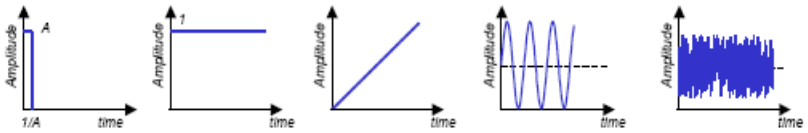
Karakteristik Dinamik

- The sensor response to a variable input is different from that exhibited when the input signals are constant (the latter is described by the static characteristics)
- The reason for dynamic characteristics is the presence of energy-storing elements
  - Inertial: masses, inductances
  - Capacitances: electrical, thermal
- Dynamic characteristics are determined by analyzing the response of the sensor to a family of variable input waveforms:
  - Impulse, step, ramp, sinusoidal, white noise…



Model Dinamika

- The dynamic response of the sensor is (typically) assumed to be linear
  - Therefore, it can be modeled by a constant-coefficient linear differential equation

$$a_k \frac{d^k y(t)}{dt^k} + \cdots a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t)$$

  - In practice, these models are confined to zero, first and second order. Higher order models are rarely applied
- These dynamic models are typically analyzed with the Laplace transform, which converts the differential equation into a polynomial expression
  - Think of the Laplace domain as an extension of the Fourier transform
    - Fourier analysis is restricted to sinusoidal signals
      - $x(t) = \sin(\omega t) = e^{j\omega t}$
    - Laplace analysis can also handle exponential behavior
      - $x(t) = e^{-\sigma t}\sin(\omega t) = e^{-(\sigma + j\omega)t}$

Review Transformasi LAPLACE

- **The Laplace transform of a time signal y(t) is denoted by**
  - L[y(t)] = Y(s)
    - The s variable is a complex number $s=\sigma+j\omega$
      - The real component σ defines the real exponential behavior
      - The imaginary component defines the frequency of oscillatory behavior
- **The fundamental relationship is the one that concerns the transformation of differentiation**

$$L\left[\frac{d}{dt}y(t)\right]=sY(s)-f(0)$$

- **Other useful relationships are**

Impulse :	$L[\delta(t)]=1$	Decay :	$L[\exp(at)]=(s-a)^{-1}$
Step :	$L[u(t)]=\frac{1}{s}$	Sine :	$L[\sin(\omega t)]=\frac{\omega}{s^2+\omega^2}$
Ramp :	$L[r(t)]=\frac{1}{s^2}$	Cosine :	$L[\cos(\omega t)]=\frac{s}{s^2+\omega^2}$

Review Transformasi LAPLACE

- **Applying the Laplace transform to the sensor model yields**

$$L\left[a_k\frac{d^k y}{dt^k}+\cdots a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_o y(t)=x(t)\right]$$

$$\Downarrow$$

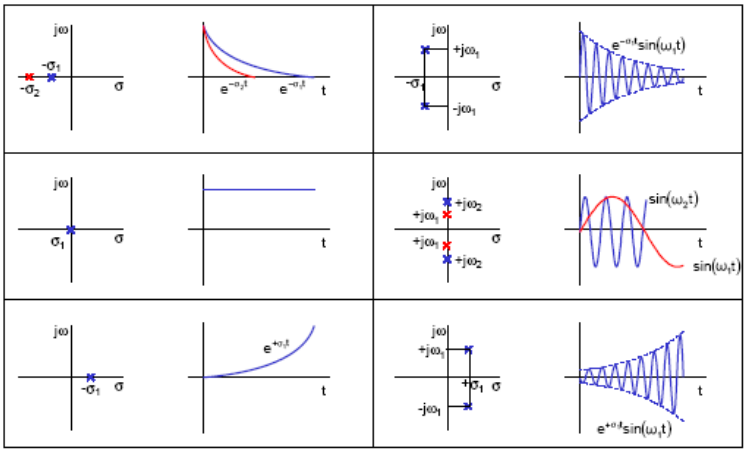
$$(a_k s^k+\cdots a_2 s^2+a_1 s+a_o)Y(s)=X(s)$$

