SISTEM MODELING

Materi 5
Mathematical Modeling

OUTLINE

- Introduction
- What is a mathematical model?
- Why do we build a mathematical model?
- How to build a mathematical model?
- An illustrative case (Case of LOD)
- Formal Approaches for finding the optimal solution

INTRODUCTION

- We use the OR/MS Methodology
- To capture the relationships between various elements of the relevant system in a mathematical model and explore its solution.

What is a mathematical model?

- A mathematical model: Express, **in quantitative term**, the relationships between various components, as defined in the relevant system for the problem (e.g. using Influence Diagram).
- Terminology:
  - **Decision variables** or the alternative courses of action (controllable inputs)
  - **Performance measure** (how well the objectives are achieved)

What is a mathematical model?

- Terminology:
  - **Objective function** (the performance measure is expressed as a function of decision variables)
  - **Uncontrollable inputs**: parameters, coefficients, or constants
  - **Constraints** –limit the range of the decision variables

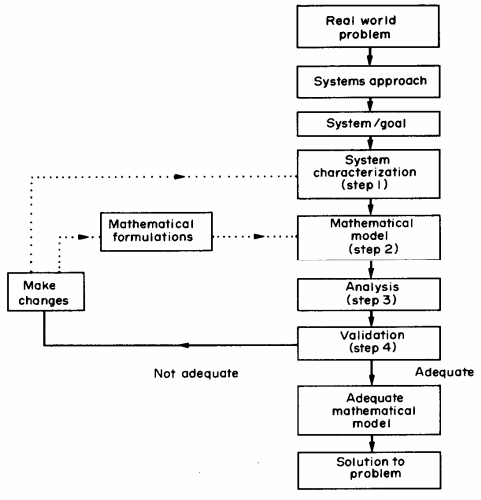
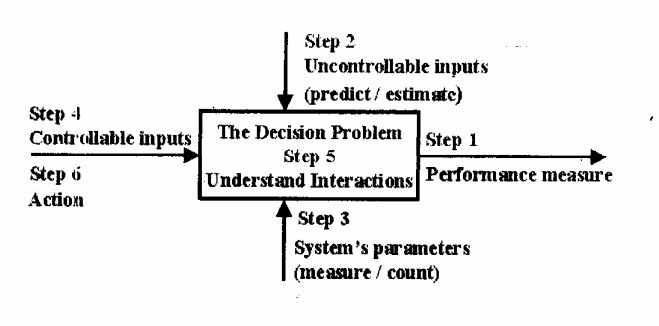


FIGURE 4.2 Mathematical modelling process (simplified).

Relationship Between Input-System-Output



Why build mathematical models?

- Real-life tests are not possible
  - Disruptive
  - Risky
  - Expensive
- Math Models are easy to manipulate
  - Quick exploration of the effect of changes in the inputs on the objective functions

Properties of Good mathematical models

- **Simple** –simple models are more easily understood by the problem owner
- **Complete** –should include all significant aspect of the problem situation affecting the measure of effectiveness
- **Easy to manipulate** –possible to obtain answer from the model
- **Adaptive** –changes in the structure of the problem situation

Properties of Good mathematical models

- **Easy to communicate** with –easy to prepare, update, and change the inputs and get answer quickly
- **Appropriate** for the situation studied – produces the relevant outputs at the lowest possible cost and in the time frame required
- Produce information that is **relevant and appropriate** for decision making –has to be useful for decision making

The Art of Modeling

- A scientific process
- More akin to art than science
- A few guidelines
- Ockham's Razor:
  - "Things should not be muliplied without good reason".
  - The modeler has to be selective in including aspects into a model

The Art of Modeling

- **An iterative process of enhancements** –begin with a very simple model and move in an evolutionary fashion toward more elaborate models
- **Working out a numerical example** – observe how variables of interest behave
- **Diagram and Graphs** –to see things in the form of graphs or other drawings expressing relationships and patterns.

Math. Model For The *LOD* Problem

- Simplification
  - Constraints (Warehouse space & mixing and filling capacities)
  - Two decision variables (cutoff point, L and order size,Q)
- First Approximation
  - Ignore the constraints
  - Involve only one decision variable, Q
- Performance measure
  - Total annual relevant cost (TAC) (per year)
  - TAC=Annual stock holding cost+Annual set up cost+Annual handling cost+Annual product values

Table 6-1: Frequency table of customer orders for product Y

Order size	Number of orders	Cumulative number
1	134	134
2	356	490
3	95	585
4	186	771
5	34	805
6	112	917
8	91	1008
9	15	1023
10	56	1079
12	85	1164
15	12	1176
16	48	1224
20	23	1247
24	19	1259
36	5	1264
48	2	1266

