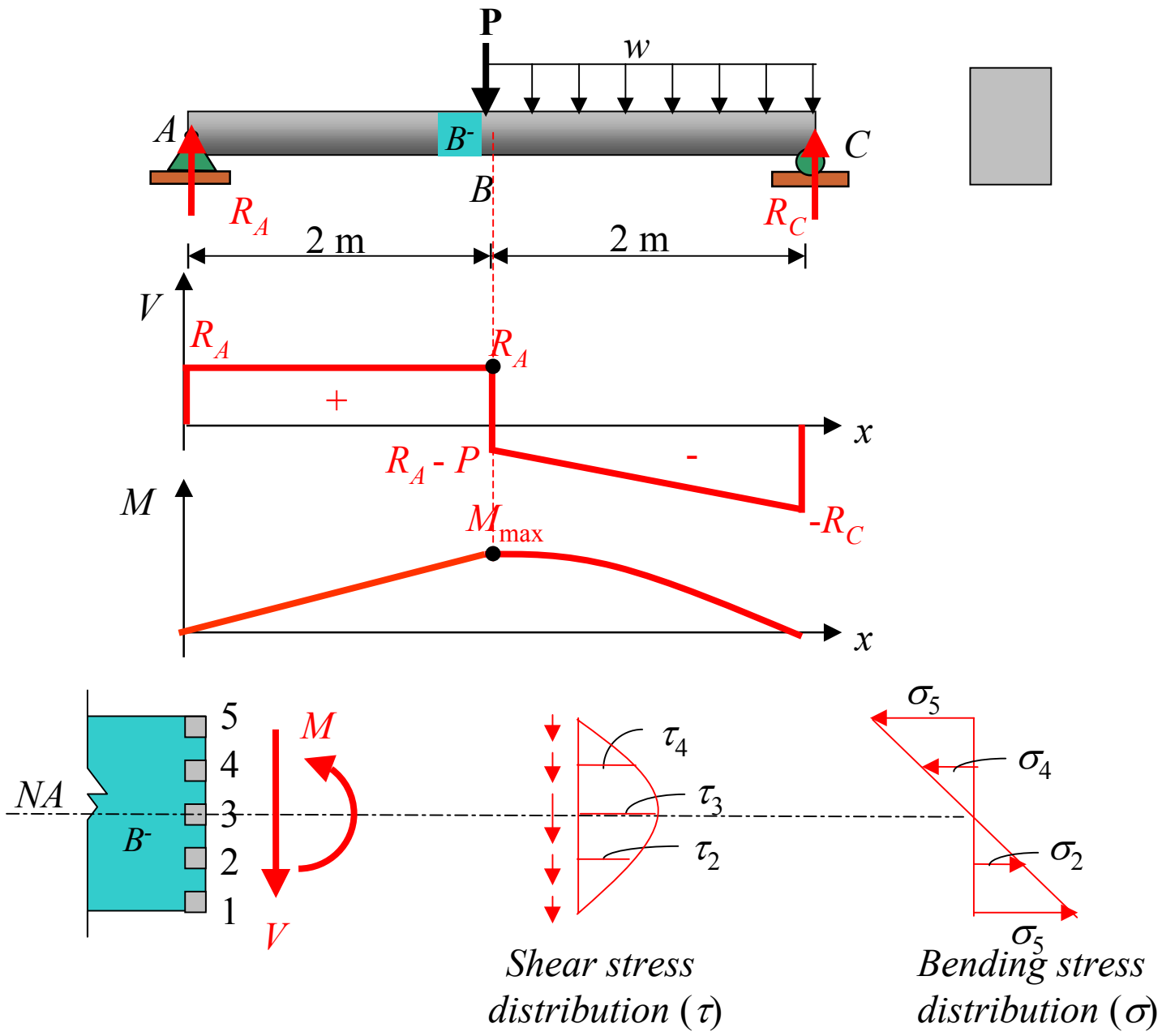
- The effects of an *internal axial force* are often neglected in design.

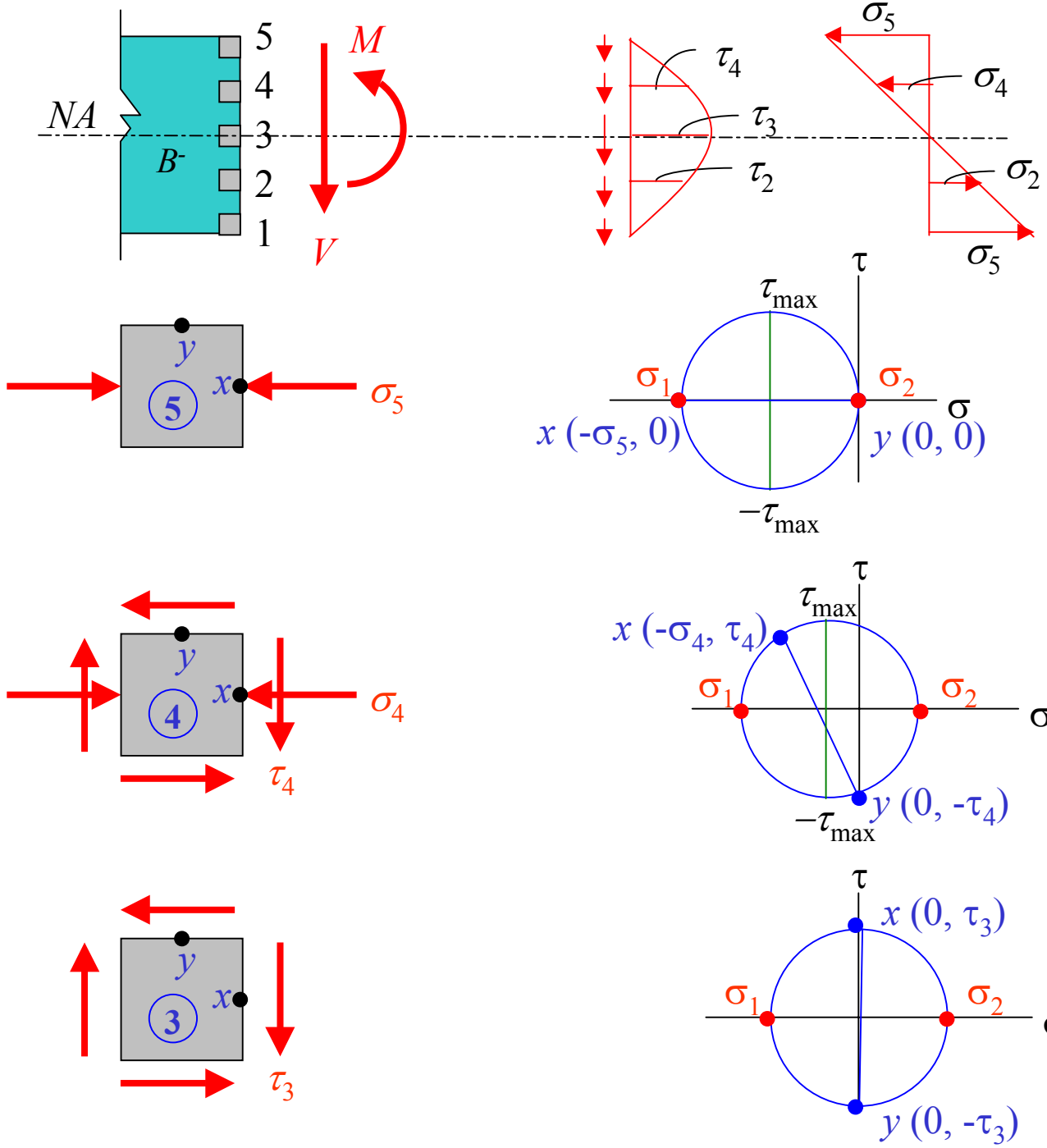
- Application of shear and flexure formulas are limited to beams made of a homogeneous material that has linear-elastic behavior.

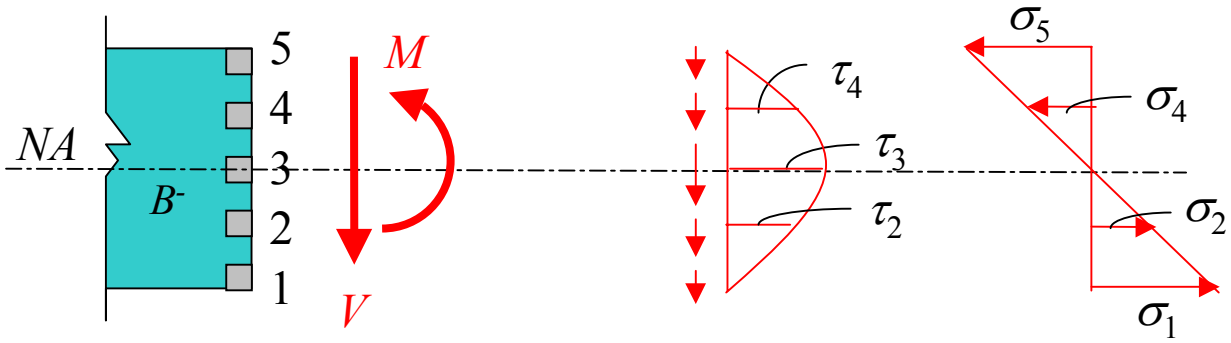
- The cross-sectional area must have an axis of symmetry in the plane of the loading.

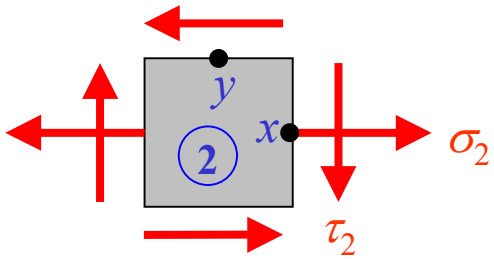
- The design does provide an adequate means of obtaining both a *safe and economical* design.

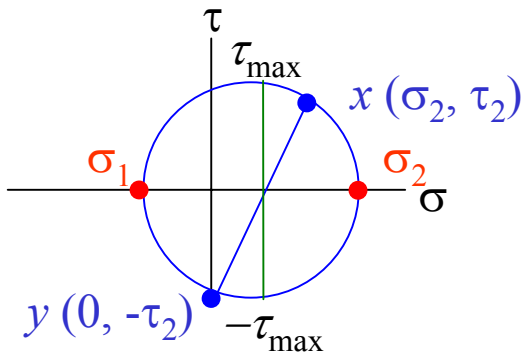
Stress Variations Throughout a Prismatic Beam

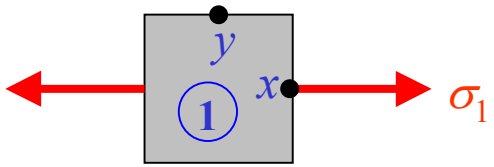


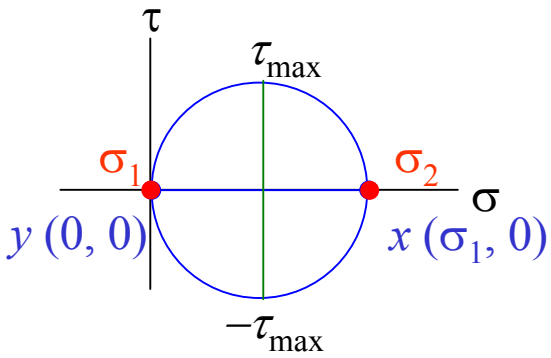

5
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NA
B-
M
V
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σ2
σ5
y
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τmax
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σ
x (-σ5, 0)
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−τmax
y
x
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τ4
τ
τmax
x (-σ4, τ4)
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σ
y (0, -τ4)
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y
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τ
x (0, τ3)
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σ
y (0, -τ3)