$$\Downarrow$$

$$G(s)=\frac{Y(s)}{X(s)}=\frac{1}{a_k s^k+\cdots a_2 s^2+a_1 s+a_o}$$

  - G(s) is called the transfer function of the sensor
- **The position of the poles of G(s) -zeros of the denominator- in the s-plane determines the dynamic behavior of the sensor such as**
  - Oscillating components
  - Exponential decays
  - Instability

Lokasi *pole* dan perilaku dinamik

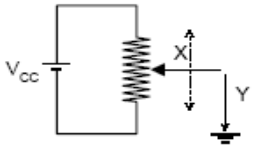


Sensor orde NOL

- **Input and output are related by an equation of the type**

$$y(t) = k \cdot x(t) \Rightarrow \frac{Y(s)}{X(s)} = k$$

- Zero-order is the desirable response of a sensor
  - No delays
  - Infinite bandwidth
  - The sensor only changes the amplitude of the input signal
- Zero-order systems do not include energy-storing elements
- Example of a zero-order sensor
  - A potentiometer used to measure linear and rotary displacements
    - This model would not work for fast-varying displacements



Sensor orde SATU

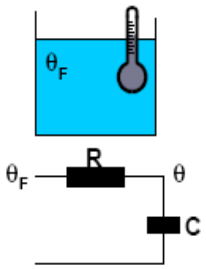
- **Inputs and outputs related by a first-order differential equation**

$$a_1\frac{dy}{dt}+a_0y(t)=x(t)\Rightarrow\frac{Y(s)}{X(s)}=\frac{1}{a_1s+a_0}=\frac{k}{\tau s+1}$$

  - First-order sensors have one element that stores energy and one that dissipates it
  - Step response
    - $y(t) = Ak(1-e^{-t/\tau})$
      - A is the amplitude of the step
      - k ($=1/a_0$) is the static gain, which determines the static response
      - τ ($=a_1/a_0$) is the time constant, which determines the dynamic response
  - Ramp response
    - $y(t) = Akt - Ak\tau u(t) + Ak\tau e^{-t/\tau}$
  - Frequency response
    - Better described by the amplitude and phase shift plots

Contoh sensor orde SATU

- **A mercury thermometer immersed into a fluid**
  - What type of input was applied to the sensor?
  - Parameters
    - C: thermal capacitance of the mercury
    - R: thermal resistance of the glass to heat transfer
    - θ_F: temperature of the fluid
    - $\theta(t)$: temperature of the thermometer
  - The equivalent circuit is an RC network



- **Derivation**
  - Heat flow through the glass $(\theta_F-\theta(t))/R$
  - Temperature of the thermometer rises as $\frac{d\theta(t)}{dt}=\frac{\theta_F-\theta(t)}{RC}$
  - Taking the Laplace transform

$$s\,\theta(s)=\frac{\theta_F(s)-\theta(s)}{RC}\Rightarrow(RCs+1)\theta(s)=\theta_F(s)\Rightarrow$$
$$\Rightarrow\theta(s)=\frac{\theta_F(s)}{(RCs+1)}\Rightarrow\theta(t)=\theta_F\left(1-e^{-t/RC}\right)$$

Sensor orde DUA

- **Inputs and outputs are related by a second-order differential equation**

$$a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y(t)=x(t)\Rightarrow\frac{Y(s)}{X(s)}=\frac{1}{a_2s^2+a_1s+a_0}$$

  - We can express this second-order transfer function as

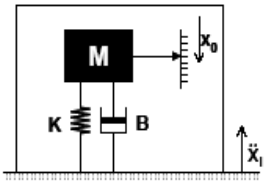
$$\frac{Y(s)}{X(s)}=\frac{k\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\text{with } k=\frac{1}{a_0},\ \zeta=\frac{a_1}{2\sqrt{a_0a_1}},\ \omega_n=\sqrt{\frac{a_0}{a_2}}$$