Math. Model For The *LOD* Problem

- Annual stock holding cost
  - (Average stock level x Unit product value) x Holding cost/$/year
- Annual set up cost
  - Setup cost per batch x Annual number of stock replenishments
- Annual handling cost
  - Product handling cost per unit x annual volume met from stock
- Annual product values
  - Unit product value x Annual volume of demand

Math. Model For The *LOD* Problem

$$T(Q) = [0.5Qvr] + [sD_1 / Q] + [h_1 D_1] + [vD_1]$$

$$T(Q, L) = [sN] + [h_2 D_2] + [0.5Qvr + sD_1 / Q] + [h_1 D_1]$$

$$T(Q|L)$$

Math. Model – *LOD*
[Second Approximation]

- Two decision variables, L and Q.
- Two additional costs
  - The annual set up cost for special production run
    - Annual volume by special prod.runs x Product handling cost per unit
  - The annual handling cost for big order
    - Production setup per batch x Annual number of special prod.runs
- Total cost =

  The annual set up cost for special production run +
  The annual handling cost for big order +
  Associated annual EOQ cost given L +
  The annual handling cost for small order.

Figure 6-4: Spreadsheet printout for T(L,Q)-model

LOD SECOND MODEL FOR STOCK CONTROL

Decision variables: Stock replenishment size
Special production run cutoff size

INPUT DATA

Total Demand	7132	drums/year
Product value	$320.00	/drum
Production setup cost	$18.00	/production run
Product handling cost	$1.10	/drum via storage
	$0.45	/drum by special production run
Investment holding cost	$0.18	/dollar invested per year

COMPUTATION OF T(L,Q) (base period: one year)

cutoff point	Demand from stock	Demand by special prod. runs	Number of special prod. runs	Setup cost special prod. runs	Total handling cost	Corresp. EOQ	Corresp. relevant EOQ cost	Total relevant cost
1	0	7132	1266	$22,788	$3,209	0.0	$0	$25,997
2	134	6998	1132	$20,376	$3,297	9.2	$527	$24,200
3	846	6286	776	$13,968	$3,759	23.0	$1,324	$19,052
4	1131	6001	681	$12,258	$3,945	26.6	$1,531	$17,734
5	1875	5257	495	$8,910	$4,428	34.2	$1,972	$15,310
6	2045	5087	461	$8,298	$4,539	35.8	$2,059	$14,896
8	2717	4415	349	$6,282	$4,975	41.2	$2,374	$13,631
9	3445	3687	258	$4,644	$5,449	46.4	$2,673	$12,765
10	3580	3552	243	$4,374	$5,536	47.3	$2,725	$12,635
12	4140	2992	187	$3,366	$5,900	50.9	$2,930	$12,196
15	5160	1972	102	$1,836	$6,563	56.8	$3,271	$11,670
16	5340	1792	90	$1,620	$6,680	57.8	$3,328	$11,628
20	6108	1024	42	$756	$7,180	61.8	$3,559	$11,494
24	6568	564	19	$342	$7,479	64.1	$3,690	$11,511
36	6856	276	7	$126	$7,666	65.5	$3,770	$11,562
48	7036	96	2	$36	$7,783	66.3	$3,820	$11,638
49	7132	0	0	$0	$7,845	66.8	$3,846	$11,691

CUSTOMER DEMAND DATA

Cust. order size	Number	Cumulative number
0	0	0
1	134	134
2	356	490
3	95	585
4	186	771
5	34	805
6	112	917
8	91	1008
9	15	1023
10	56	1079
12	85	1164
15	12	1176
16	48	1224
20	23	1247
24	12	1259
36	5	1264
48	2	1266
49	0	1266

Deriving A Solution To The Model

- Enumeration
- Search Methods
- Algorithmic Solution Methods
- Classical Methods of Calculus
- Heuristic Solution Methods
- Simulation

Deriving A Solution To The Model

- Enumeration
  - Number of alternatives of action is relatively small.
  - Computational effort is relatively minor
  - Optimal solution is obtained by evaluating the performance measure for each alternatives.
- Search Methods
  - e.g. Golden section search
- Algorithmic Solution Methods
  - A set of logical and mathematical operations performed repeatedly in a specific sequence
  - Iteration
  - Stopping rules.

Deriving A Solution To The Model

- Classical Methods of Calculus
- Heuristic Solution Methods
  - Impossible to find the optimal solution with the computational means currently available (intractable)
  - If the optimal solution is possible to obtain, but the potential benefit do not justify the computational effort needed.
  - Heuristic methods: to find a good solutions or to improve an existing solutions (out put based techniques)
- Simulation
  - For complex dynamic systems
  - To identify good policies rather than the optimal one.

Figure 6-7: A function and its derivative