NA
B-
5
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x (σ2, τ2)
σ1
σ2
σ
y (0, -τ2)
−τmax


y
x
1
σ1


τ
τmax
σ1
σ2
σ
y (0, 0)
x (σ1, 0)
−τmax

Prismatic Beam Design

- **Section Modulus (*S*)**

$$\sigma = \frac{Mc}{I}$$

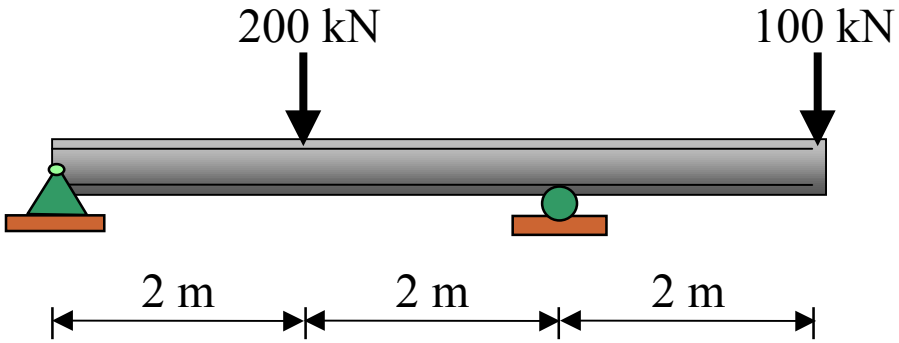
$$= \frac{M}{I/c}$$

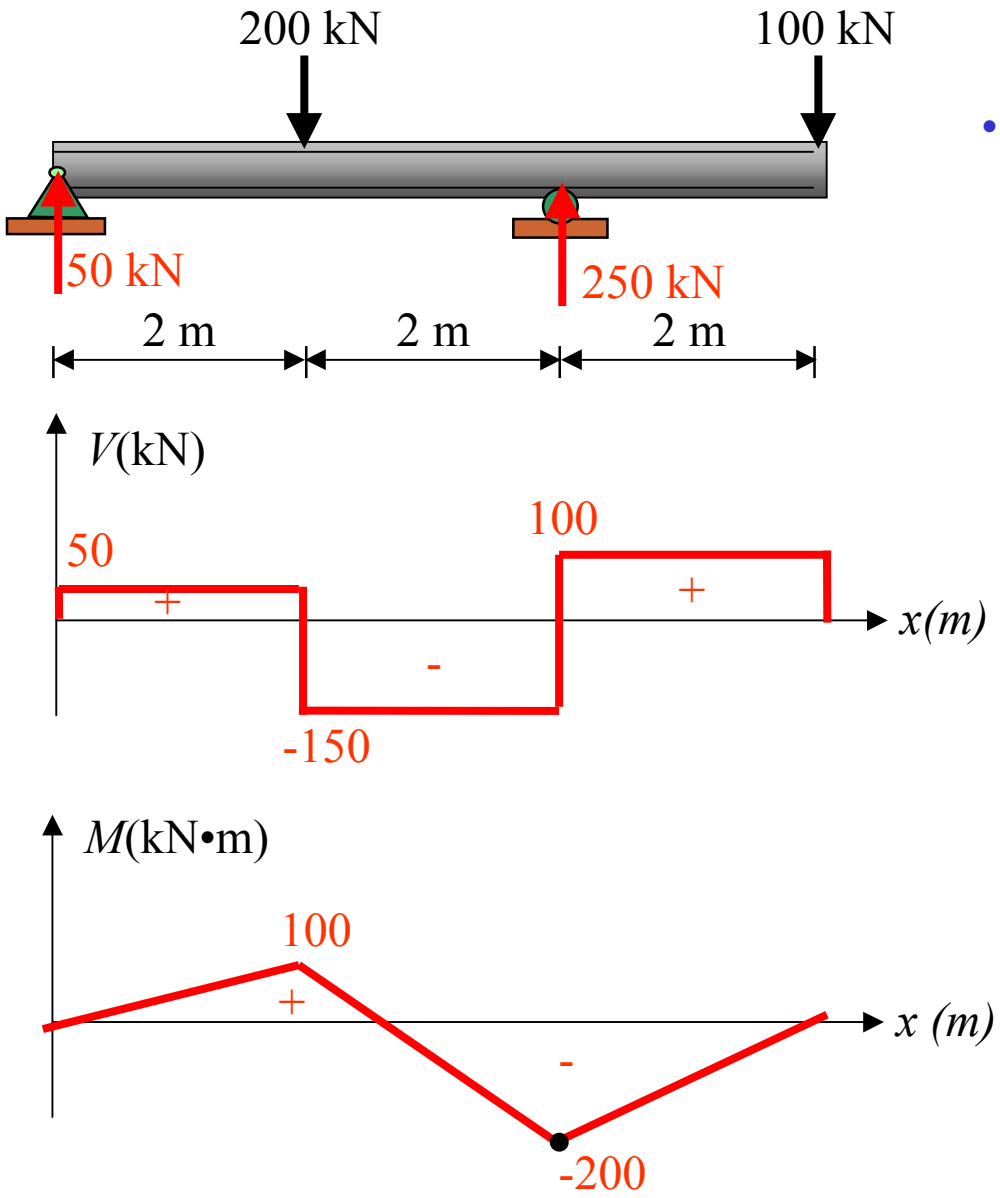
$$= \frac{M}{S}$$

$$S_{req'd} = \frac{M}{\sigma_{allow}}$$

Example 1

A beam is to be made of steel that has an allowable bending stress of σ_{allow} = 150 MPa and an allowable shear stress of τ_{allow} = 100 MPa. Select an appropriate *W* shape that will carry the loading shown.





- **Allowable normal stress:** $\sigma_{allow} = 150$ MPa

$$\sigma = \frac{M}{S}$$

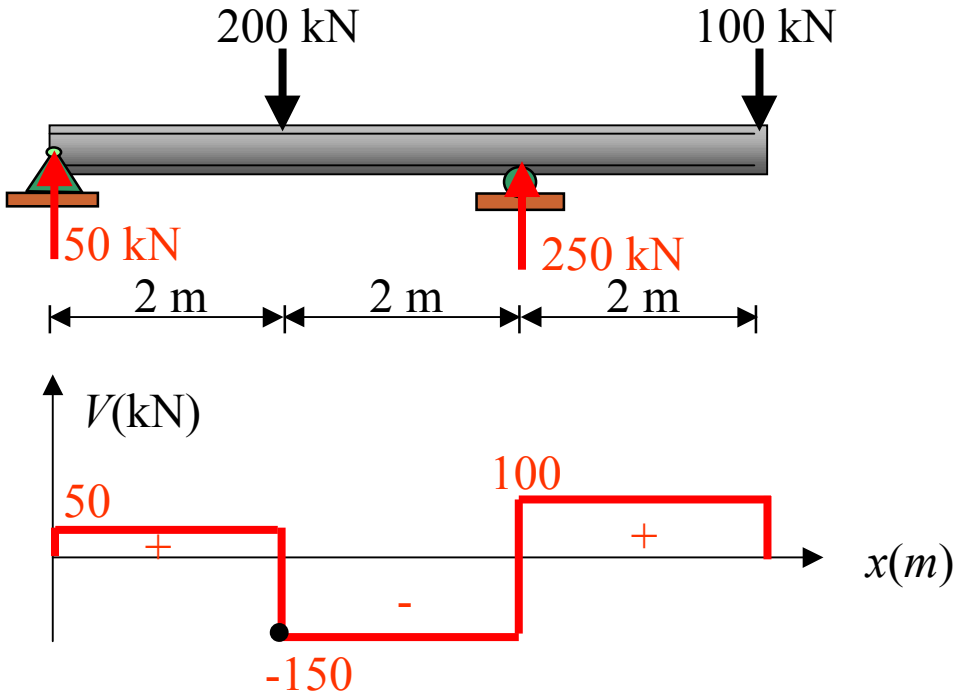
$$S_{req'd} = \frac{M}{\sigma_{allow}}$$

$$= \frac{(200\text{x}10^3)}{150\text{x}10^6} = 1333\text{x}10^3 \text{ mm}^3$$

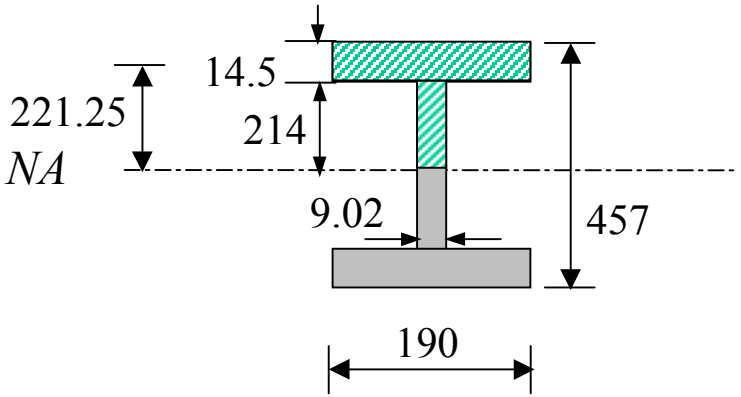
Using the table in **Appendix B**, the following beams are adequate:

W 610x82	$S = 1870\text{x}10^3$ mm^3
W 460x97	$S = 1910\text{x}10^3$ mm^3
W 460x89	$S = 1770\text{x}10^3$ mm^3
W 460x82	$S = 1610\text{x}10^3$ mm^3
W 460x74	$S = 1460\text{x}10^3$ mm^3
W 410x85	$S = 1510\text{x}10^3$ mm^3

Use *W* 460x74 because the beam having the least weight.



***W* 460x74** : $A = 9460$ mm^2, $d = 457$ mm,
$t_w = 9.02$ mm, $b_f = 190$ mm,
$t_f = 14.5$ mm, $I_{NA} = 333\text{x}10^6$ mm^4,



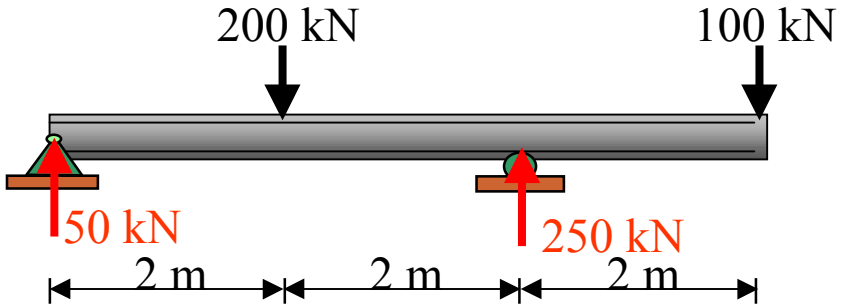
- **Check allowable shear stress:** $\tau_{allow} = 100$ MPa

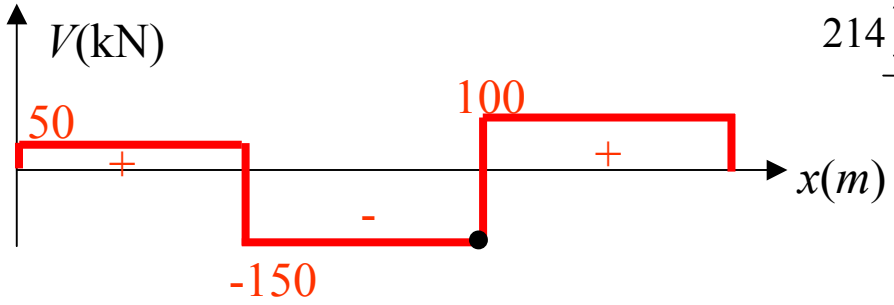
$$Q_{NA} = (221.25)(190\text{x}14.5) + (214/2)(214\text{x}9.02)$$