  - Where
    - k is the static gain
    - ζ is known as the damping coefficient
    - ω_n is known as the natural frequency

Contoh sensor orde DUA

- **A thermometer covered for protection**
  - Adding the heat capacity and thermal resistance of the protection yields a second-order system with two real poles (overdamped)
- **Spring-mass-dampen accelerometer**
  - The armature suffers an acceleration
    - We will assume that this acceleration is orthogonal to the direction of gravity
  - x_0 is the displacement of the mass M with respect to the armature
  - The equilibrium equation is:



$$M(\ddot{x}_i-\ddot{x}_0)=Kx_0+B\dot{x}_0$$

$$\Downarrow$$

$$Ms^2X_i(s)=X_0(s)\left[K+Bs+Ms^2\right]$$

$$\Downarrow$$

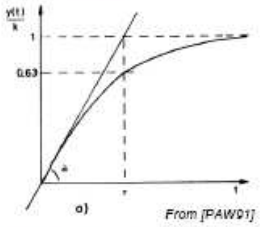
$$\frac{X_0(s)}{s^2X_i(s)}=\frac{M}{K}\frac{K/M}{s^2+s(B/M)+K/M}$$

IDENTIFIKASI KARAKTERISTIK SENSOR

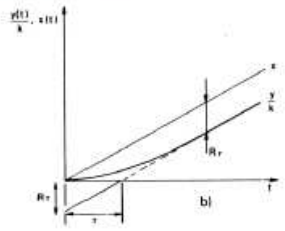


Respon sensor orde SATU

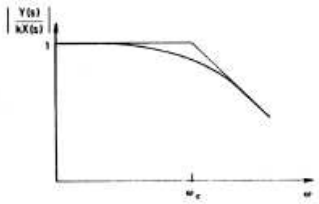
- **Step response**

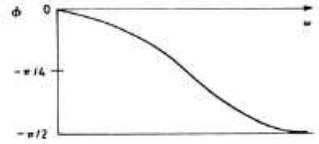


- **Ramp response**



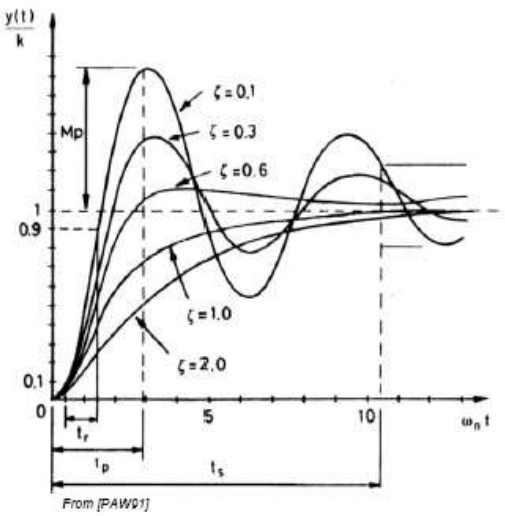
- **Frequency response**
  - Corner frequency $\omega_c = 1/\tau$
  - Bandwidth





Respon STEP orde DUA

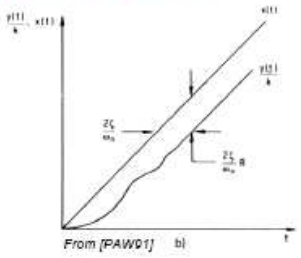
- **Response types**
  - Underdamped (ζ<1)
  - Critically damped (ζ=1)
  - Overdamped (ζ>1)
- **Response parameters**
  - Rise time (t_r)
  - Peak overshoot (M_p)
  - Time to peak (t_p)
  - Settling time (t_s)



From [PAW91]

Respon orde DUA

- **Ramp response**



- **Frequency response**