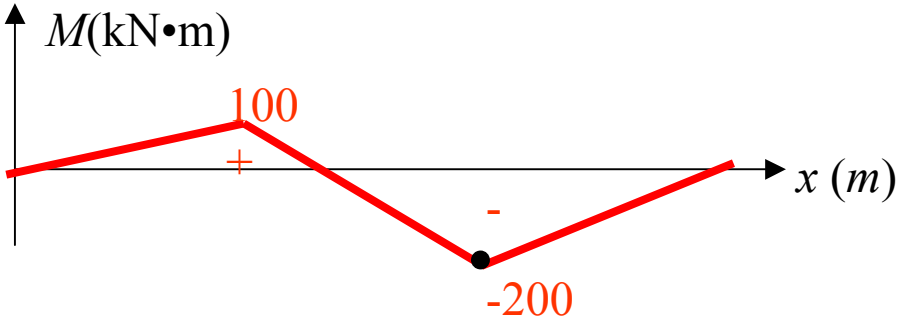
$$= 816.08\text{x}10^{-6}\ \text{m}^3$$

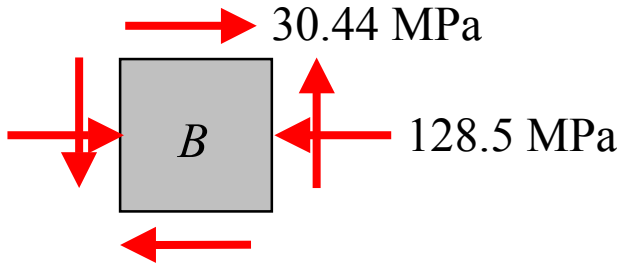
$$\tau_{max} = \frac{VQ}{It} = \frac{V(816.08\text{x}10^{-6})}{(333\text{x}10^{-6})(0.00902)} = 271.7V = (271.7)(150\text{x}10^3)$$

$$= 32.6\ \text{MPa} < 100\ \text{MPa},\ O.K.$$

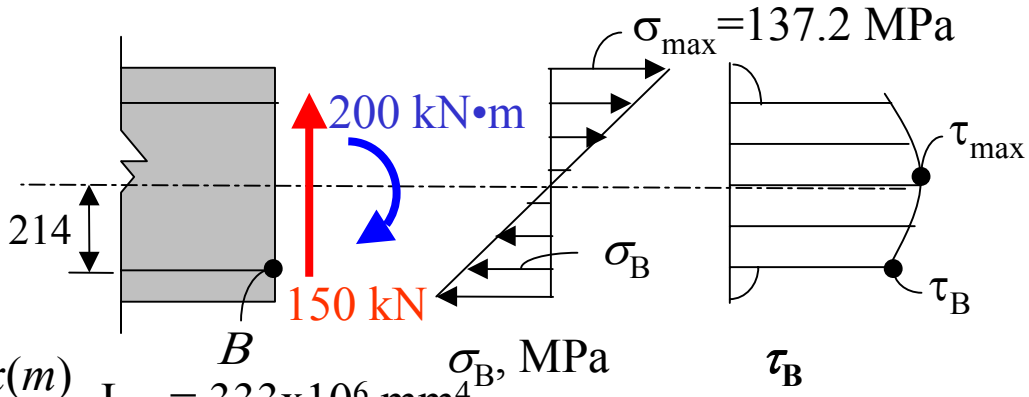








- **Check shear and normal stress at B**



$I_{NA} = 333\text{x}10^6\ \text{mm}^4$

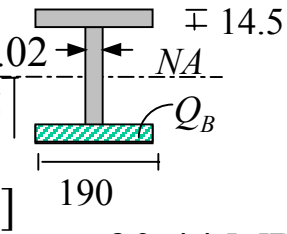
- Bending: σ_B

$$\sigma_B = \frac{My}{I} = \frac{(-200\text{x}10^3)(0.214)}{(333\text{x}10^{-6})}$$

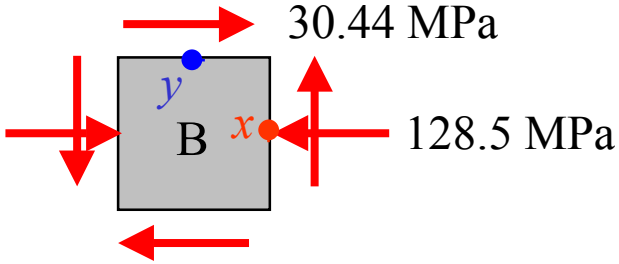
$$= -128.5\ \text{MPa}$$

- Shear: τ_B

$$\tau_B = \frac{V_{\max} Q_B}{It}$$



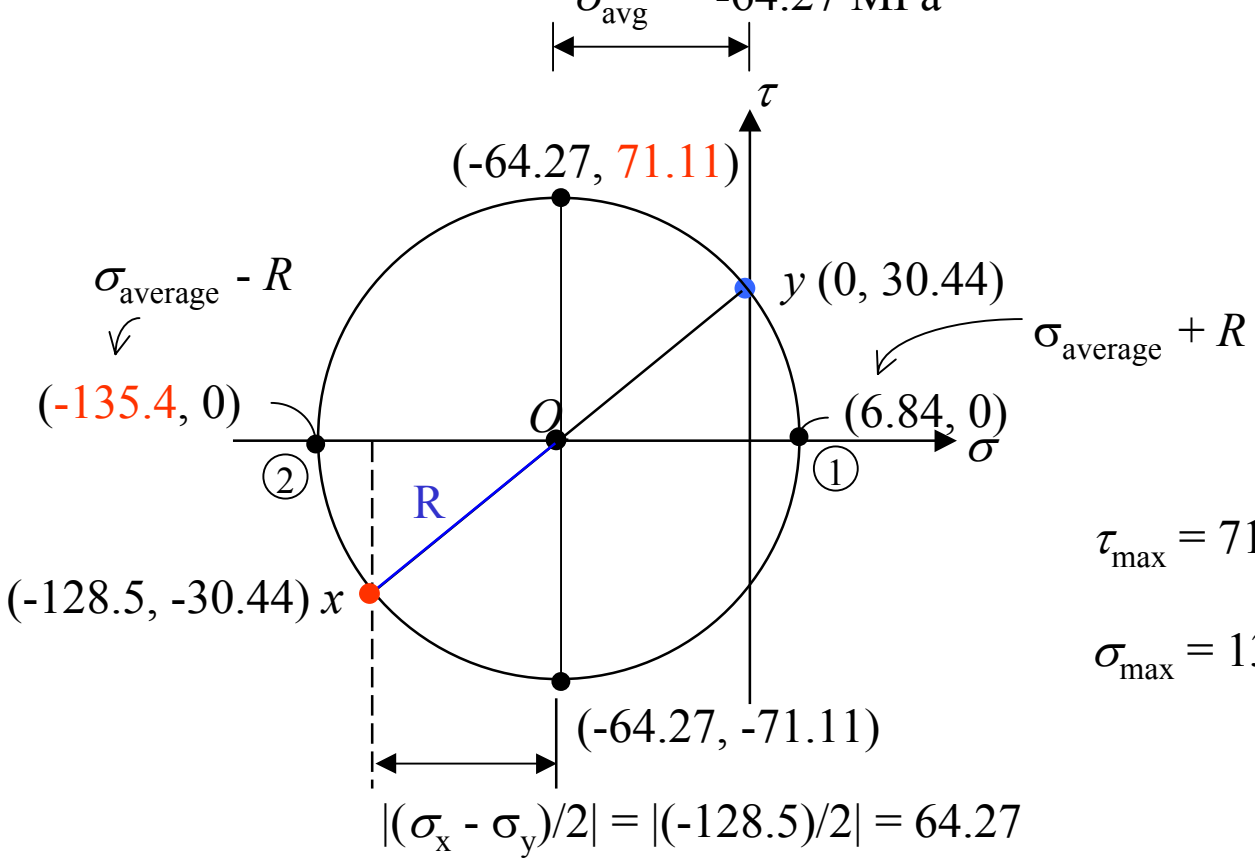
$$= \frac{(150\text{x}10^3)[(0.2213)(0.19\text{x}0.0145)]}{(333\text{x}10^{-6})(0.00902)} = 30.44\ \text{MPa}$$



$$\sigma_{average} = \frac{\sigma_x + \sigma_y}{2} = \frac{-128.53+0}{2} = -64.27 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

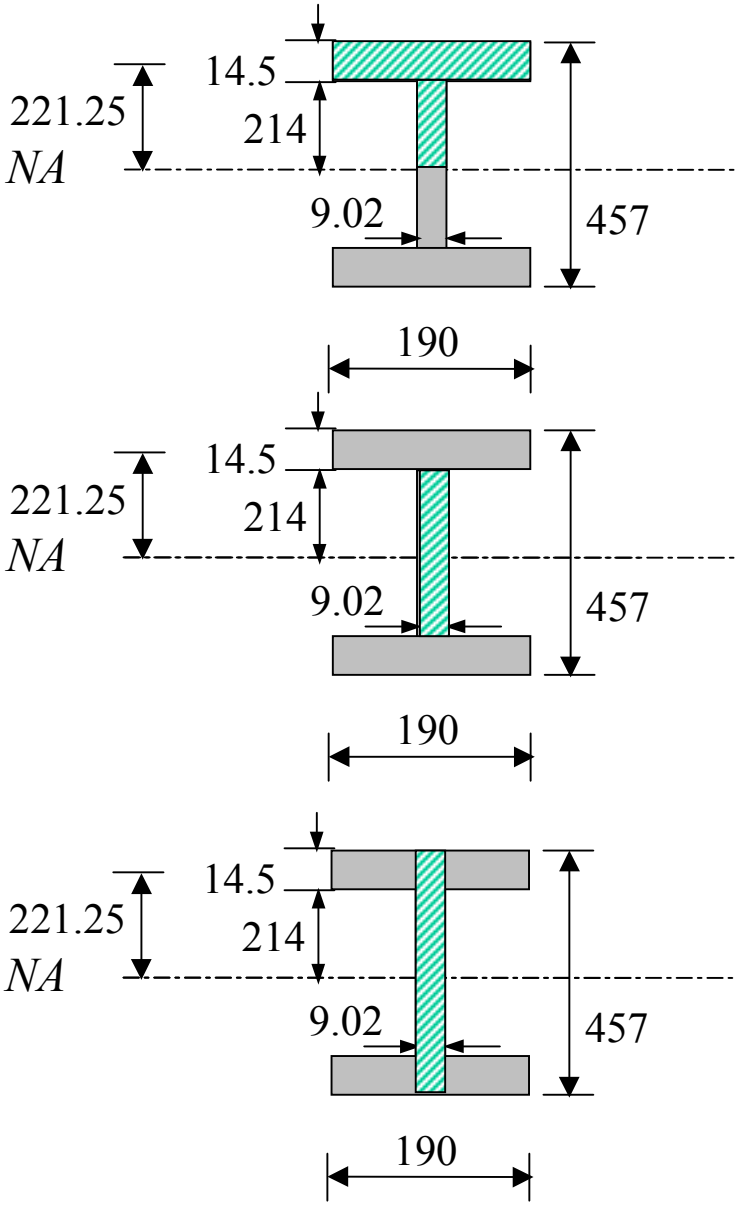
$$= \sqrt{(64.27)^2 + (30.44)^2} = \pm 71.11 \text{ MPa}$$



$\tau_{max} = 71.11$ MPa $<$ 100 MPa, *O.K.*

$\sigma_{max} = 135.4$ MPa $<$ 150 MPa, *O.K.*

Comments



$$Q = \bar{y}A$$

$$Q_{NA} = (221.25)(190x14.5) + (214/2)(214x9.02)$$

$$= 816083.71 \text{ mm}^3 = 816.08x10^{-6} \text{ m}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{V(816.08x10^{-6})}{(333x10^{-6})(0.00902)} = 271.7V$$

$$\tau_{avg} = \frac{V_{max}}{A_{web}} = \frac{V}{(0.457 - 2x0.0145)(0.00902)}$$

$$= 259.03V \longrightarrow 4.7\ \%\ \text{off}$$

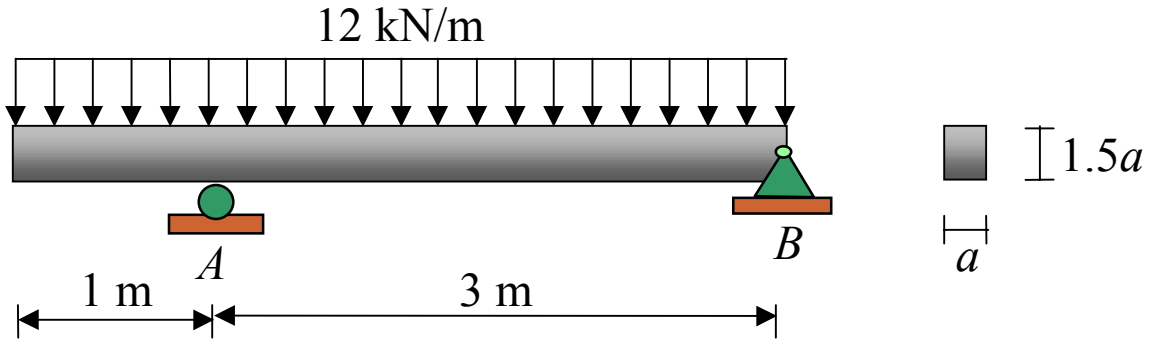
$$\tau_{avg} = \frac{V_{max}}{A_{web}} = \frac{V}{(0.457)(0.00902)}$$

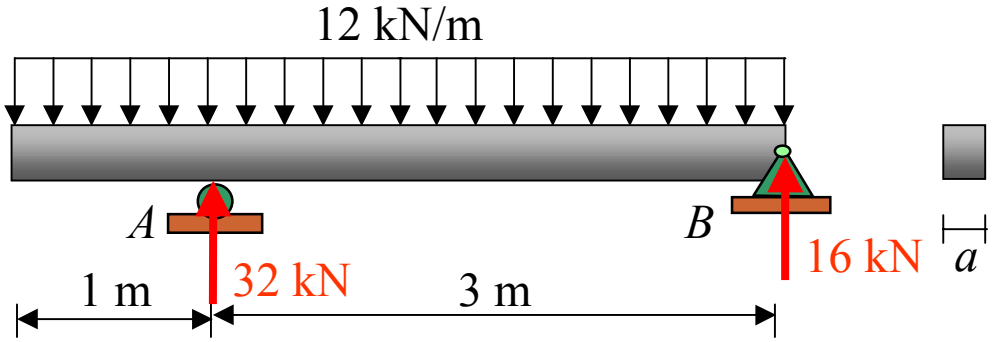
$$= 242.59V \longrightarrow 10.7\ \%\ \text{off}$$

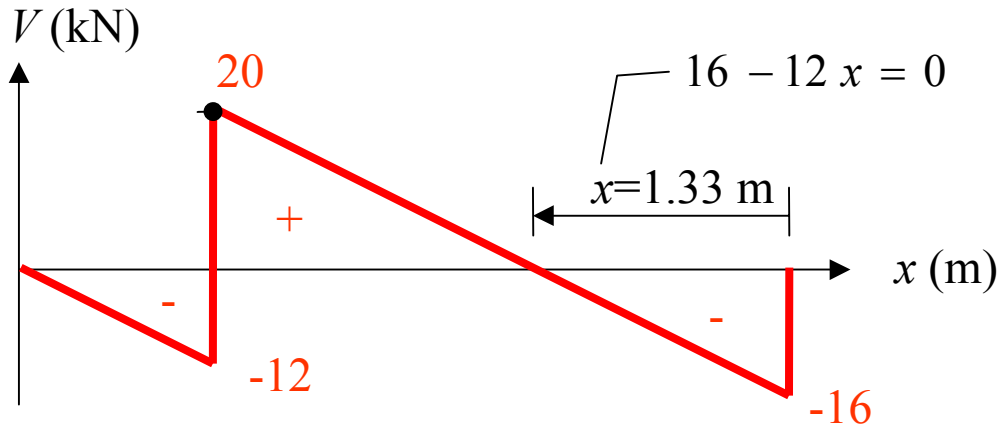
Note: dimension in mm $I_{NA} = 333x10^6 \text{ mm}^4$,

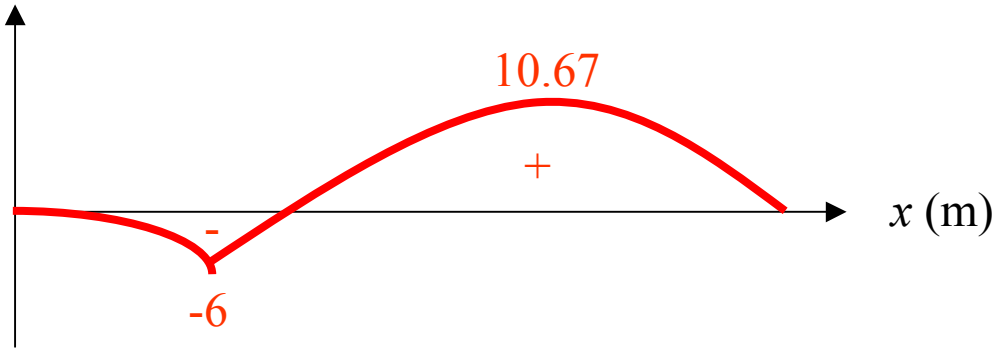
Example 2

The laminated wooden beam shown supports a uniform distributed loading of 12 kN/m. If the beam is to have a height-to-width ratio of 1.5, determine its smallest width. The allowable bending stress is σ_{allow} = 9 MPa and the allowable shear stress is τ_{allow} =0.6 MPa. Neglect the weight of the beam.









- **Check allowable shear stress**

$$\tau_{allow} = 0.6 \text{ MPa}$$

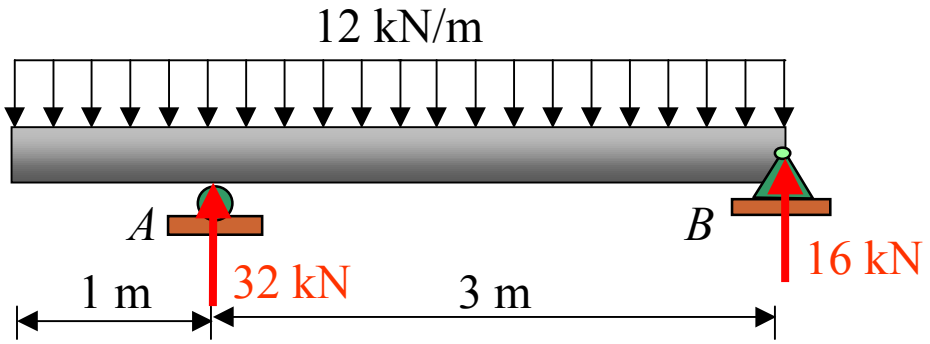
$$\tau = \frac{VQ}{It}$$

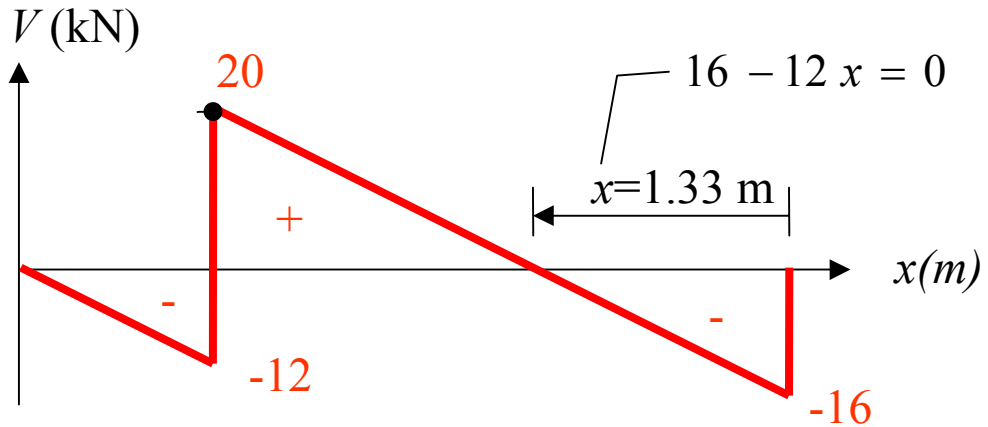
$$\tau = 1.5 \frac{V}{A}$$

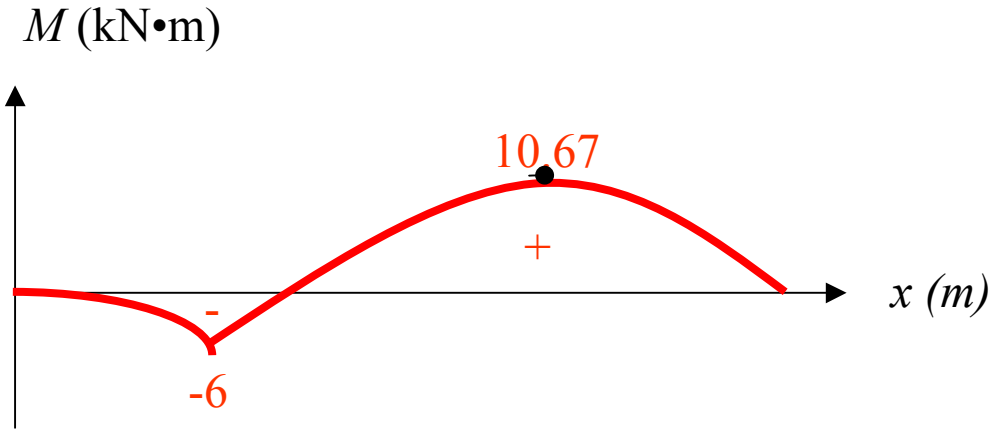
$$0.6 \times 10^6 = 1.5 \frac{20 \times 10^3}{1.5(a^2)}$$

$$a = 0.183 \text{ m} = 183 \text{ mm}$$

Use a = 185 mm









$1.5a = 277.5$

$a = 185$ mm

- **Check allowable normal stress**

$$\sigma_{allow} = 9 \text{ MPa}$$

$$\sigma_{max} = \frac{Mc}{I}$$

$$= \frac{(10.67 \times 10^3)(278/2)}{(1/12)(185)(278^3)}$$

$$= 4.48 \text{ MPa}$$

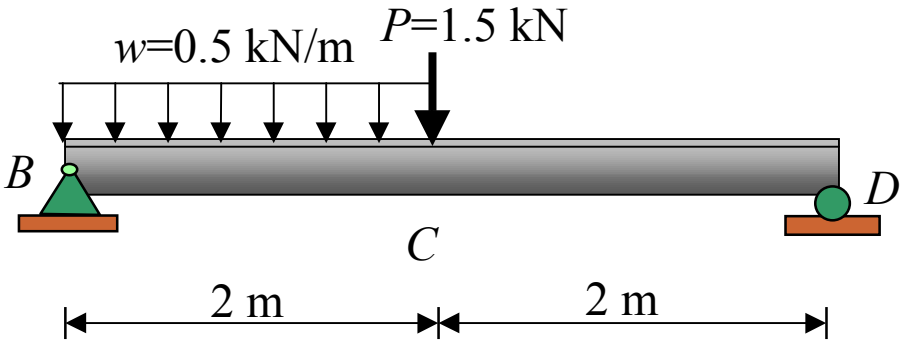
4.48 MPa < 9 MPa , *O.K.*

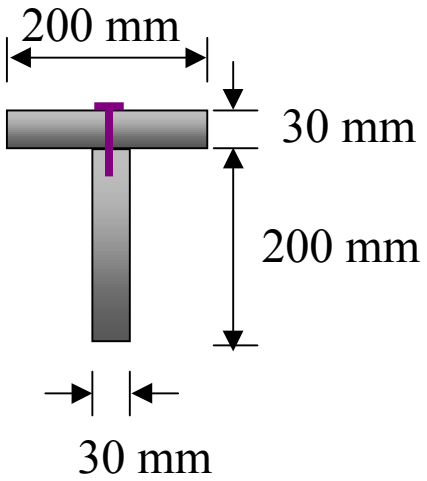
Use $a = 185$ mm

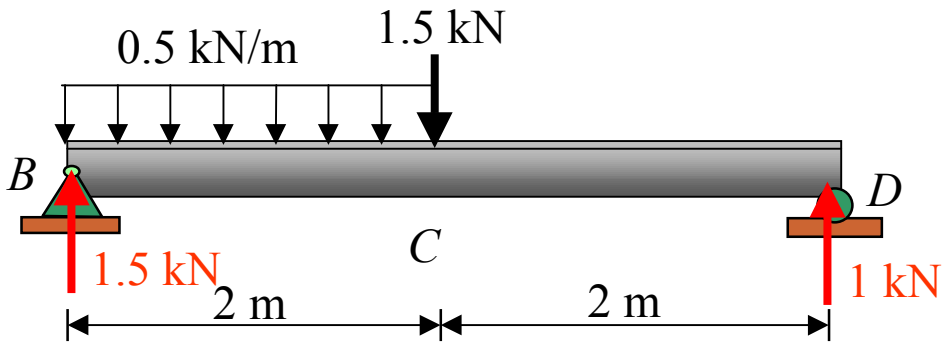
Example 3

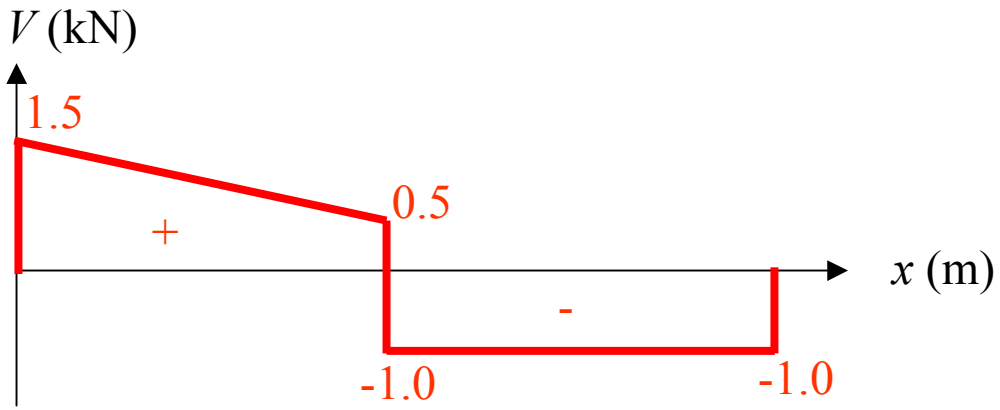
The wooden T-beam shown is made from two 200 mm x 30 mm boards. If the allowable bending stress is σ_{allow} =12 MPa and the allowable shear stress is τ_{allow} = 0.8 MPa, determine if the beam can safely support the loading shown without having the deflection more than L/240. The spacing of nails specified to hold the two boards together if each nail can resist 1.50 kN in shear are: between *BC* use 125 mm, and *CD* use 250 mm. Take *E* = 12 GPa, and the formula for deflection at the the mid-span:

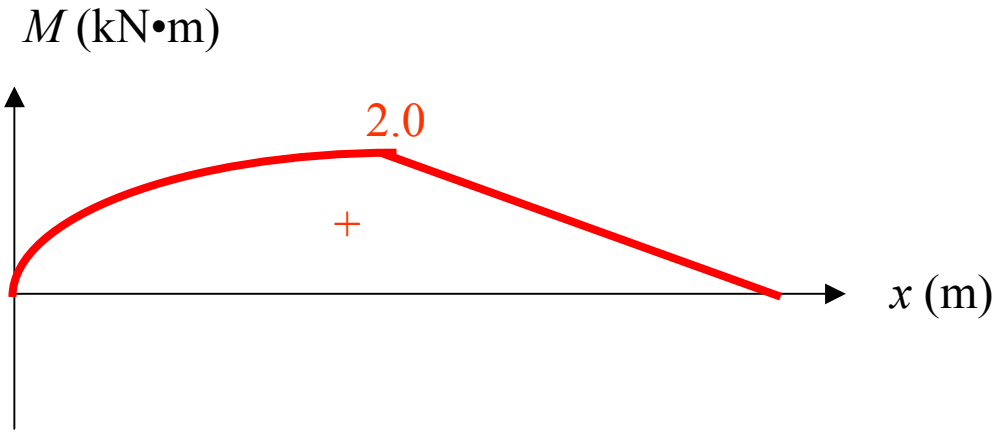
$$\upsilon_C = \frac{-5wL^4}{768EI} + \left(\frac{-PL^3}{48EI}\right)$$



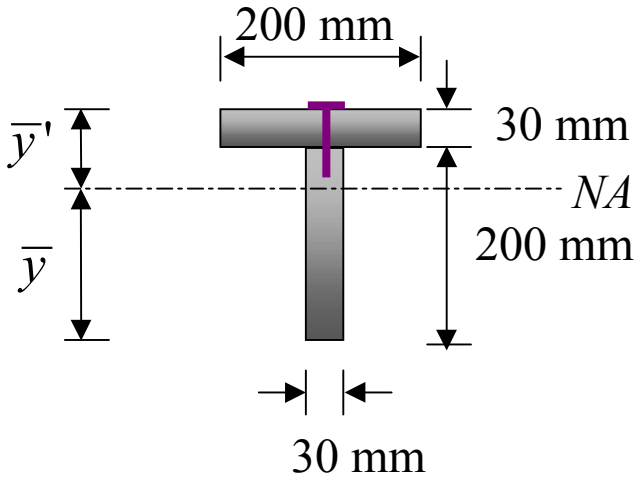



0.5 kN/m
1.5 kN
B
D
C
1.5 kN
1 kN
2 m
2 m


V (kN)
1.5
0.5
+
x (m)
-
-1.0
-1.0


M (kN•m)
2.0
+
x (m)

- **Section property**



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

$$= \frac{(100)(200 \times 30) + (215)(200 \times 30)}{2(200 \times 30)}$$

$$= 157.5 \text{ mm}$$

$$\bar{y}' = 230 - 157.5 = 72.5 \text{ mm}$$

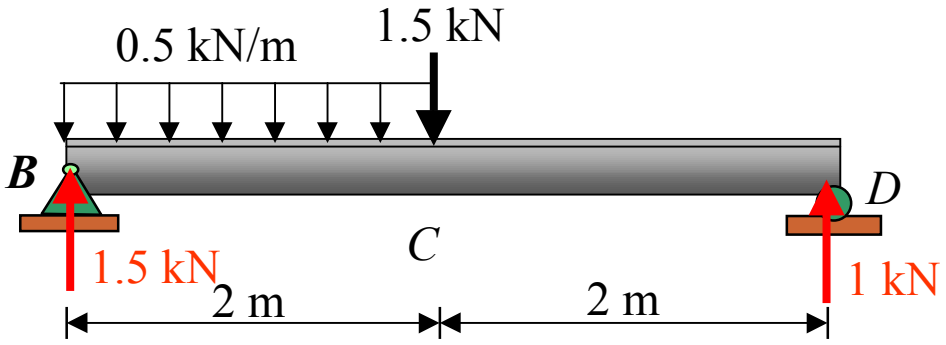
$$I = \Sigma(\bar{I} + Ad^2)$$

$$= (\bar{I} + Ad^2)_{\text{web}} + (\bar{I} + Ad^2)_{\text{flang}}$$

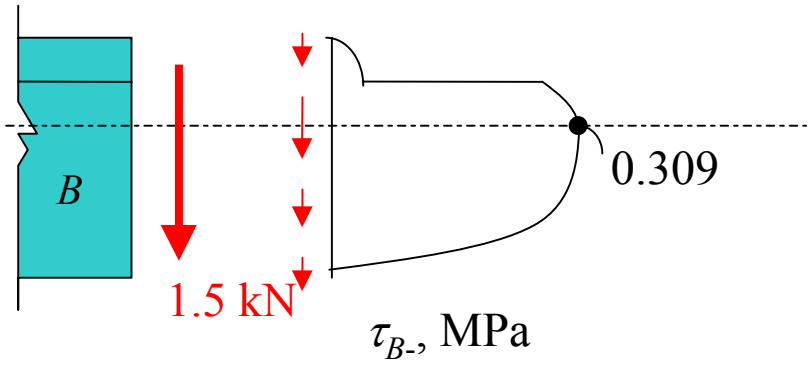
$$= [(1/12)(30)(200^3) + (30\text{x}200)(157.5\text{-}100)^2]$$

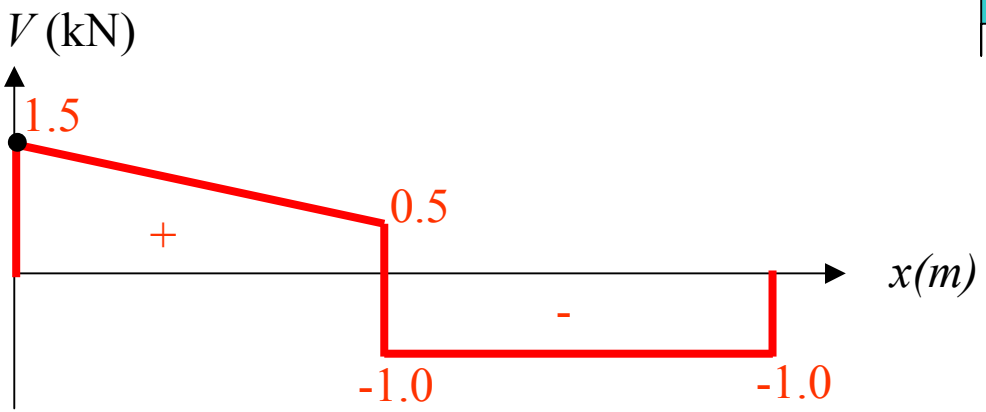
$$+ [(1/12)(200)(30^3) + (200\text{x}30)(215\text{-}157.5)^2]$$

$$= 60.125\text{x}10^6 \text{ mm}^4 = 60.12\text{x}10^{-6} \text{ m}^4$$



- Stress distribution at point *C*



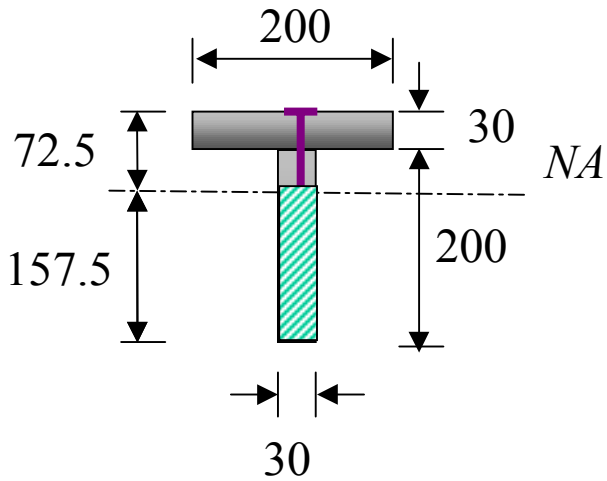


- **Check allowable shear stress:** $\tau_{allow} = 0.8$ MPa

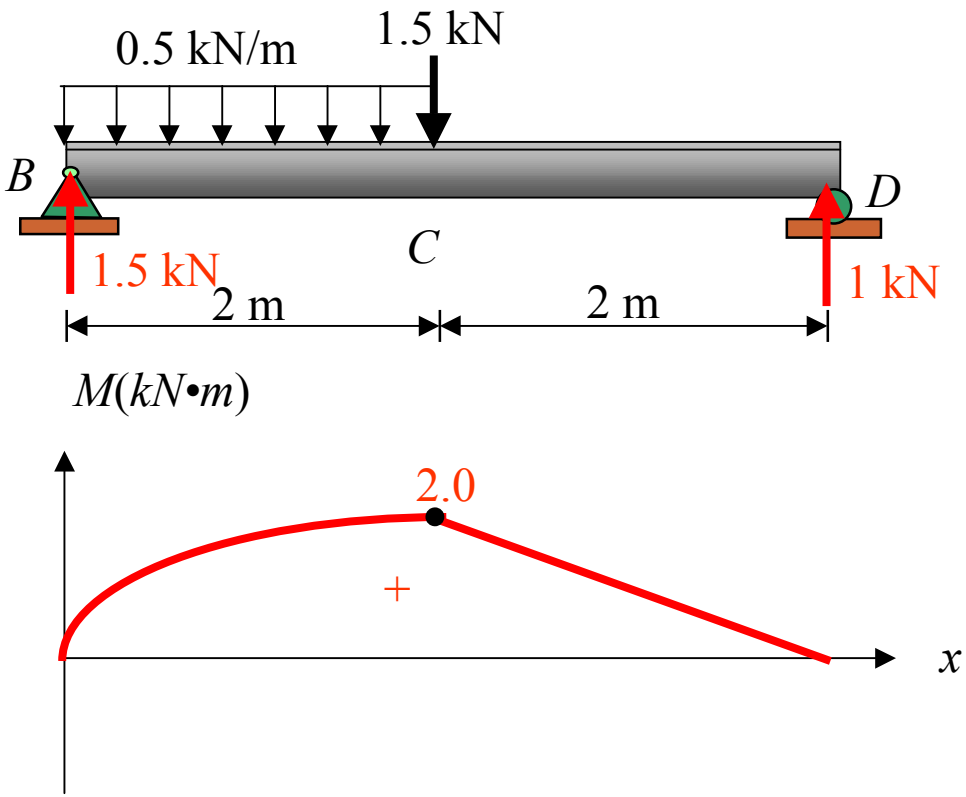
$$\tau_{max} = \frac{V_{max}Q}{It} = \frac{(1.5\text{x}10^{3})[(0.07875)(0.03\text{x}0.1575)]}{(60.12\text{x}10^{-6})(0.03)}$$

$$= 0.309 \text{ MPa}$$

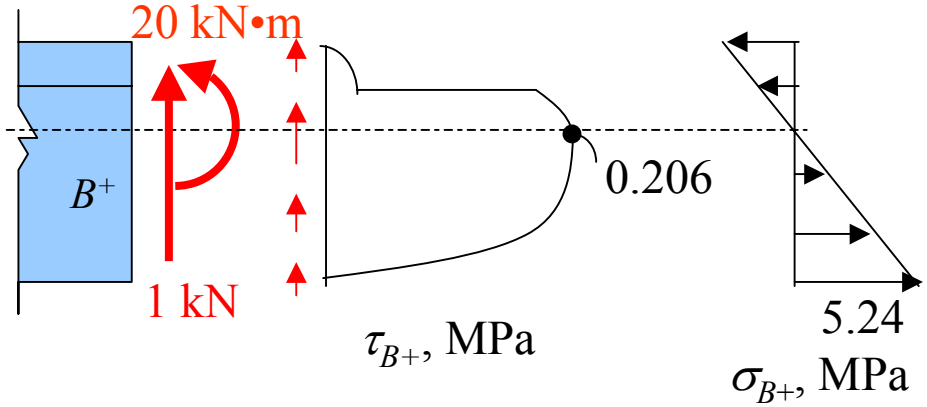
0.309 MPa < 0.8 MPa , O.K.



$I = 60.12\text{x}10^{-6}$ m^4



- Stress distribution at point C^+

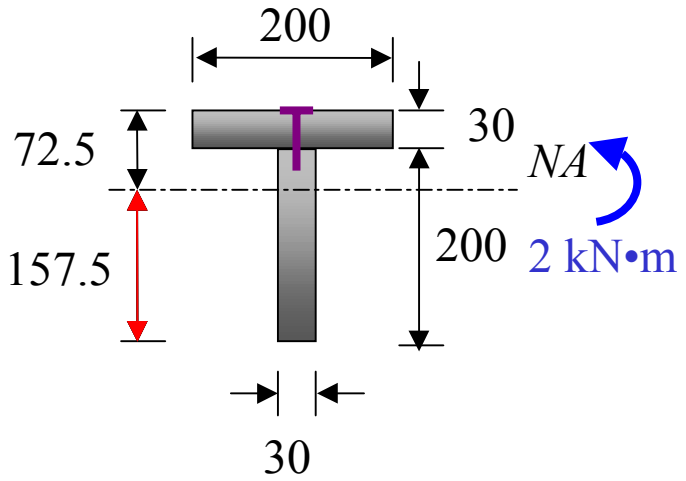


- **Check allowable normal stress:** $\sigma_{allow} = 12$ MPa

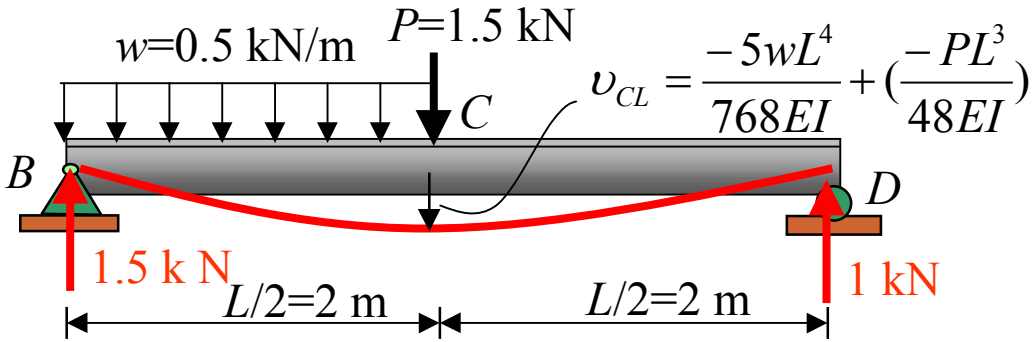
$$\sigma_{max} = \frac{Mc}{I} = \frac{(2\text{x}10^3)(0.1575)}{(60.12\text{x}10^{-6})}$$

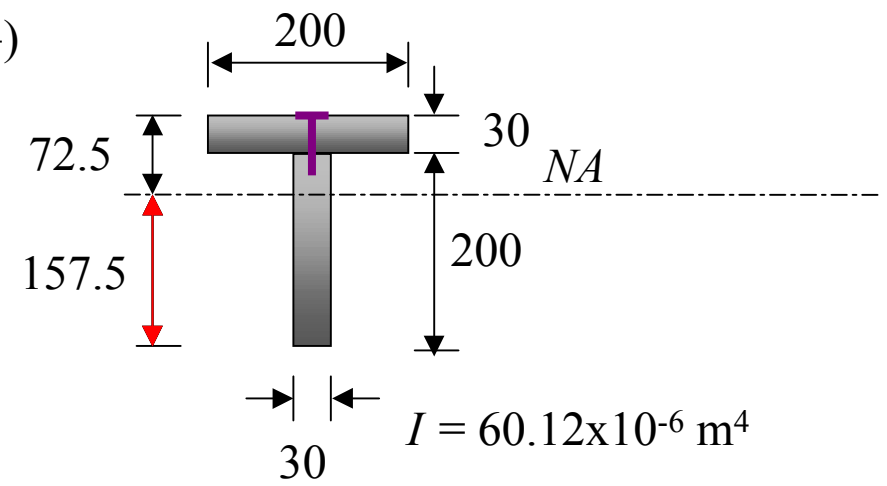
$$= 5.24 \text{ MPa}$$

$5.24 \text{ MPa} < 12 \text{ MPa}$, O.K.



$I = 60.12\text{x}10^{-6}$ m^4

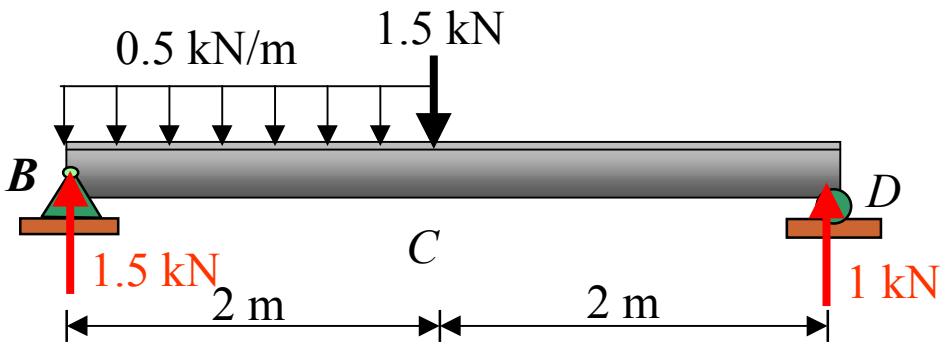


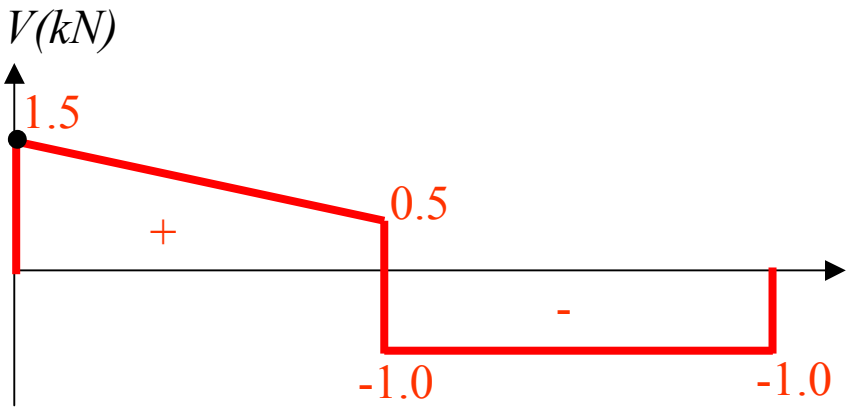


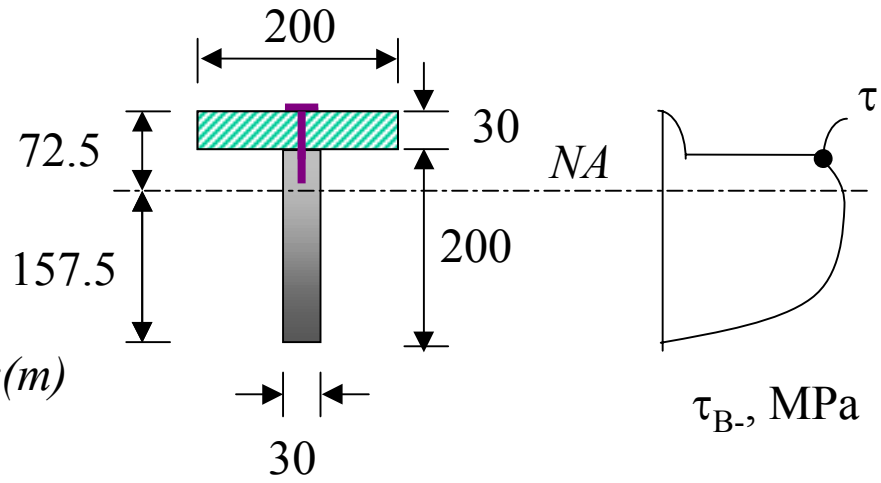
- **Check maximum deflection:** $\upsilon_{allow} = L/240$

$$\upsilon_{CL} = (\frac{-5wL^4}{768EI}) + (\frac{-PL^3}{48EI}) = -\frac{5(.5)4^4}{768(12\times10^9)(60.12\times10^{-6})} - \frac{(1.5)4^3}{48(12\times10^9)(60.12\times10^{-6})})$$

$$= -0.00115 - 0.00277 = -0.00392\ m, (\downarrow) < \frac{4}{240} = 0.0167\ m, O.K.$$







$I = 60.125\text{x}10^{-6}\ \text{m}^4$

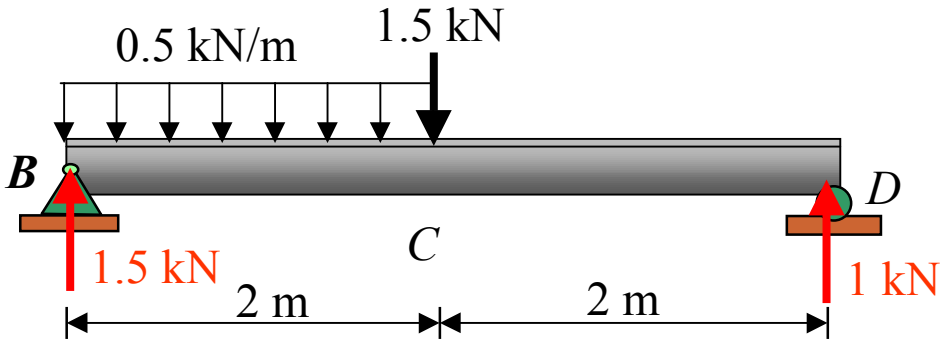
- **Nail spacing:** $s_{BC} = 125$ mm, and $s_{CD} = 250$ mm; $(F_s)_{allow} = 1.5$ kN

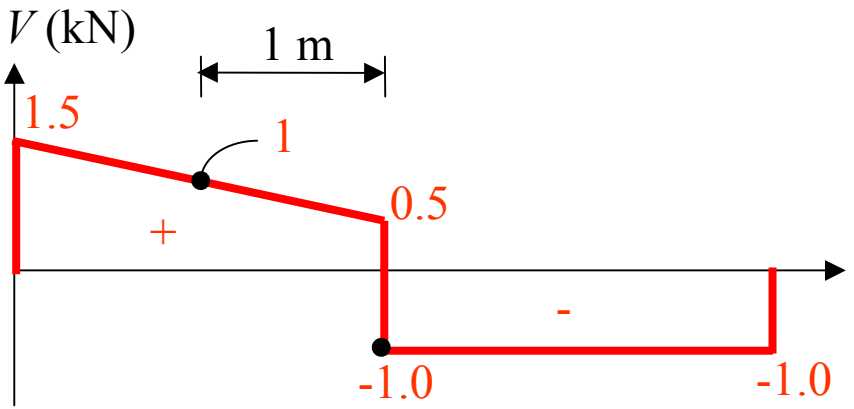
- Segment BC

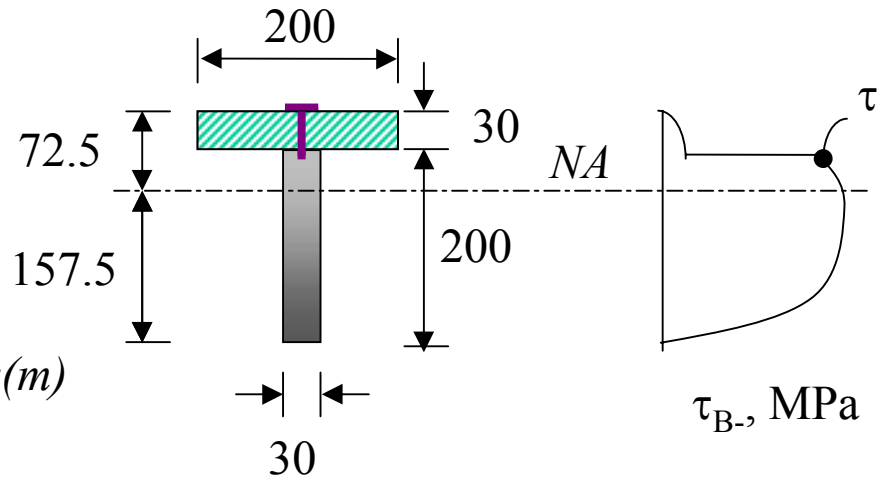
$$F_{nail} = \tau A_{shear} = \left(\frac{VQ}{It}\right)(s_{BC}\text{x}0.03)$$

$$F_{nail} = \frac{(1.5\text{x}10^3)[(0.0575)(0.2\text{x}0.03)]}{(60.12\text{x}10^{-6})(0.03)}(0.125\text{x}0.03)$$

$$= 1.076\ \text{kN} < 1.5\ \text{kN},\ O.K.$$







$I = 60.125\text{x}10^{-6}\ \text{m}^4$

- **Nail spacing:** $s_{BC} = 125$ mm, and $s_{CD} = 250$ mm; $(F_s)_{allow} = 1.5$ kN

- Segment CD

$$F_{nail} = \tau A_{shear} = \left(\frac{VQ}{It}\right)(s_{CD}\text{x}0.03)$$

$$F_{nail} = \frac{(1\text{x}10^3)(0.0575)(0.2\text{x}0.03)}{(60.12\text{x}10^{-6})(0.03)}(0.25\text{x}0.03)$$

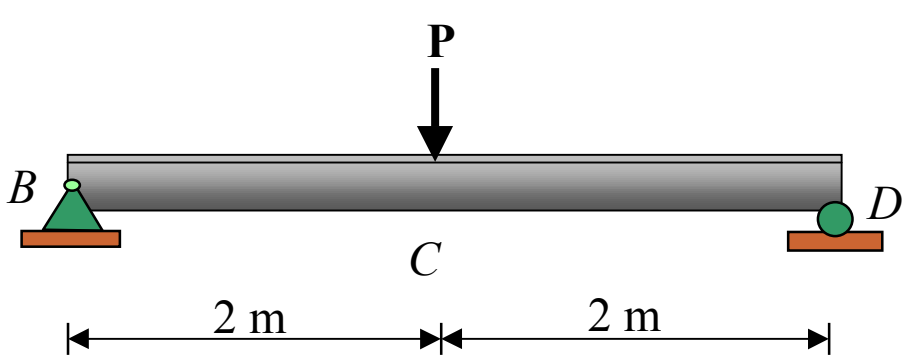
$$= 1.434\ \text{kN} < 1.5\ \text{kN},\ O.K.$$

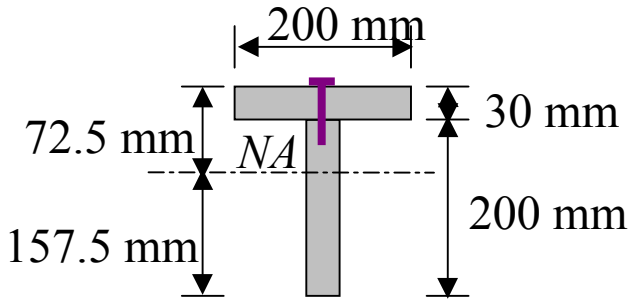
Example 4

From the beam shown, determine the largest load *P* that can be applied to the beam.
Requirements specified:

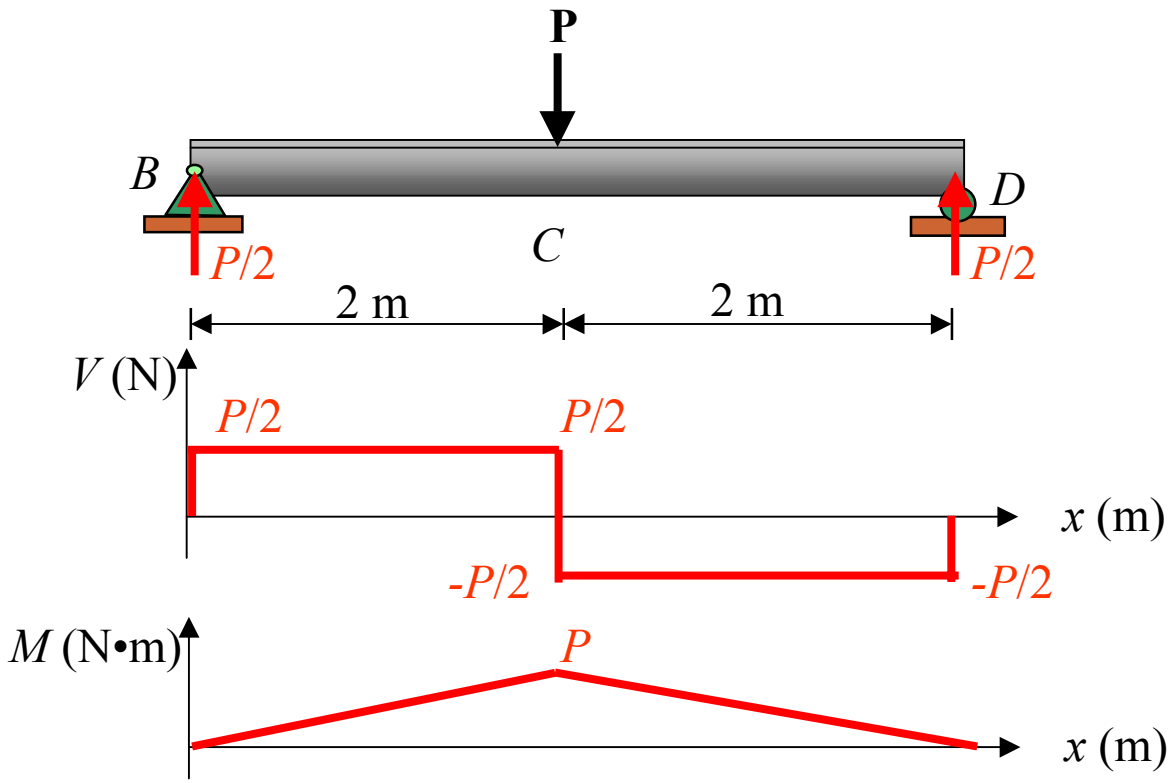
- σ_{allow} = 12 MPa
  τ_{allow} = 0.8 MPa

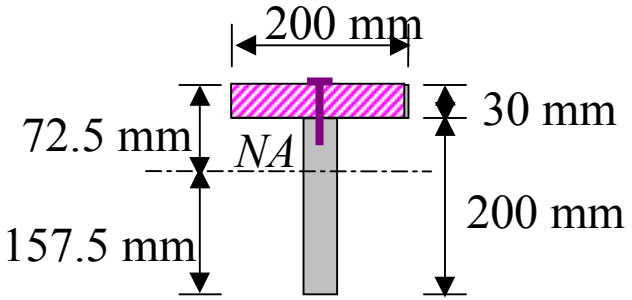
- Nail spacing @ 150 mm
  ϕ = 3 mm
  τ_{allow} = 200 MPa





$I_{NA} = 60.12 \times 10^{-6}$ mm^4





$I_{NA} = 60.125\text{x}10^{-6}\ \text{mm}^4$

$$Q_{neck} = A_{neck}\bar{y}$$

$$= (0.03\times 0.2)(72.5 - \frac{30}{2})$$

$$= 0.345\ m^3$$

- **Maximum shear force on Nail:** Nail spacing @ 150 mm, $\phi = 3$ mm, $\tau_{\text{allow}} = 200$ MPa

$$F_s = (\tau_{\text{allow}})_{\text{nail}}(A_{\text{nail}})$$

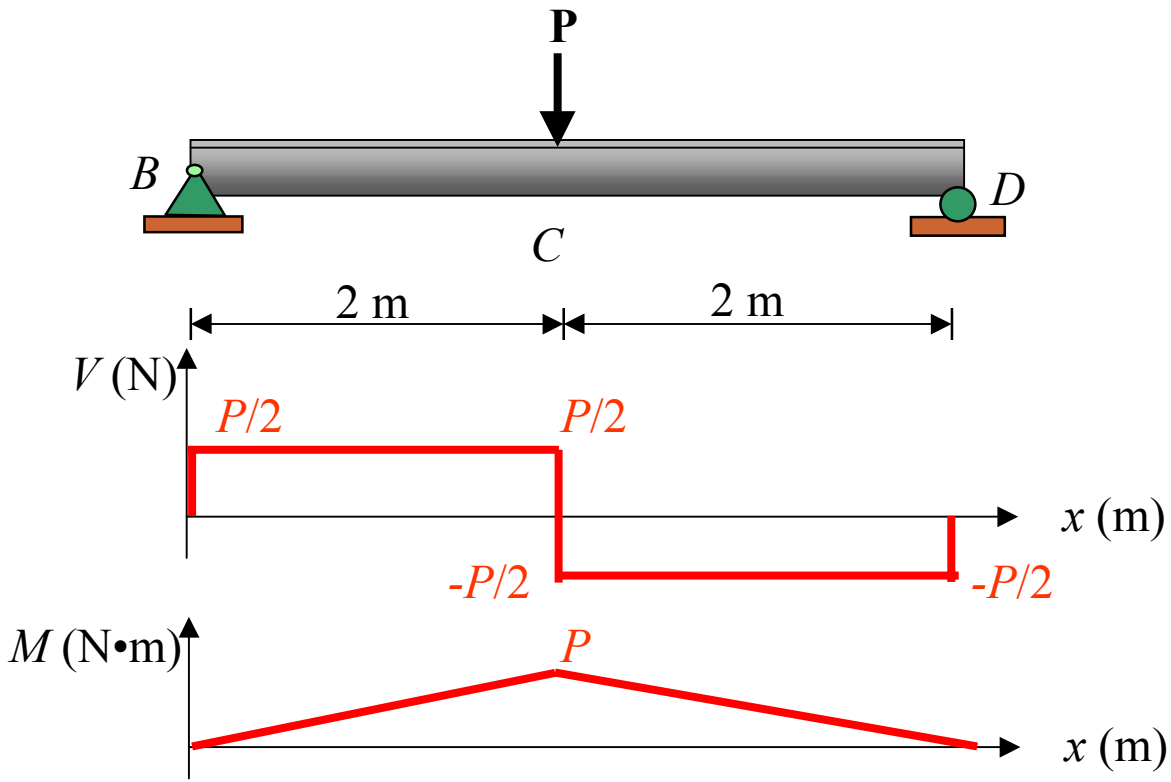
$$= (200\text{x}10^6)(\pi\text{x}0.0015^2)$$

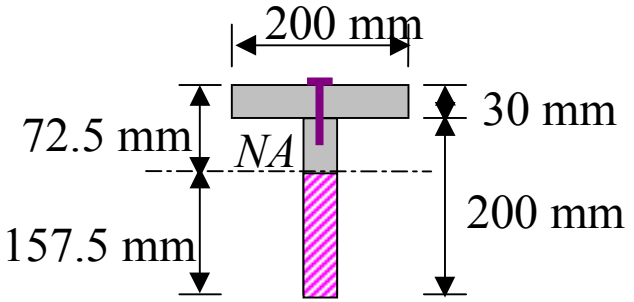
$$= 1414\ \text{N}$$

$$1414 = (\frac{VQ}{It})(t \bullet s)$$

$$1414 = \frac{(\frac{P}{2})(0.345)}{(60.125\times 10^{-6})})(0.150)$$

$$P = 3286\ \text{N}$$





$I_{NA} = 60.125\text{x}10^{-6}\ \text{mm}^4$

$$Q_{\max} = A_{\max}\bar{y}$$

$$= (0.03\times 0.1575)(\frac{0.1575}{2})$$

$$= 0.372\times 10^{-3}\ m^3$$

•**Bending:** $\sigma_{\text{allow}} = 12$ MPa

$$\sigma = \frac{Mc}{I}$$

$$12\times 10^{6} = \frac{P(0.1575)}{60.125\times 10^{-6}}$$

$$P = 4580\ \text{N}$$

•**Shear :** $\tau_{\text{allow}} = 0.8$ MPa

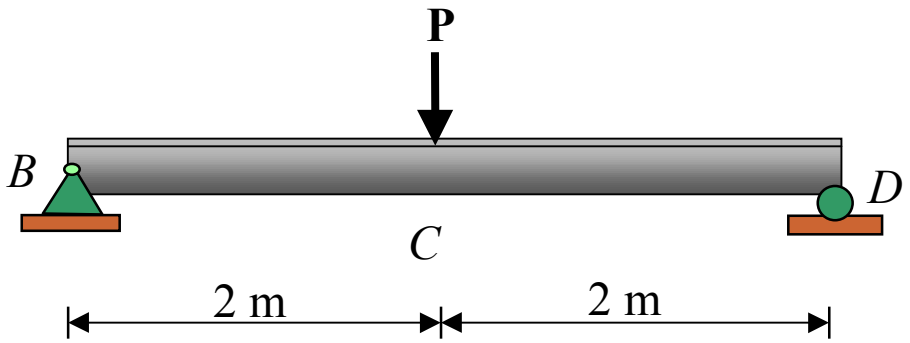
$$\tau = \frac{VQ}{It}$$

$$0.8\times 10^{6} = \frac{(\frac{P}{2})(0.372\times 10^{-3})}{(60.125\times 10^{-6})(0.03)}$$

$$P = 3879\ \text{N}$$

- σ_{allow} = 12 MPa
  τ_{allow} = 0.8 MPa

- Nail spacing @ 150 mm
  ϕ = 3 mm
  τ_{allow} = 200 MPa



- **Summary:**

Nail , shear 3.28 kN, use 3 kN ⇦

Beam , bending 4.58 kN

Beam , shear 3.88 kN

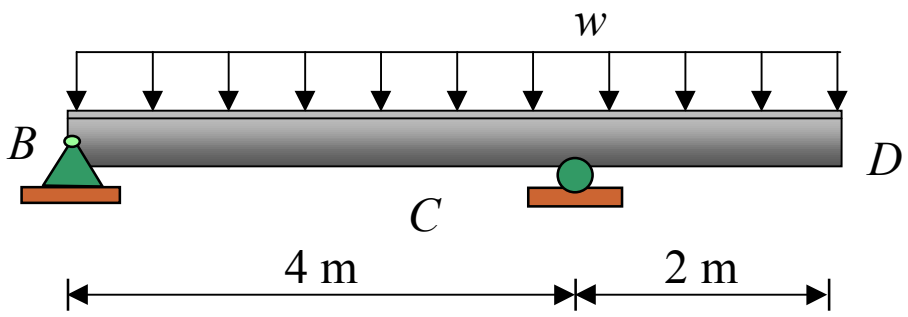
Example 5

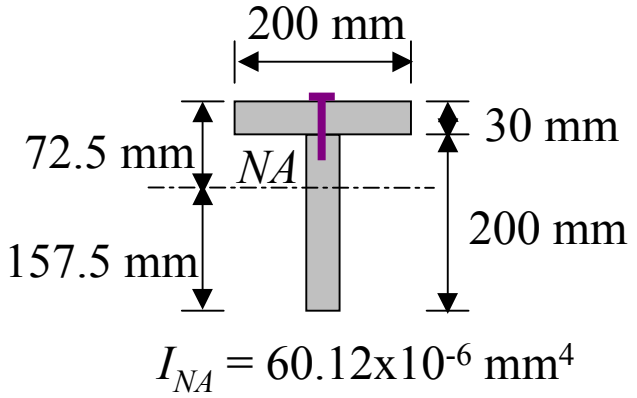
From the beam shown, determine the largest load *w* that can be applied to the beam.

Conditions specified: Wood

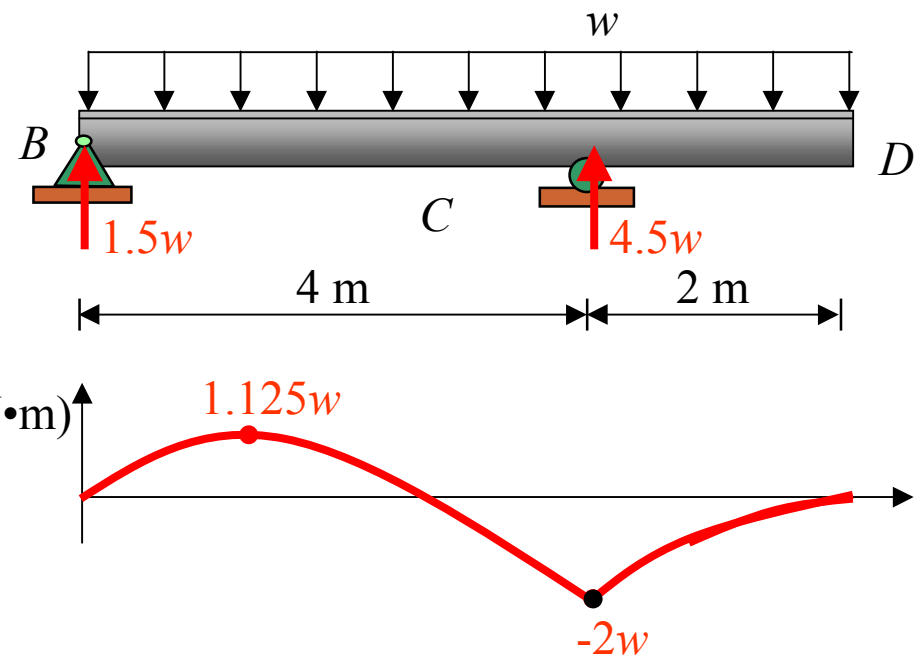
- $\sigma_{allow} = +12$ MPa in tension
  $\sigma_{allow} = 8$ MPa in compression
  $\tau_{allow} = 0.8$ MPa

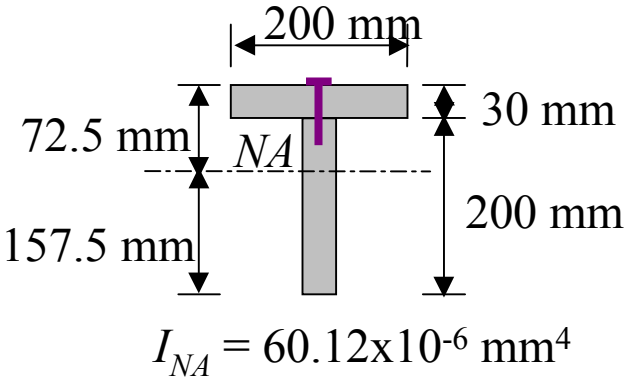
- For each nail, $(F_{nail})_{allow} = 1.5$ kN @ 100-mm spacing





$I_{NA} = 60.12 \times 10^{-6}$ mm^4





$I_{NA} = 60.12\text{x}10^{-6}\ \text{mm}^4$

- **Beam : Bending :** $\sigma_{allow} = +12$ MPa in tension, $\sigma_{allow} = 8$ MPa in compression

- Compression

$$8\times10^{6} = \frac{(2w)(0.1575)}{60.12\times10^{-6}}$$

$$w = 1.527\ \text{kN/m}\ \bullet$$

$$8\times10^{6} = \frac{(1.125w)(0.725)}{60.12\times10^{-6}}$$

$$w = 5.897\ \text{kN/m}$$

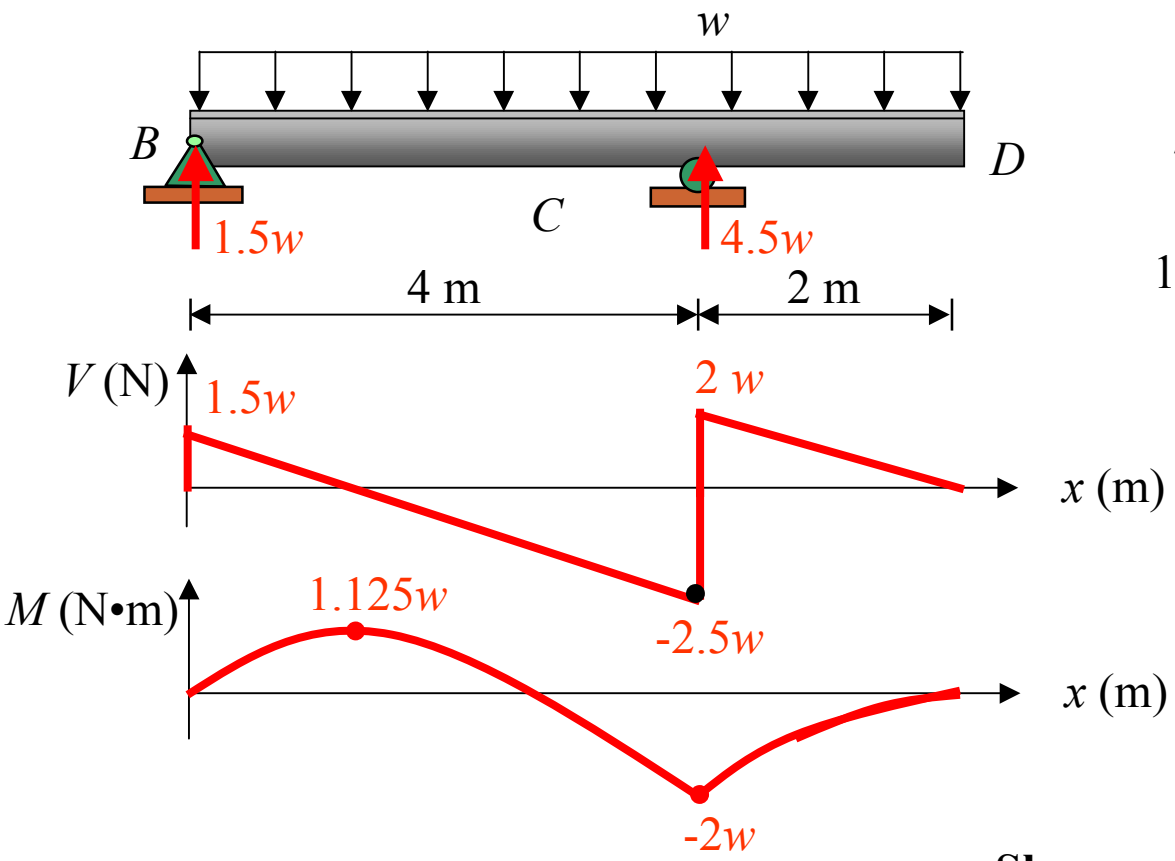
- Tension

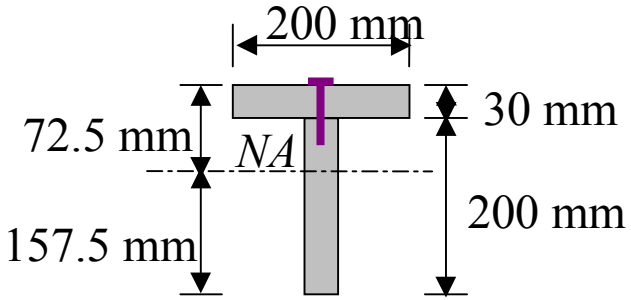
$$12\times10^{6} = \frac{(1.125\text{w})(0.1575)}{60.125\times10^{-6}}$$

$$w = 4.071\ \text{kN/m}\ \bullet$$

$$12\times10^{6} = \frac{(2w)(0.725)}{60.12\times10^{-6}}$$

$$w = 4.975\ \text{kN/m}$$





$I_{NA} = 60.12\text{x}10^{-6}$ mm^4

- **Beam: Shear @ *NA*,** $\tau_{allow} = 0.8$ MPa

$$\tau = \frac{VQ}{It}$$

$$0.8\times10^{6} = \frac{(2.5w)(0.1575\times0.03\times\frac{0.1575}{2})}{(60.125\times10^{-6})(0.03)}$$

$w = 1.55$ kN/m

- **Shear on nail :** $(F_{nail})_{allow} = 1.5$ kN
  spacing = 100 mm

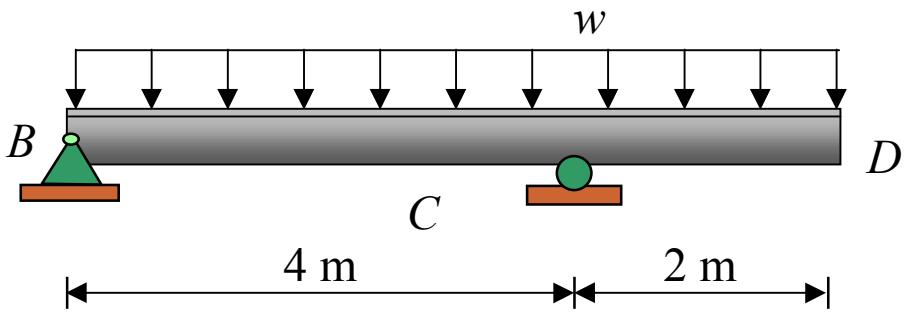
$$F_s = (\tau_{allow})_{nail}(A_s) \quad = (\frac{VQ}{It})(t \bullet s)$$

$$1.5 = \frac{(2.5w)(0.2\times0.03\times0.0575)}{(60.125\times10^{-6})(t)}(t)(0.1)$$

$w = 1.04$ kN/m

σ_{allow} = +12 MPa in tension
σ_{allow} = 8 MPa in compression
τ_{allow} = 0.8 MPa

$(F_{nail})_{allow}$ = 1.5 kN
Nail spacing = 100 mm



- **Summary:**

Beam bending, compression 1.53 kN/m
tension 4.07 kN/m

Beam shear, 1.55 kN/m

Nail shear 1.04 kN/m, use 1 kN/m ⇦